IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 18, Issue 6 Ser. II (Nov. – Dec. 2022), PP 26-30
www.iosrjournals.org

A Result on Common Fixed Point in S-Metric Spaces

Duduka Venkatesh[1], V.Nagaraju[2]
[1](Department of Mathematics, Osmania Univerisity, Hyderabad, India)
[2](Department of Mathematics, Osmania University, Hyderabad, India)

Abstract: *The purpose of this paper is to prove a common fixed point theorem for four self maps in complete S-metric space using compatibility of type (A).*
Key Word*: Fixed Point, S-metric space, Compatible mappings, Compatible mappings of type (A).*

Date of Submission: 24-11-2022 Date of Acceptance: 08-12-2022

I. Introduction

Fixed point is also known as an invariant point. Banach Principle of contraction[11] on metric spaces is the paramont importance cause in the field of invariant points and non linear analysis. During 1922, Stefan Banachconcived the concept of contraction and established well known Banach contraction theorem. Literatures are brought out new outcomes that are related to prove the generalization of metric space and to acquire a refinement about the contractive condition. In the year 2006, B Sims and Mustafa[12], established theory on G-metric spaces, that is an extention of metric spaces and established some properties. Later, A.Aliouche, S.Sedghi, N.Shobe[3] initiated S-metric spaces, it is a generalization of G-metric spaces in the year 2012. In 2014, S.Radojevic, N.V.Dung and N.T.Hieu[9] proved by examples shows S-metric spaces are not a generalization of G-metric spaces and controvisely.In 1986, Jungck[1] introduced the notion of compatible mappings in metric spaces. In 1993, Jungck et al. [2] introduced the concept of compatible mappings of type(A) in metric spaces and proved that compatible mappings and compatible mappings of type(A) are equivalent under some conditions. Fixed points of contractive maps on S-metric spaces were studied by [5-10].In this paper, we define compatible mappings of type(A) in S metric space and prove a common fixed point theorem for four self maps.

II. Preliminaries

In this section,we present some definitions and results which will be used in the main result.

Definition 2.1.[3] Let Ω be a non-empty set. Then a function S: $\Omega^3 \to [0,\infty)$ is said to be S-metric on Ω if for each x,y,z,a in Ω,we have
S(x,y,z)=0 ⇔ x=y=z and S(x,y,z)≤ S(x,x,a)+ S(y,y,a)+ S(z,z,a).
The pair (Ω,S) is called an S-metric space.

Example 2.2. [3] Let R be the set of all real numbers. Then S(x,y,z)=|x-y|+|y-z| for all x,y,z∈Ris an S-metric on R.

Lemma 2.3.[5] Let (Ω,S) be an S-metric space. Then for all x,y∈Ω, we have
S(x,x,y)=S(y,y,x).

Definition 2.4.[6] Let (Ω,S) be an S-metric space.Then
(i) a sequence $\{x_\eta\} \subset \Omega$is said to converge to x in Ωif $S(x_\eta, x_\eta, x) \to 0$ as $\eta \to \infty$.
That is, for each $\varepsilon > 0$, there exists a positive integer k such that for all $\eta \geq k$, $S(x_\eta, x_\eta, x) < \varepsilon$.
In this case,we write $\lim_{\eta\to\infty} x_\eta = x$.
(ii) a sequence $\{x_\eta\} \subset \Omega$is called a Cauchy sequence if $S(x_\eta, x_\eta, x_m) \to 0$ as η,m→∞. That is, for each $\varepsilon > 0$, there exists a positive integer k such that for all η,m ≥ k, we have $S(x_\eta, x_\eta, x_m) < \varepsilon$.
(iii)We say that an S-metric space (Ω,S) is complete if every Cauchy sequence is convergent.

Definition 2.5.[4] Let (Ω,S) and (Ω', S') be S-metric spaces. Then a function f:(Ω,S)→(Ω', S') is said to be continuous at a point c in Ω if for every sequence $\{x_\eta\} \subset \Omega$, $S(x_\eta, x_\eta, c) \to 0 \Rightarrow S'(f(x_\eta), f(x_\eta), f(c)) \to 0$.
We say that a function f is continuous on Ω if it is continuous at each point of Ω.

Definition 2.5.[7] A pair (A,B) of self maps of an S-metric space (Ω,S) is said to be compatible if $\lim_{\eta\to\infty} S(ABx_\eta, ABx_\eta, BAx_\eta) = 0$ whenever $\{x_\eta\}$ is a sequence in Ω such that $\lim_{\eta\to\infty} Ax_\eta = \lim_{\eta\to\infty} Bx_\eta = t$, for some t∈Ω.

Definition 2.6. A pair (A,B) of self maps of an S-metric space (Ω,S) is said to be compatible of type(A) if $\lim_{\eta\to\infty} S(ABx_\eta, ABx_\eta, BBx_\eta) = 0$ and $\lim_{\eta\to\infty} S(BAx_\eta, BAx_\eta, AAx_\eta) = 0$ whenever a sequence $\{x_\eta\} \subset \Omega$such that $\lim_{\eta\to\infty} Ax_\eta = \lim_{\eta\to\infty} Bx_\eta = t$, for some t ∈Ω.

Lemma 2.7.[6] Let (Ω,S) be an S-metric space. If there are two sequences $\{x_\eta\}$ and $\{y_\eta\}$ in Ω such that $\lim_{\eta\to\infty} x_\eta$ = x and $\lim_{\eta\to\infty} y_\eta = y$, then $\lim_{\eta\to\infty} S(x_\eta, x_\eta, y_\eta)$ = S(x,x,y).

Proposition 2.8. Suppose P and Q are two self maps of S-metric space (Ω, S). Let P and Q be compatible mappings of type(A) and $\lim_{\eta\to\infty} Px_\eta = \lim_{\eta\to\infty} Qx_\eta = t$, for some t∈Ω.Then $\lim_{\eta\to\infty} QPx_\eta = Pt$, provided P is continuous.

Proof: Since P and Q are compatible of type(A) ,we have $\lim_{\eta\to\infty} S(PQx_\eta, PQx_\eta, QQx_\eta) = 0$ and $\lim_{\eta\to\infty} S(QPx_\eta, QPx_\eta, PPx_\eta) = 0$ whenever a sequence $\{x_\eta\}$ in Ω such that $\lim_{\eta\to\infty} Px_\eta = \lim_{\eta\to\infty} Qx_\eta = t$,for some t∈Ω. Since P is continuous, we have $\lim_{\eta\to\infty} PPx_\eta = \lim_{\eta\to\infty} PQx_\eta = Pt$.

Now, by triangle inequality we have

$S(QPx_\eta, QPx_\eta, Pt) \le 2S(QPx_\eta, QPx_\eta, PPx_\eta) + S(Pt, Pt, PPx_\eta)$

Letting η →∞,we get $\lim_{\eta\to\infty} S(QPx_\eta, QPx_\eta, Pt) = 0$ which implies that $\lim_{\eta\to\infty} QPx_\eta = Pt$.

Proposition 2.9. Supose P and Q are two self maps of S-metric space (Ω, S). Let P and Q be compatible mappings of type(A) andPt = Qt for some t in Ω. Then PQt = QQt = QPt = PPt.

Proof: Since P and Q are compatible of type(A),we have $\lim_{\eta\to\infty} S(PQx_\eta, PQx_\eta, QQx_\eta) = 0$ and $\lim_{\eta\to\infty} S(QPx_\eta, QPx_\eta, PPx_\eta) = 0$

whenever a sequence $\{x_\eta\}$ in Ω such that $\lim_{\eta\to\infty} Px_\eta = \lim_{\eta\to\infty} Qx_\eta = t$,for some t∈Ω.

Let x_η = t for η = 1,2,3,4,……….. .Then $\lim_{\eta\to\infty} Px_\eta = \lim_{\eta\to\infty} Qx_\eta = Qt$,

sinceQt = Pt.

Now S(PQt,PQt,QQt) = $\lim_{\eta\to\infty} S(PQx_\eta, PQx_\eta, QQx_\eta) = 0$ .Hence PQt = QQt.

Also S(QPt,QPt,PPt) = $\lim_{\eta\to\infty} S(QPx_\eta, QPx_\eta, PPt) = 0$.

So QPt=PPt. Since Qt=Pt, it follows that PQt=QQt=QPt=PPt.

The following theorem was proved by Sedghi et al. in 2018.

Theorem 2.10.[7] Let A,B,U and V be self maps of an S-complete metric space (Ω,S) such that

i.A(Ω)⊆V(Ω), B(Ω)⊆U(Ω)

ii. (A,U) and (B,V) are compatible mappings

iii. S(Ax,Ay,Bz)≤ b_1 S(Ux,Uy,Vz) +b_2 S(Ax,Ay,Vz)
+ b_3 S(Ux,Uy,Bz) + b_4 S(Ay,Ay,Vz) +b_5 S(Bz,Bz,Vz)

where $b_i \ge 0$,i=1,2,3,4,5 are real constants such that $b_1+3b_2+3b_3+3b_4+b_5 < 1$.

iv. U and V are continuous.

Then A,B,U and V have a unique common fixed point.

III. Main Results

Theorem 3.1. Let A,B,U and V be self maps of an S-complete metric space (Ω,S) such that

3.1.1 A(Ω)⊆V(Ω), B(Ω)⊆U(Ω)

3.1.2. The pairs (A,U) and (B,V) are compatible mappings of type(A)

3.1.3 S(Ax,Ay,Bz)≤ b_1 S(Ux,Uy,Vz) +b_2 S(Ax,Ay,Vz)
+ b_3 S(Ux,Uy,Bz) + b_4 S(Ay,Ay,Vz) +b_5 S(Bz,Bz,Vz)

where $b_i \ge 0$,i=1,2,3,4,5 are real constants such that $b_1+3b_2+3b_3+3b_4+b_5 < 1$

3.1.4. one of A,B,U and V is continuous.

Then A,B,U and V have a unique common fixed point.

Proof: Let x_0∈Ω. Since A(Ω)⊆V(Ω), B(Ω)⊆U(Ω) ,we can find x_1∈Ωsuch that

$Ax_0=Vx_1$ and for this x_1 there exists $x_2\in\Omega$such that $Bx_1=Ux_2$ and so on. Repeating this way, we obtain a sequence $\{r_\eta\}$ such that $r_{2\eta} = Ax_{2\eta} = Vx_{2\eta+1}$ and $r_{2\eta+1} = Bx_{2\eta+1}=Ux_{2\eta+2,}$ for $\eta \geq 0$.
We first claim that $\{r_\eta\}$ is Cauchy.
From 3.1.3, we have

$S(r_{2\eta},r_{2\eta},r_{2\eta+1}) = S(Ax_{2\eta}, Ax_{2\eta}, Bx_{2\eta+1})$
$\leq b_1S(Ux_{2\eta},Ux_{2\eta},Vx_{2\eta+1}) + b_2S(Ax_{2\eta}, Ax_{2\eta}, Vx_{2\eta+1})$
$+ b_3S(Ux_{2\eta},Ux_{2\eta},Bx_{2\eta+1}) + b_4S(Ax_{2\eta}, Ax_{2\eta},Vx_{2\eta+1})$
$+ b_5S(Bx_{2\eta+1},Bx_{2\eta+1},Vx_{2\eta+1})$
$= b_1 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) + b_2S(r_{2\eta},r_{2\eta},r_{2\eta}) + b_3 S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta+1})$
$+ b_4S(r_{2\eta},r_{2\eta},r_{2\eta}) + b_5 S(r_{2\eta+1}, r_{2\eta+1}, r_{2\eta})$
$= b_1 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) + b_3 S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta+1}) + b_5 S(r_{2\eta+1}, r_{2\eta+1}, r_{2\eta})$
$\leq b_1S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) + b_3 [2S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) + S(r_{2\eta+1}, r_{2\eta+1}, r_{2\eta})]$
$+ b_5S(r_{2\eta+1}, r_{2\eta+1}, r_{2\eta})$

Hence,we have

$S(r_{2\eta},r_{2\eta},r_{2\eta+1})\leq b_1S(r_{2\eta-1},r_{2\eta-1},r_{2\eta})+2b_3 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) + (b_3 + b_5)S(r_{2\eta}, r_{2\eta}, r_{2\eta+1})$ (1)

Now we prove that $S(r_{2\eta},r_{2\eta},r_{2\eta+1}) \leq S(r_{2\eta-1},r_{2\eta-1},r_{2\eta})$ for all $\eta \in N$.
If possible, assume that $S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) < S(r_{2\eta},r_{2\eta},r_{2\eta+1})$ for some $\eta \in N$.
From (1), we have

$S(r_{2\eta},r_{2\eta},r_{2\eta+1}) \leq b_1 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) + 2b_3 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta}) + (b_3 + b_5)S(r_{2\eta}, r_{2\eta}, r_{2\eta+1})$
$< (b_1 + 3b_3 + b_5) S(r_{2\eta},r_{2\eta},r_{2\eta+1})$
$<S(r_{2\eta},r_{2\eta},r_{2\eta+1})$ which is a contradiction. So we must have
$S(r_{2\eta},r_{2\eta},r_{2\eta+1}) \leq S(r_{2\eta-1},r_{2\eta-1},r_{2\eta})$ for all $\eta \in N$.

Therefore, from (1) we have

$S(r_{2\eta},r_{2\eta},r_{2\eta+1}) \leq (b_1 + 3b_3 + b_5) S(r_{2\eta-1},r_{2\eta-1},r_{2\eta})$ (2)

Consider,

$S(r_{2\eta},r_{2\eta},r_{2\eta-1}) = S(Ax_{2\eta}, Ax_{2\eta}, Bx_{2\eta-1})$
$\leq b_1S(Ux_{2\eta},Ux_{2\eta},Vx_{2\eta-1}) + b_2S(Ax_{2\eta}, Ax_{2\eta},Vx_{2\eta-1})$
$+ b_3S(Ux_{2\eta},Ux_{2\eta},Bx_{2\eta-1}) + b_4S(Ax_{2\eta}, Ax_{2\eta},Vx_{2\eta-1})$
$+ b_5S(Bx_{2\eta-1},Bx_{2\eta-1},Vx_{2\eta-1})$
$= b_1 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta-2}) + b_2S(r_{2\eta},r_{2\eta},r_{2\eta-2}) + b_3 S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta-1})$
$+ b_4S(r_{2\eta},r_{2\eta},r_{2\eta-2}) + b_5 S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta-2})$
$= b_1 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta-2}) + (b_2 + b_4) S(r_{2\eta},r_{2\eta}, r_{2\eta-2})$
$+ b_5S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta-2})$
$\leq b_1 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta-2}) + (b_2 +b_4)[2S(r_{2\eta},r_{2\eta}, r_{2\eta-1})+S(r_{2\eta-2}, r_{2\eta-2}, r_{2\eta-1})]$
$+ b_5S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta-2})$

Hence, we get

$S(r_{2\eta},r_{2\eta},r_{2\eta-1}) \leq b_1 S(r_{2\eta-1},r_{2\eta-1},r_{2\eta-2}) + (2b_2 +2b_4)S(r_{2\eta},r_{2\eta}, r_{2\eta-1})$
$+(b_2+ b_4 + b_5)S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta-2})$ (3)

Similarly, if $S(r_{2\eta-1}, r_{2\eta-1}, r_{2\eta-2}) <S(r_{2\eta},r_{2\eta},r_{2\eta-1})$, for some $\eta \in N$, then (3) gives
$S(r_{2\eta},r_{2\eta},r_{2\eta-1}) \leq (b_1 +3 b_2 +3 b_4 + b_5) S(r_{2\eta},r_{2\eta},r_{2\eta-1})$
$<S(r_{2\eta},r_{2\eta},r_{2\eta-1})$ which is a contradiction. So, we must have
$S(r_{2\eta},r_{2\eta},r_{2\eta-1}) \leq S(r_{2\eta-1},r_{2\eta-1},r_{2\eta-2})$, for all $\eta \in N$.

Hence from (3) , we have

$S(r_{2\eta},r_{2\eta},r_{2\eta-1}) \leq (b_1 +3 b_2 +3 b_4 + b_5) S(r_{2\eta-1},r_{2\eta-1},r_{2\eta-2})$ (4)

From (2) and (4), we get
$S(r_\eta,r_\eta,r_{\eta-1}) \leq \beta S(r_{\eta-1},r_{\eta-1},r_{\eta-2})$, for all $\eta \geq 2$,
where$\beta = \min\{ b_1 + 3b_3 + b_5 , b_1 +3 b_2 +3 b_4 + b_5 \}$.
Hence, for $\eta \geq 2$, we have

$S(r_\eta,r_\eta,r_{\eta-1}) \leq \beta^{\eta-1} S(r_1,r_1,r_0)$ (5)

For η >m, we have

$S(r_\eta,r_\eta,r_m) \leq S(r_m,r_m,r_\eta)$
$\leq 2 S(r_m,r_m,r_{m+1}) + S(r_\eta,r_\eta,r_{m+1})$
$\leq 2 S(r_m,r_m,r_{m+1}) + 2S(r_{m+1},r_{m+1},r_{m+2})+ S(r_\eta,r_\eta,r_{m+2})$

Continuing in this way ,

$S(r_\eta,r_\eta,r_m) \leq 2 S(r_m,r_m,r_{m+1}) + 2S(r_{m+1},r_{m+1},r_{m+2})+\ldots..+ S(r_{\eta-1},r_{\eta-1},r_\eta)$

Hence,

$S(r_\eta,r_\eta,r_m) \leq 2(\beta^m + \beta^{m+1} + ... + \beta^{\eta-1})S(r_0,r_0,r_1)$

$\leq 2\beta^m(1+\beta+\beta^2+.....)$ S(r_1,r_1,r_0)

$= \dfrac{2\beta^m}{1-\beta}$ S(r_1,r_1,r_0) $\to 0$ as m$\to\infty$, since $\beta<1$.It follows that

{ r_η }is a Cauchy sequence in Ω and Ω is complete,we can find r∈Ω such that { r_η }→r as η →∞.

Hence, $\lim_{\eta\to\infty} Ax_{2\eta} = \lim_{\eta\to\infty} Vx_{2\eta} = \lim_{\eta\to\infty} Bx_{2\eta} = \lim_{\eta\to\infty} Ux_{2\eta}$ = r.

Claim: r is a common fixed point of A,B,U and V.

Suppose U is continuous.Since (A,U) is compatible of type(A), it follows from Proposition 2.8 that $UUx_{2\eta+2}$, $AUx_{2\eta+2}$ → Ur as η →∞.

Put x=y=$Ux_{2\eta+2}$, z= $x_{2\eta+2}$ in 3.1.3, we get

$S(AUx_{2\eta+2},AUx_{2\eta+2},Bx_{2\eta+1})\leq b_1S(UUx_{2\eta+2},UUx_{2\eta+2},Vx_{2\eta+1})+b_2S(AUx_{2\eta+2},AUx_{2\eta+2},Vx_{2\eta+1})$
$+b_3S(UUx_{2\eta+2},UUx_{2\eta+2},Bx_{2\eta+1})$
$+b_4S(AUx_{2\eta+2}, AUx_{2\eta+2},Vx_{2\eta-1})+b_5 S(Bx_{2\eta+1},Bx_{2\eta+1},Vx_{2\eta+1})$

Letting η →∞, we get

$S(Ur,Ur,r) \leq b_1S(Ur,Ur,r) +b_2 S(Ur,Ur,r)+ b_3 S(Ur,Ur,r)$
$+ b_4S(Ur,Ur,r)+ b_5S(r,r,r)$
$\leq (b_1+ b_2 + b_3 + b_4)\, S(Ur,Ur,r)$
$\leq (b_1+ 3b_2 + 3b_3 + 3b_4 + b_5)S(Ur,Ur,r)$

As $b_1+ 3b_2 + 3b_3 + 3b_4 + b_5 <1$, we get

S(Ur,Ur,r)< S(Ur,Ur,r), which is a contradiction.

Therefore, Ur=r. (6)

Putting x=y=r, z= $x_{2\eta+1}$ in 3.1.3, we get

$S(Ar,Ar,Bx_{2\eta+1}) \leq b_1S(Ur,Ur,Vx_{2\eta+1}) + b_2S(Ar,Ar,Vx_{2\eta+1})+ b_3S(Ur, Ur, Bx_{2\eta+1})$
$+ b_4S(Ar,Ar,Vx_{2\eta+1}) + b_5S(Bx_{2\eta+1},Bx_{2\eta+1},Vx_{2\eta+1})$

Letting η →∞, we get

$S(Ar,Ar,r) \leq b_1S(Ur,Ur,r) + b_2S(Ar, Ar,r) + b_3S(Ur,Ur,r)$
$+ b_4S(Ar, Ar\ r) + b_5S(r,r,r)$
$\leq (b_1+ b_4)S(Ar, Ar, r)$ [by using (6)]

$S(Ar, Ar, r) \leq (b_1+ 3b_2 + 3b_3 + 3b_4 + b_5)S(Ar, Ar, r)$

S(Ar, Ar, r) < S(Ar, Ar, r), since $b_1+ 3b_2 + 3b_3 + 3b_4 + b_5 <1$,

which is a contradiction.Therefore Ar=r.

Since Ar=r and A(Ω) ⊆ V(Ω) , there exists u∈Ω such that r=Ar=Vu

Claim : Bu=r.

Putting x=y=r, z=u in 3.1.3,then

$S(Ar,Ar,Bu) \leq b_1 S(Ur,Ur,Vu)+ b_2 S(Ar, Ar, Vu) + b_3 S(Ur, Ur, Bu)$
$+ b_4S(Ar, Ar,Vu) + b_5 S(Bu,Bu,Vu)$

$S(r,r,Bu) \leq (b_3+ b_5)\, S(r,r,Bu)$
$< (b_1+ 3b_2 + 3b_3 + 3b_4 + b_5)\, S(r,r,Bu)$

S(r,r,Bu) < S(r,r,Bu), since $b_1+ 3b_2 + 3b_3 + 3b_4 + b_5<1$,which is a contradiction.

Therefore Bu=r.Hence, Bu=Vu=r.

Now by Proposition 2.9, we have BVu=VBu and so Br=Vr.

Finally, we prove that Br=r.

Put x=y=z=r in 3.1.3, we get

$S(Ar,Ar,Br) \leq b_1 S(Ur, Ur,Vr)+ b_2 S(Ar, Ar, Vr) + b_3 S(Ur, Ur, Br)$
$+ b_4S(Ar, Ar,Vr) + b_5 S(Br,Br,Vr)$

$S(r,r,Br) \leq (b_1+ b_2+ b_3+ b_4)\, S(r,r,Br)$
$\leq (b_1+ 3b_2 + 3b_3 + 3b_4 + b_5)\, S(r,r,Br)$

S(r,r,Br) <S(r,r,Br), since $b_1+ 3b_2 + 3b_3 + 3b_4 + b_5<1$, which is a contradiction.

Therefore Br=r=Vr and hence Ar=Br=Ur=Vr=r.

Similarly, we can prove that the mapping A,B,U& V have a common fixed point, when any one of the maps A,B and V is continuous.

Uniqueness:

Let w be any other common fixed point. Then Aw=Bw=Uw=Vw=w.

Put x=y=r & z=w in 3.1.3, we get

$S(Ar,Ar,Bw) \leq b_1 S(Ur, Ur,Vw)+ b_2 S(Ar, Ar, Vw) + b_3 S(Ur,Ur, Bw)$
$+ b_4S(Ar, Ar,Vw) + b_5 S(Bw,Bw,Vw)$

$S(r,r,w) \leq (b_1+ b_2+ b_3+ b_4)\, S(r,r,w)$
$\leq (b_1+ 3b_2 + 3b_3 + 3b_4 + b_5)\, S(r,r,w)$

$S(r,r,w) < S(r,r,w)$, since $b_1 + 3b_2 + 3b_3 + 3b_4 + b_5 < 1$, which is a contradiction.
Therefore r=w and the proof is complete.

If U = V in the Theorem 3.1.,we get
Corollary 3.2. Let A,B and V be self maps of a S-complete metric space (Ω,S) such that
3.2.1 $A(\Omega) \subseteq V(\Omega)$, $B(\Omega) \subseteq V(\Omega)$
3.2.2. (A,V) and (B,V) are compatible mappings of type(A)
3.2.3. $S(Ax,Ay,Bz) \leq b_1 S(Vx,Vy,Vz) + b_2 S(Ax,Ax,Vz) + b_3 S(Vx,Vy,Bz) + b_4 S(Ay,Ay,Vz) + b_5 S(Bz,Bz,Vz)$
where $b_i \geq 0$, i=1,2,3,4,5 are real constants with $b_1 + 3b_2 + 3b_3 + 3b_4 + b_5 < 1$.
3.2.4. One of A,B and V is continuous. Then A,B and V have a unique common fixed point.

If A=B and U=V in the Theorem 3.1.,we get
Corollary 3.3. Let A and V be self maps of a S-complete metric space (Ω,S) such that
3.3.1. $A(\Omega) \subseteq V(\Omega)$
3.3.2. The pair (A,V) is compatible mappings of type(A)
3.3.3. $S(Ax,Ay,Az) \leq b_1 S(Vx,Vy,Vz) + b_2 S(Ax,Ax,Vz) + b_3 S(Vx,Vy,Az) + b_4 S(Ay,Ay,Vz) + b_5 S(Az,Az,Vz)$,
where $b_i \geq 0$, i=1,2,3,4,5 are real constants with $b_1 + 3b_2 + 3b_3 + 3b_4 + b_5 < 1$.
3.3.4. Either A or V is continuous.
Then A and V have a unique common fixed point.

IV. Conclusion

In this paper, we defined compatible mappings of type(A) in S-metric spaces and obtained a common fixed point theorem for two pairs of such mappings.

References

[1]. Jungck , (1986), Compatible mappings and common fixed points, Internat.J. Math. Math.Sci.,Vol. 9, pp. 771-778.https://doi.org/10.1155/S0161171286000935
[2]. Jungck , Murthy P.P and Cho Y.J., (1993), Compatible mappings of type (A) and common fixed points, Math.Japonica,Vol .38, pp. 381-391.
[3]. Sedghi, S., Shobe, N., Aliouche, A.,(2012), A generalization of fixed point theorems in S-metric spaces. Mat. Vesn. Vol. 64, No. 3, pp. 258–266.
[4]. Kyu Kim, J., Sedghi, S., Gholidahneh, A., Mahdi Rezaee, M., (2016), Fixed point theorems in S-metric spaces. East Asian Math. J. Vol. 32, No. 5, pp. 677–684.
[5]. Sedghi, S., Altun, I., Shobe, N., Salahshour, M.A, (2014),Some properties of S-metric spaces and fixed point results. KyungpookMath. J. Vol. 54, No. 1, pp. 113–122.
[6]. Sedghi, S., Dung, N.V., (2014), Fixed point theorems on S-metric spaces, Mat. Vesn. Vol. 66, No. 1, pp. 113–124.
[7]. Sedghi S,Shobkolaei N,Shahraki M and Dosenovic T, (2018), Common fixed point of four maps in S-metric spaces,Mathematical sciences, Vol. 12, pp. 137-143.https://doi.org/10.1007/s40096-018-0252-6
[8]. Mojaradi, J., Afra, (2015), double contraction in S-metric spaces, Int. J. Math. Anal. Vol. 9, No. 3, pp. 117-125.
[9]. N.V.Dung, N.T.Hieu and S.Radojevic, S., (2014), Fixed point theorems for g-monotone maps on partially ordered S-metric spaces, Filomat, Vol. 28, No. 9, pp. 1885-1898.
[10]. Gupta, A., (2013), Cyclic contraction on S-metric spaces. Int. J. Anal. Appl. Vol.3, No.2, pp. 119-130.
[11]. S.Banach, (1922), Sur les operations dans les ensembles abstraits et leur application aux equations integrals, Fundam.Math., pp. 133-181.
[12]. ZeadMustafa and BraileySims, (2006), A new approach to generalized metric spaces," Journal of Nonlinear and convex Analysis, vol. 7, No. 7, pp. 289-297.

Duduka Venkatesh. et. al. "A Result on Common Fixed Point in S-Metric Spaces." *IOSR Journal of Mathematics (IOSR-JM)*, 18(6), (2022): pp. 26-30.