IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 19, Issue 1 Ser. I (Jan. – Feb. 2023), PP 52-60
www.iosrjournals.org

On The Greek Rho of Asian Option and Best of Asset Option, A Malliavin Calculus Approach

Akeju Adeyemi. O[1], Ayoola. E. O[2]
Department of Mathematics, University of Ibadan, Nigeria.
Corresponding Author: Akeju Adeyemi.O

Abstract:
In this paper, we consider the greek rho of Asian option and Best of asset option. These type of options are options with more than one underlying assets. The greeks which are represented by the sensitivities are obtained with Malliavin calculus. We use the Malliavin calculus to derive weight function of the Greeks for both Asian and Best of Asset Options. The weight function were used to derive expressions for the Greeks which represent the sensitivities of the two options with respect to the interest rate of the underlying process.
Key Word**: **Asian Option, Best of Asset options, Brownian motion, Greeks, Malliavin Calculus

---

Date of Submission: 02-02-2023 Date of Acceptance: 14-02-2023

I. Introduction

We examine in this work, the Greek rho of an Asian option (AO) and Best of Asset option (BOA). Greeks are generally the price sensitivity of a derivative to a change in an underlying parameter. Rho measures an options sensitivity to changes in the risk free rate of interest. It represent the amount of money an option will gain or lose with a 1% change in interest rate. Interest rate impact on the option value can impact the cost of carrying the position over time. The fact that this impact is on the cost, interest rate changes impact longer term options than the options with short term. As the price of the stock increases over time towards the expiration, the higher the sensitivity to changes in interest rate and the higher the absolute Rho values. Rho is positive for a long call (right to buy) and it increase with the price of the stock. Rho is negative for long put (right to sell) and it approaches zero as the stock prices increases. Rho is positive for short put (obligation to buy) and negative for short call (obligation to sell). Asian option and Best of Asset option are option types whose payoff are defined with respect to multiple underlying assets. To determine the Rho sensitivity, we use the principles of Malliavin calculus, a calculus which involves the integration by part technique of the stochastic of variation as discussed in [7, 11]. We use this calculus to derive the expectation of the payoff function of both Asian and Best of Asset Options. The study of Malliavin calculus and the applications in finance involve the use of integration by part formula to give a mathematical approach to the computation of the price sensitivities [3, 4, 8].The Malliavin calculus is applicable when dealing with random variables with unknown density functions and when there are options with non-smooth payoffs [11].

Options are derivative contracts which gives its holder the right to buy or to sell a given number of derivatives (which can be a financial stock, a currency etc.) at a given and agreed price and at a particular time $\tau < T$ which are fixed in the contract.

Let S_τ represent the market price of the underlying asset at any time τ, C_τ represent the Call option value at time τ and P_τ represent the Put option value at any time τ, where τ satisfies the condition $0 \le \tau \le T$, then the values of the Call and Put option can be defined respectively at the time of exercise as

$C_T = max\ ((S_T - \mathbf{K}), 0)$
and
$P_T = max\ ((\mathbf{K} - S_T), 0)$

Due to the variations associated with the underlying assets, investors have opportunity to several investment plans and strategies. One important feature of this type of option contract is the possibility to customize it to meet up with the investor risk tolerance. This will enable the investor to achieve a set desired profit.

Asian option considered the average of the assets underlying the contract over a certain period of time to determine if there is profit when compared with the strike price.

Best of Asset option is the type that considered the maximum of the underlying assets prices in comparison with the strike price to determine the profitability of the contract.

The dynamics of pricing and hedging of options is such that at maturity time, a flow of the payoff h (S_T) can be guaranteed by the option owner. Then the option owner can purchase with the premium, a portfolio that has

equal flow of price with one of the options. This process is known as the portfolio hedging or dynamic strategy of buying and selling of options [5, 11].
We shall denote, at any time τ the value of the hedged portfolio simply as $\curlyvee\tau$, $0 \leq \tau \leq T$ and the possibility of not having arbitrage is such that
$P (\curlyvee T > 0) > 0$ $\curlyvee 0 = 0$
This means that, the possibility that the portfolio will always be replicated is positive at every time τ.

II. Methods

In this section, we provide some definitions and important concepts with respect to the theory of Malliavin calculus and its properties.
Definition 2.1 (Stochastic Process): A random variable X is said to be a stochastic process if $X = \{X(t), t \in [0, T]\}$ is a collection of random variables on a common probability space indexed by parameter $t \in T \subset \mathbb{R}+$. Stochastic process can be formulated as a function that is, $X: T \times \Omega \longrightarrow \mathbb{R}$, such that $X(t,.)$ is A- measurable for each $t \in T$ where Ω is a non-empty set, A is σ-algebra generated by Ω. $X(t)$ can be written also as X_t.
Definition 2.2 (Filtered Probability Space):Let Ω be a non-empty set, let A, a σ-algebra, be the collection of subsets of Ω, let P be a probability measure, if there exists $(A_t, t \in [0,T])$, a family of sub σ-algebra of A, then (Ω, A, P, A_t) is referred to as a filtered probability space.

Remark:
1. A sequence $(f_n, n \in \mathbb{N})$ of σ-algebra is called filtration if $f_n \subset f_{n-1} \subset A$ for every $n \in \mathbb{N}$ where $A \subset \Omega$
2. $(F_t, t \in [0,T])$ is called filtration of the probability space (Ω, F, P) if and only if
(i) F_0 contains all subsets of any P- null set.
(ii) F_s is a sub σ-algebra of F_t, $t \geq s$

Filtration can always be used with the property $P(\Omega)$ which represents the power set of Ω such that;
(1) $F_0 = (\emptyset, \Omega)$: At the beginning, there is no information.
(2) $F_T = P(\Omega)$: At the end, there is full information.
(3) $F_0 \subset F_1 \subset ... \subset F_T$: The information available increases over time.

Filtration are used to model the flow of information over time. At time t, we can decide if the event $A \in F_t$ has occurred or not.
Definition 2.3 (Adapted Processes): A sequence $(X_t, t \geq 0)$ of random variables is said to be adapted to a filtration F_t if for each t, the random variable (X_t is F_t- measurable, that is, for any t, F_t contains all the information about X_t.
Definition 2.4 (Black Scholes Financial Market): A market, in the Black-Scholes sense is made up of an asset that is risk free A and an asset that is risky S. The price of the risk free asset A is expected to satisfy the differential equation

$$dA(\tau) = rA(\tau)\, d\tau \quad A(0) = 1 \tag{2.1}$$

which is an ordinary differential equation, provided the interest rate r is constant. The solution of equation (2.1) is $A(\tau) = A_\tau = e^{r\tau}$, which satisfies the price process of the risk free asset. If the interest rate r is a non-negative adapted process, then r will satisfy the condition that

$$\int_0^T r_\tau d\tau < \infty$$

The price of the asset that is risky S is expected to have the dynamics

$$dS(\tau) = \kappa S(\tau)d\tau + \sigma S(\tau)dB(\tau) \qquad S(0) = S_0,\ S(\tau) = S_\tau, \qquad \tau \in [0,T] \tag{2.2}$$

a stochastic differential equation (SDE).
The solution $S(\tau) = S_0 \exp((\kappa - \frac{\sigma^2}{2})\tau + \sigma B(\tau))$ of the stochastic differential equation (2.2) shown in [6], satisfy the price process of the risky asset S where S_0 represents the initial price of the asset S, κ is the drift term which is taken to be constant, σ represents the volatility of the process which is also known as the noise term, this volatility is also assumed to be constant, $B = \{B(\tau), \tau \in [0,T]\}$ represents a Brownian motion defined on a filtered probability space (Ω, A, P, A_τ), and $\{A_\tau, \tau \in [0,T]\}$ is a filtration, that is the flow of available information determined by the Brownian motion.
Definition 2.5: Suppose that an investor holds a Call option with strike price K. If $\tau = 0$ is the time when the Call option was acquired and $S(\tau)$ is the price of the underlying asset at time τ, then, if at maturity time T,
$S(T) > K$, then, the option is in the money.
$S(T) = K$, then, the option is at the money.

$S(T) < K$, then, the option is out of the money.

Malliavin Calculus for Gaussian Processes

Malliavin, [7] studied the solution of stochastic differential equation generated by Brownian noise by considering the regularity of the law of functional of the Brownian motion. The calculus can be adapted to both finite dimensional space, like R^n and infinite dimensional space like the Wiener space. Malliavin Calculus helps us to obtain the derivative of the functions of Brownian motion and this derivative is referred to as Malliavin derivative.

Definition 2.6: $B: = (B_\tau)_{\tau\in[0,T]}$ is a standard Brownian motion with respect to a right continuous filtration $(A_\tau)_{\tau\in[0,T]}$ if

(i) B is adapted with respect to $(A_\tau)_{\tau\in[0,T]}$

(ii) $B_0 = 0$

(iii) B possess a stationary Independent increments

(iv) B is a Gaussian process that has Variance τ $\forall\ 0 \le \tau_0 \le \tau_1 \le \cdots \le \tau_n \le T$, the random vector $(B_{\tau 1} - B_{\tau 0},\ldots,B_{\tau n} - B_{\tau n-1})$ is Centered Gaussian with Covariance matrix $Diag(\tau_1 - \tau_0,\ldots,\tau_n - \tau_{n-1})$.The Brownian motion can be described in the setting of isonormal Gaussian process.

Let R: $= L^2([0, T], d\tau)$ be the space of deterministic functions $h: [0, \tau] \to$ R such that $\int_0^\tau h(s)^2 ds < \infty$. then define $Z(r) := \int_0^T r(s) dB_s\ ,\ r \in \mathcal{R}$ where the stochastic integral is defined in the sense of Ito calculus. By linearity of the Ito stochastic integral, we have that

- Z is a linear map
- $\mathbb{E}[\int_0^T h(s) dB_s] = 0\ \forall\ h \in \mathcal{R}$
- Z is Centered Gaussian random variable with variance $\int_0^T h(s)^2 ds\ \forall\ h \in \mathcal{R}$
- $\mathbb{E}[\int_0^T g(s) dB_s \int_0^T h(s) dB_s] = \int_0^T g(s)h(s) ds = \langle g, h\rangle_{\mathcal{R}}\ \forall\ (g, h) \in \mathcal{R}^2$.

Suppose the Hilbert space H be represented as $L^2(B, \mathrm{B}, \mu)$ such that (B, B) represent a measurable space and a σ - finite measure μ, i.e. the Gaussian process Z is characterized by the family of random variables $\{Z(A), A \in \mathrm{B}, \mu(A) < \infty\}$ where $Z(A) = Z(1_A)$. We assume Z(A) to be an $L^2(\Omega, A, P)$-valued measure on the measurable space (B, B), which takes independent values on any family of disjoint subsets of B such that any random variable W(A) has the distribution $N(0,\mu(A))$, where $\mu(A) < \infty$.

This measure is also known as the white noise. To this end, for any function $h \in L^2(B)$, we shall dene the stochastic integral $W(h)$ as $W(h) = \int_B hdW$

It is possible to expressed as multiple stochastic integral the nth Wiener chaos H_n with respect to W. Next, the multiple stochastic integral $I_n(f)$ is dene in what follows;

For a function $f \in L^2(B^k, \mathrm{B}^k, \mu^k)$, $k \ge 1$, a stochastic integral is defined where B^k is the k-times product of space B and μ^k is the corresponding product measure. Let E_k represent the set of simple functions defined as

$$f(\tau_1 \ldots \tau_k) = \sum_{i_1 \ldots i_k}^{n} a_{i_1 \ldots i_k} 1_{A_{i_1} \times \ldots \times A_{i_k}}(t_1, \ldots, t_k)$$

such that whenever we have any two equal indices, the coefficient $a_{i1\ldots ik}$ vanish and the set $A_1 \ldots A_k$ are pairwise disjoint in β_0. So,

$$I_k(f) = \sum_{i_1 \ldots i_k = 1}^{n} a_{i_1 \ldots i_k} W(A_{i_1}) \ldots W(A_{i_k})$$

defined the multiple-stochastic integral.

Remarks:

The multiple stochastic integral $I_k(f)$ has the following properties

(1) $I_k(f)$ is linear.

(2) Let

$$\widehat{f}(\tau_1 \ldots \tau_k) = \frac{1}{k!} \sum_{\sigma} f(\tau_{\sigma(1)} \ldots \tau_{\sigma(k)})$$

be the symmetrization of f and σ run over all permutation of $\{1,\ldots, k\}$ then $I_k(f) = I_k(\widehat{f})$

Skorohod Integral

Consider a Hilbert space H defined as H = $L^2(D, A, \kappa)$, an L^2-space where κ is dene on a measurable space (D, A). Here, the square integrable processes are members of $Dom\delta \subset L^2(T \times \Omega)$, and the Skorohod stochastic integral is represented as $\delta(v)$ of the process $v = v(\tau, \varpi)$ $\tau \in T$, $\varpi \in \Omega$.

Definition 2.7: Suppose the stochastic process $u(\tau)$ is measurable such that $\tau \in [0, T]$. If

$$\mathbb{E}\left[\int_0^T v^2(\tau)d\tau\right] < \infty$$

then $v(\tau)$ is A_τ - measurable.
Suppose for $f_n(\cdot,\tau) \in \hat{L}([0,T]^n)$, we defined Wiener Ito expansion as

$$v(\tau) = \sum_{n=0}^{\infty} J_n(f_n(\cdot,\tau))$$

then,

$$\delta(v) := \int_0^T v(\tau)dB(t) := \sum_{n=0}^{\infty} I_{n+1}(\hat{f}_n)$$

defined the Skorohod integral of u where the symmetrization of $f_n(.,t)$ is represented as $\hat{f}_n$
More so,

$$||\delta(v)||^2_{L^2(P)} = \sum_{n=0}^{\infty}(n+1)!||\widehat{f}_n||_{L^2([0,T])^{n+1})} < \infty$$

We can write $f_{n,\tau}(\tau_1...\tau_n) = f_n(\tau_1,...,\tau_n,\tau)$ since $f_n(\cdot,\tau) = f_{n,\tau}(.)$ is a function of the parameter τ.
Since the function f_n is symmetric with respect to its first n variables then f_n and the symmetrization $\widehat{f}_n$ are function of n+1 variables $\tau_1,...,\tau_n,\tau$ where the symmetrization with $\tau_{n+1} = \tau$ is given by,

$$\widehat{f}_n(t_1,...,t_{n+1}) = \frac{1}{n+1}\left[f_n(t_1...t_{n+1}) + ... + f_n(t_1...t_{i-1},t_i,t_{i+1}...t_{n+1}) + ... + f_n(t_2...t_{n+1},t_1)\right]$$

where the sum is taken over those permutations σ of the indices (1..... , n+1) which inter- change the last component with one of the others and leave the rest in place.

The Skorohod integral satisfies the following properties

- It is a linear operator.
- Its expectation is zero i.e. $E[\delta(v)] = 0$
- If v, $Xv \in Dom(\delta)$ then,

$$\int_0^T Xv(\tau)\delta B(\tau) \neq X\int_0^{\infty} v(\tau)\delta B(\tau)$$

provided the random variable X is an A_τ-measurable.

Theorem 2.8 [4]:
The Ito-integral can be extended to the Skorohod integral i.e.
Let $E\left[\int_0^T v^2(t)d\tau\right] < \infty$ where the stochastic process $v(\tau)$, $\tau \in [0, T]$ is a $\mathcal{A}_t$- adapted measurable process then

$$\int_0^T v(\tau)\delta B(\tau) = \int_0^T v(\tau)dB(\tau)$$

i.e. v is Skorohod integrable and it is also Ito integrable.

Proposition (2.9) [8]:
If in $L^2(\Omega)$, the series

$$\delta(v) = \sum_{n=0}^{\infty} I_{n+1}\widehat{f}_n$$

converges and v can be expanded as

$$v(\tau) = \sum_{n=0}^{\infty} J_n(f_n(\cdot,\tau))$$

where $v \in L^2(T \times \Omega)$, then v is in $Dom\delta$.
Theorem 2.10: [2]
Suppose $v(\tau,\varpi)$ is a A_τ-adapted stochastic process and
$E\left[\int_0^T v^2(\tau,\varpi)d\tau\right] < \infty$ where $\tau \in [0, T]$ then

$$\int_0^T v(\tau,\varpi)\delta B(\tau) = \int_0^T v(\tau,\varpi)dB(\tau)$$

and $v \in Dom(\delta)$

Let A represent a σ-field generated by B and let (A, A, P) represent a complete probability space on which a Hilbert space R is defined, then we can represent by $Z = \{Z(r), r \in R\}$ an Isonormal Gaussian process.

The space of infinitely continuously differentiable functions f: $\mathbb{R}^n \to \mathbb{R}$ is represented as $C_b^{\infty}(\mathbb{R}^n)$ (respectively $C_p^{\infty}(\mathbb{R}^n)$) such that its partial derivatives are bounded (respectively have polynomial growth). We represent also $C_0^{\infty}(\mathbb{R}^n)$ as the space of all infinitely continuously differentiable functions with compact support.

Definition 2.11: Let Y: $\Omega \to \mathbb{R}$ and let denote by S the set of smooth random variables, if there is a function y in $C_p^{\infty}(\mathbb{R}^n)$, then

$$Y = y\,(Z\,(r_1)...Z\,(r_n)) \tag{2.3}$$

for $n \geq 1$ and elements $r_1,...,r_n \in \mathbb{R}$

Integration by Part Formula

We use the Malliavin derivative and the relation between it and Skorohod integral to obtain an integration by part formula which play an important role in the calculation of the Greeks. The integration by part formula is very essential in the study of smoothness of random variables and the absolutely continuity of the Malliavin calculus. This is fundamental in application to finance.

Proposition 2.12: [8]

Let $r \in \mathbb{R}$ and let Y be a smooth random variable of the form (2.3), then

$E\,[\langle DY, r\rangle_{\mathbb{R}}] = E[Y\,Z(r)]$

the integration by parts formula holds.

Proposition 2.13: [10] Suppose that $(DY_n)_n$ converges to η, a stochastic process in $L^P(\Omega\ ,\mathbb{R})$ such that the sequence $\{Y_n\}_{n\in\mathbb{N}}$ of smooth random variables, $n \to\infty$ converges to zero in $L^p(\Omega)$. Then, $\eta = 0$ and D, the Malliavin derivative operator is closable from $L^P(\Omega)$ to $L^P(\Omega, \mathbb{R})$

Proposition 2.14 [8]:Suppose ϱ: $\mathbb{R}^m \to \mathbb{R}$ is a function, where $x, y \in \mathbb{R}^m$ and $k > 0$ then ϱ is a Lipchitz function provided $|\varrho(x) - \varrho(y)| \leq k\|x - y\|$. Given a random vector $Y = (Y^{'},...,Y^{m})$ such that $Y^{i} \in D^{1,P}$, $P \geq 1$, if there exist random variables X^i and $\varrho(Y)$ which belongs to $D^{1,P}$ then

$$D(\varphi(Y)) = \sum_{i=1}^{m} X^i DY^i$$

In addition, if Y is an absolutely continuous random variable on $\mathbb{R}^m$then$\mathcal{G}^i = \frac{d\varphi}{dx_i}(Y)$. Note that since ϱ is Lipchitz, $\frac{d\varrho}{dx_i}(x)$ exist for almost all x in $\mathbb{R}^m$.

Proposition 2.15 [8, 9, 10]: Given the function y $\in C^1$ with bounded derivative and two random variables Y and X where Y$\in \mathbb{D}^{1,2}$. Suppose $Xv(< DY, v >_R)^{-1} \in \mathrm{Dom}\delta$ and $< DY, v >_R \neq 0$ where v is an $\mathbb{R}$-valued random variable, then,

$$E\,[y^{'}(Y)\,X] = E[\,f\,(Y)\,H(Y, X)] \tag{2.4}$$

and

$$H(Y, X) = \delta(X\,v(< DY, v >_R)^{-1}) \tag{2.5}$$

Remarks: In application to finance,

1) If v = DY, then $E\,[y^{'}(Y)\,X] = E[y(Y)\,\delta(\frac{XDY}{||DY||_R^2})]$ 2.6

2) Suppose $X(< DY, v >_R)^{-1}) \in \mathbb{D}^{1,2}$.such that $Xv(< DY, v >_R)^{-1}) \in \mathbb{D}^{1,2} \subset Dom\delta$, then v is a deterministic process

III. Result

We consider here, an Asian option which is an example of a Rainbow Option. Rainbow options are options or derivatives exposed to two or more sources of uncertainty. Apart from it been a path dependent option [3], that is, options whose value depend both on the price of the underlying assets, and the path that the asset took during some part or all the life of the option, it is also an option contract linked to the performance of two or more underlying assets. They can speculate on the best performer in the group or minimum performance of all the underlying assets at any time. Each underlying may be called a color so the sum of all these factors makes up a rainbow. Rainbow options sometimes has many moving paths and all the underlying assets in a rainbow option have to move in the right direction so that the investment will pay o eventually. The measure of the sensitivity analysis refers to the greeks, and the greeks are quantities that describe the sensitivities of financial derivative with respect to the different parameters of the model. They are vital tools in risk management and hedging.

Definition 3.1: Suppose $V(t)$ represent the pay o process of some derivatives where $t \in [0, T]$, then

$$\rho = rho = \frac{\partial V}{\partial r}$$

. This measures the changes in V in terms of the prevailing rate of interest r.
The computation of the greeks are sometime difficult to express in closed form depending on the pay-off function, and so, they require numerical methods for their computation.

Malliavin calculus is suitable in calculating greeks especially when the pay-off function is strongly discontinuous [4].
Greeks are the measure of changes of financial derivative with respect to its parameters. They are important when considering stability of the quantity under variation, which is the chosen parameter. If the price of an option is calculated using the measure Q as
$V = \mathrm{E}\,[exp\,(-r\,(T-\tau))\,\varphi(\mathrm{s}(\tau))]$
where the pay-off function is represented as $\varphi(x)$, then under the same measure as the price, the greek will be calculated, so that the
$Greek = \mathrm{E}\,[e^{-r\tau}\varphi\,((s\,(t)))*\psi(x)]$
where $\psi(x)$ represent the weight function called Malliavin weight.
We consider the stochastic process $\mathrm{S}(t)$ defined on $(\Omega, \mathrm{A}, P, \mathrm{A}_\tau)$, the filtered probability space where $\tau \in [0, T]$
So, if $\mathrm{S}(\tau)$ satisfies equation
$\mathrm{S}(\tau) = \mathrm{S}_0 \exp((\kappa - \frac{\sigma^2}{2})\tau + \sigma B(\tau)),$
Then

$$\frac{\partial S_T}{\partial \kappa} = S_0 T \exp\left(\left(\kappa - \frac{\sigma^2}{2}\right)T + \sigma\, B(T)\right) = T S_T$$

Greeks generally measure the sensitivity of the financial quantity in terms of the changes in the parameter, and these can be calculated using Malliavin calculus integration by part technique defined in equation (2.4)

$$\mathrm{E}\,[y'(Y)\,X] = \mathrm{E}[y(Y)\,\delta\,(Xv\,(D^v Y)^{-1})]$$

Theorem 3.2 (Greek Rho): [1]
Suppose the value of the Rainbow option is represented by V: $[0, T] \times \mathbb{R} \longrightarrow \mathbb{R}$, where the dynamics of the option underlying asset $\mathrm{S}\,(\tau)$ is given by
$dS\,(\tau) = \kappa s\,(\tau)\,d\tau + \sigma s\,(\tau)\,dB\,(\tau) \quad \tau \in [0, T]$
where κ and σ are constant, $B(\tau)$ is defined on the filtered probability space $(\Omega, \mathrm{A}, P, \mathrm{A}_\tau)$, with filtration A_τ, then greek rho is given by
$\rho = e^{-rT}\,\mathrm{E}\,(\varphi\,(\mathrm{S}_T)\,\psi(x))$

Proof

$$\rho = \frac{\partial V}{\partial \kappa}, \qquad V_0 = \mathbb{E}(e^{-rT}\varphi(\mathrm{S}_T))$$

$$\rho = \frac{\partial \mathbb{E}(e^{-rT}\varphi(\mathrm{S}_T))}{\partial \kappa}$$

$$= e^{-rT}\frac{\partial \mathbb{E}(\varphi(\mathrm{S}_T))}{\partial \kappa}$$

$$= e^{-rT}\mathbb{E}(\varphi'(\mathrm{S}_T)\frac{\partial \mathrm{S}_T}{\partial \kappa})$$

$$= e^{-rT}\mathbb{E}(\varphi'(\mathrm{S}_T)T\mathrm{S}_T)$$

Here, using
$\varphi = y,\ S_T = Y, \quad X = TS_T\ v = 1$
in equation (2.2)
$\mathrm{E}\,(y'(Y)\,X = \mathrm{E}(y(Y)\,\delta(Xv(D^v Y)^{-1}))$
we have

$$\mathbb{E}(y'(Y)X) = \mathbb{E}(y(\mathrm{S}_T)\delta(\frac{T\mathrm{S}_T}{\sigma T \mathrm{S}_T}))$$

$$= \mathbb{E}(\varphi(\mathrm{S}_T)\delta(\frac{1}{\sigma}))$$

$$= \mathbb{E}(\varphi(\mathrm{S}_T)\frac{B_T}{\sigma})$$

So

$$\rho = \frac{e^{-rT}}{\sigma}\mathbb{E}(\varphi(\mathrm{S}_T), B_T)$$

The weight function is
$\psi = \frac{B_T}{\sigma}$
For Asian options whose pay-off is described as
$\varphi(\mathtt{S}_T) = \frac{1}{T}\int_0^T \mathtt{S}_T d\tau,$
Then

$$\rho = \frac{e^{-rT}}{\sigma}\mathbb{E}\left(\frac{1}{T}\int_0^T \mathtt{S}_T d\tau B_T\right.$$

For a best of asset call whose pay-off is described as
φ (S_T) = max$(S_i - [K])$, i = *1,2...*

So
$\rho = \frac{e^{-rT}}{\sigma}\mathbb{E}\ \ max(\mathtt{S}_i - \mathbf{K})\mathbf{1}_{\mathbf{S}_i > \mathbf{S}_j\ \ i\neq j,\ \ i,j=1,2,\ldots n} B_T\big)$

3.1 **Computation and Analysis**

The greeks play a major role when hedging a financial derivatives. It provides the tool for risk management which help investor in taking right and appropriate decisions concerning their investment. We discretize the investment period and express the underlying asset price in discrete form by the Euler-Maruyana method.

Definition 3.3 [Call Option] If the holder of a certain option is given a right in the option contract to buy the option at a specified time τ at a fixed strike price **K**, such an option is known as a call option. The call option has a pay-off described by

Payoff = max $[(S_T - \boldsymbol{K}), 0]$

S_T is the price of the underlying asset at the expiration date or time

Definition 3.4 [Put Option] An option is called put if the option at a particular time τ gives the holder the right to sell at specified strike price **K** but not the obligation. The put option has a pay-off described by

Payoff = max $[(\boldsymbol{K} - S_T), 0]$

S_T is the price of the underlying asset at the expiration date or time.

Rho Let $C_E = max\ [(S_T - \mathbf{K}), 0]$ be the pay o process of an European call and suppose $V(\tau)$ represent the option value where $\tau \in [0, T]$, then the measures of changes in V in terms of rate of interest is given as

$\rho = \frac{\partial V}{\partial r}$

$\rho_1 = \frac{e^{-rT}}{\sigma}\mathbb{E}_Q[(\mathtt{S}_T - \mathbf{K})^+)B_T]$

so,

$\rho_1 = \frac{e^{-rT}}{\sigma}[(\mathtt{S}_j + a\mathtt{S}_j h - \mathbf{K})B_0]$

Let $C_A = [Max(\frac{1}{T}\int_0^T \mathtt{S}_T d\tau - \mathbf{K}), 0]$ be the pay o process of an Asian call and suppose $V(\tau)$ represents the option value where $\tau \in [0, T]$, then the measures of changes in V in terms of rate of interest is given as

$$\rho = \frac{\partial V}{\partial r}$$

$\rho_2 = \frac{e^{-rT}}{\sigma}\mathbb{E}_Q[(\frac{1}{T}\int_0^T \mathtt{S}_\tau d\tau - \mathbf{K})B_T]$

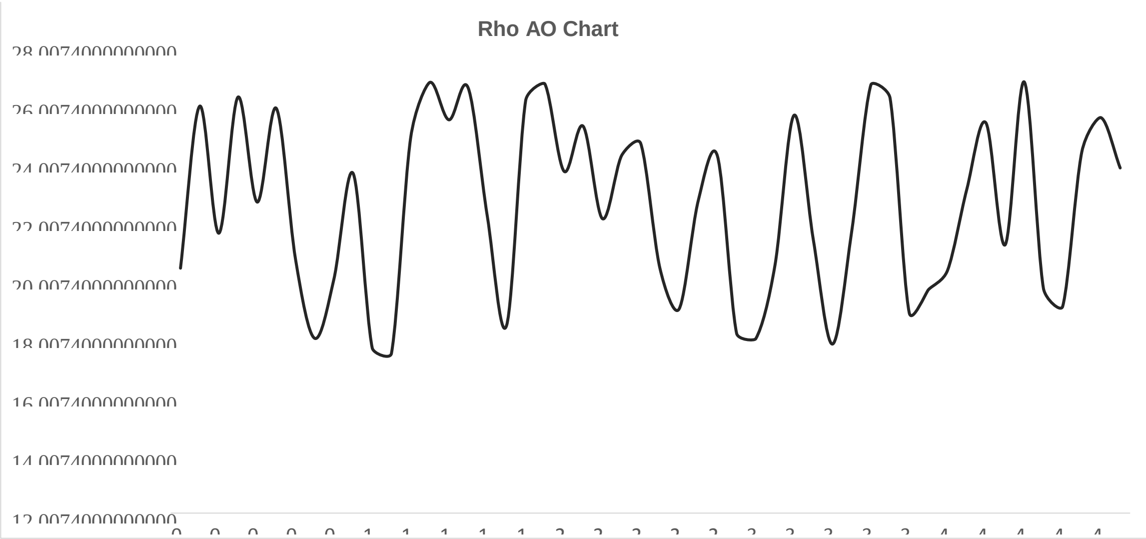

Figure 1: Rho AO Graph

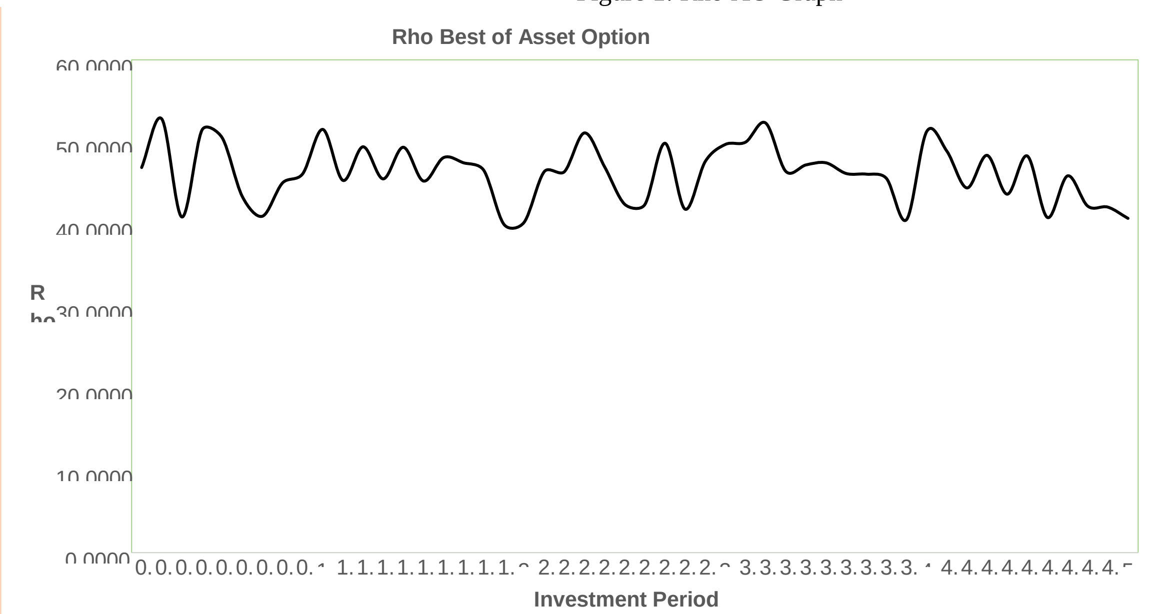

Figure 2: Rho Graph BAO

$$\rho_2 = \frac{e^{-rT}}{\sigma}[(\frac{1}{m}\sum_{j=1}^{m}(\mathtt{S}_j + a\mathtt{S}_j h - \mathbf{K})B_0]$$

Let $C_B = [Max(\mathrm{S}_i - \mathbf{K}),0]\mathbf{1}_{Si>Sj \ I\neq j,\ i,j=1,2,\ldots n}$ be the pay-off process of Best of Assets call option and let $V(\tau), \tau \in [0,T]$ be the value of the option at time τ, then the measures the sensitivity of the option with respect to changes in the rate of interest is given as

$$\rho = \frac{\partial V}{\partial r}$$

$$\rho_3 = \frac{e^{-rT}}{\sigma}\mathbb{E}[Max(\mathtt{S}_i - \mathbf{K}),0]\mathbf{1}_{\mathtt{S}_i>\mathtt{S}_j \quad i\neq j}B_T]$$

so,

$$\rho_3 = \frac{e^{-rT}}{\sigma}[(Max(\mathtt{S}_i + a\mathtt{S}_i h - \mathbf{K})B_0]$$

IV. Discussion

In this section, we summarize and discuss the results obtained for the various Greeks and their implications to an investors

Rho measured the effect of changes in the interest rate on the value of the option. When the interest rate is high, the holder of a Call is happy because the condition is favorable to him or her. This is because, the value of Call will increase, but this position is not favorable to the holder of a Put option. This high interest rate will lead to high value of rho, and this is only attainable when underlying asset value is high compare to the strike price.

In figure 1, we used the following values for the computation, $\sigma = 0.2$, $r = 0.01$, $S_0 = 70$, $\kappa = 0.3$, $h = 0.1$, $B_0 = 0.5$, $T = 5$, and $K = 71$. Rho is highest with value 27.23274. This value is obtained when the underlying asset values are respectively 82.45160, 87.94837, 93.44514, and 71.45805. The difference between these values and the strike price is the highest, and when this happened, the holder of a Call option is at advantage because the condition is favorable.

In figure 2, we used the following values for the computation, $\sigma = 0.2$, $r = 0.01$, $S_0 = 70$, $\kappa = 0.3$, $h = 0.1$, $B_0 = 0.5$, $T = 5$, and $K = 71$. Rho is highest with value 53.6837. This value is obtained when the underlying asset values are respectively 82.5657, 88.0701, 93.5733, 71.5569, and 77.0613. The difference between these values and the strike price is the highest, and when this happened, the holder of a Call option is at advantage because the condition is favorable.

This is expected for a Call option because, as the underlying asset value increases, the difference between the underlying asset value and the strike price increases also. This is what an investor wants since this increment is likely to be positive. This positive difference is like making profit on the investment.

References

[1]. Akeju Adeyemi. O (2021). Malliavin Calculus Approach to Pricing and Hedging of Options with More than One Underlying Assets. Ph.D. Thesis. University of Ibadan, Ibadan. repository.pgcollegeui.com.

[2]. Da Prato. G. (2007). Introduction to Stochastic Analysis and Malliavin Calculus, Vol. 6 of Appunti Scuola Normale Superiore di Pisa.

[3]. Giulia Di Nunno. (2002). Stochastic Integral representation, Stochastic derivatives and minimal variance hedging. Stoch. Rep, 73 (1-2):181-198.

[4]. Giulia Di Nunno et al. (2009). Malliavin Calculus for Levy Processes with Application to Finance. Springer-Verlag Berlin Heidelberg.

[5]. Klebaner. F. C. 2005. Introduction to Stochastic Calculus with Applications. 2nd Edition, Imperial College Press.

[6]. Kloeden. P. E and Platen, E. (1999). Numerical Solutions of Stochastic Differential Equations.

[7]. Springer-Verlag, Berlin Heidelberg New York.

[8]. Malliavin. P. 1978. Stochastic Calculus of Variation and hypo elliptic Operators. Proceedings of the international Symposium on Stochastic Differential Equations. Res. Inst. Math. Sci. 169213 Kyoto Univ. Kyoto.

[9]. Nualart. D. (2006). The Malliavin Calculus and related topics, Probability and its applications. New York Springers-verlag, Berlin. Second edition.

[10]. Oksendal. B. et al. (2000). White Noise Generalization of the Clark-Haussmann-Ocone Theorem with Application to Mathematical Finance. Finance and Stochastic,4: 465-496.

[11]. Oksendal. B. (2003). Stochastic Differential Equations. Universitaxt, Springer-Verlag, Berlin, Sixth Edition.

[12]. Pascucci. A. 2010. PDE and Martingale Methods in Option Pricing. Bocconi and Springer series.

Akeju Adeyemi. et. al. " On The Greek Rho of Asian Option and Best of Asset Option, A Malliavin Calculus Approach." *IOSR Journal of Mathematics (IOSR-JM)*, 19(1), (2023): pp. 52-60.