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Solutions of couples of nonlinear partial differential equations by using modified Adomian decomposition methods

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Abstract: *In this paper, we introduce the solution of systems of nonlinear partial differential equations subject to the general initial conditions by using Adomian decomposition method (ADM) and Modified decomposition method(MDM).Four illustrated examples has been introduced, and the method has shown a high-precision, fast approach to solve nonlinear system of partial differential equations(PDEs) with initial conditions. The steps of the method are easy implemented and high accuracy.*

Keywords: *Systems of nonlinear partial differential equations,Adomian decomposition method, Modified decomposition method.*

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I. Introduction

Systems of partial differential equations(PDEs) have been use to described many important models in real life, such as contamination, distribution of shallow water, heat, waves contamination and the chemical reaction distribution model [1-4]. The general ideas and key characteristics of these systems are generally applicable [5]. In recent years, many authors have focused on solving non-linear systems of PDEs using various methods such that HAM [6], VIM [7], DTM [8], HPM [9,10], ADM [11,12], coupled laplace decomposition method [13],and semi analytic technique[14]. Recently, decomposition method and its modifications have been used in wider scope to solve different types of PDEs. In 2001 Wazwaz and Al-sayed [15-25] presented a modification of the ADM for nonlinear operator, that is replaced the process of dividing f into two parts by infinite series of components.

Here we used ADM and MDM for solving systems of nonlinear PDEs with initial conditions.This paper is arranged as follows. In Section 2, the,Adomian decomposition method. In Section 3, The Modified Decomposition Method. In Section 4, numerical examples. The conclusions appear in Section 5.

II. The Adomian decomposition method

In this section of nonlinear partial differential equations will be examined by using Adomian decomposition method. Systems of nonlinear partial differential equations arise in many scientific models such as the propagation of shallow water waves and the Brusselator model of chemical reaction-diffusion model. To achieve our goal in handling systems of nonlinear partial differential equations , we write a system in an operator form by

$$L_t u + L_x v + N_1(u,v) = g_1$$

$$L_t u + L_x v + N_2(u,v) = g_2 \qquad (1)$$

With initial data

$$u(x,0) = f_1(x),$$

$$v(x,0) = f_2(x), \qquad (2)$$

Where $L_t$and L_x are considered, without loss of generality, first order partial differential operators,N_1 and N_2 are nonlinear operators, and $g_1$and g_2 are source terms.Operating with the integral operator L_t^{-1} to the system (1) and using the initial data (2) yields

$$u(x,t) = f_1(x) + L_t^{-1}g_1 - L_t^{-1}L_x v - L_t^{-1}N_1(u,v),$$

$$v(x,t) = f_2(x) + L_t^{-1}g_2 - L_t^{-1}L_x v - L_t^{-1}N_2(u,v), \qquad (3)$$

The linear unknown functions $u(x,t)$ and $v(x,t)$ can be decomposed by infinite series of components

$$u(x,t) = \sum_{n=0}^{\infty} u_n\,(x,t),$$

$v(x,t) = \sum_{n=0}^{\infty} v_n\,(x,t)$(4)

However, the nonlinear operators $N_1(u,v)$ and $N_2(u,v)$ should be represented by using the infinite series of the so-called Adomian polynomials A_n and B_n
As follows:

$$N_1(u,v) = \sum_{n=0}^{\infty} A_n,$$

$N_2(u,v) = \sum_{n=0}^{\infty} B_n,$ (5)

Where $u_n(x,t)$ and $v_n(x,t), n \geq 0$ are the components of $u(x,t)$ and $v(x,t)$ respectively that will be recurrently determined, and A_n and $B_n, n \geq 0$ are Adomian polynomials that can be generated for all forms of nonlinearity. Substituting (4) and (5) into (3) gives

$$\sum_{n=0}^{\infty} u_n\,(x,t) = f_1(x) + L_t^{-1} g_1 - L_t^{-1} L_x (\sum_{n=0}^{\infty} v_n) - L_t^{-1} (\sum_{n=0}^{\infty} A_n),$$

$\sum_{n=0}^{\infty} v_n\,(x,t) = f_2(x) + L_t^{-1} g_2 - L_t^{-1} L_x (\sum_{n=0}^{\infty} u_n) - L_t^{-1} (\sum_{n=0}^{\infty} B_n).$ (6)

Two recursive relations can be constructed from (1.6) given by

$$u_0(x,t) = f_1(x) + L_t^{-1} g_1,$$

$u_{k+1}(x,t) = -L_t^{-1}(L_x v_k) - L_t^{-1}(A_k),$ $k \geq 0,$ (7)

$$v_0(x,t) = f_2(x) + L_t^{-1} g_2,$$

$v_{k+1}(x,t) = -L_t^{-1}(L_x u_k) - L_t^{-1}(B_k),$ $k \geq 0,$ (8)

It is an essential feature of the decomposition method that the zeroth components$u_0(x,t)$ and $v_0(x,t)$ are defined always by all terms that arise from initial data and from integrating the source terms. Having defined the zeroth pair (u_0, v_0), the remaining pair $(u_k, v_k) k \geq 1$ can be obtained in a recurrent manner by using (7) and (8) . Additional pairs for the decomposition series solutions normally account for higher accuracy. Having determined the components of $u(x,t)$ and $v(x,t)$, the solution (u,v) of the system follows immediately in the form of a power series expansion upon using (4).

III. The Modified Decomposition Method

The modified decomposition method will further accelerate the convergence of the series solution. It is to be noted that the modified decomposition method will be applied, wherever it is appropriate, to all partial differential equations of many order. To give a clear description of the technique, we consider the partial differential equation in an operator form

$$Lu + Ru = g, \qquad (9)$$

Where L is the highest order derivative, R is a linear differential operator of less order or equal order to L, and g is the source term. Operating with the inverse operator on (9) we obtain

$$u = f - L^{-1}(Ru), \qquad (10)$$

Where f represents the terms arising from the given initial condition and form grating the source term g. Define the solution u as an infinite sum of components defined by

$u = \sum_{n=0}^{\infty} u_n$ (11)

The aim of the decomposition method is to determine the components $u_n, n \geq 0$
Recurrently and elegantly. To achieve this goal, the decomposition method admits the use of the recursive relation

$$u_0 = f,$$

$u_{k+1} = -L^{-1}(Ru_k), \quad k \geq 0.$ (12)

In view of (12), the components $u_n, n \geq 0$ are readily obtained.

The modified decomposition method introduces a slight variation to the recursive relation(12) that will lead to the determination of the components of u in a faster and easier way. For specific cases, the function f can be set as the sum of two partial functions, namely f_1 and f_2.In other words, we can set

$$f = f_1 + f_2 \qquad (13)$$

Using (13), we introduce a qualitative change in the formation of the recursive relation (12). To reduce the size of calculations, we identify the zeroth component u_0 by one part of f , namely f_1 or f_2. The other part of f can be added to the componentu_1 among other terms. I n other words, the modified recursive relation can be identified by

$$u_0 = f_1,$$
$$u_1 = f_2 - L^{-1}(Ru_0),$$
$$u_{k+1} = -L^{-1}(Ru_k), k \geq 1. \quad (14)$$

Tow important remarks related to the modified method can be made here.
First, by proper selection of the functions $f_1$and f_2, the exact solution u may be obtained by using very few iterations, and sometimes by evaluating only two components. The success of this modification depends only on the choice of $f_1$and f_2, and this can be made through trials . Second , if f consists of one term only, the standard decomposition method should be employed in this case.

IV. Numerical Examples

In this section, we apply the modified decomposition method (MDM) to solve systems of partial differential equations. Numerical results are very encouraging.

Example1. Consider the nonlinear system

$u_t + u_x v_x = 2,$
$v_t + u_x v_x = 0,$ (15)

With the conditions $u(x,0) = x,\ v(x,0) = x.$ (16)

Solution.

Operating with L_t^{-1} we obtain

$u(x,t) = x + 2t - L_t^{-1}(u_x v_x),$
$v(x,t) = x - L_t^{-1}(u_x v_x).$ (17)

The linear terms$u(x,t)$ and $v(x,t)$ can be represented by the decomposition series

$$u(x,t) = \sum_{n=0}^{\infty} u_n\,(x,t),$$
$$v(x,t) = \sum_{n=0}^{\infty} v_n\,(x,t), \quad (18)$$

And the nonlinear terms $u_x v_x$ and $u_x v_x$by an infinite series of polynomials

$$u_x v_x = \sum_{n=0}^{\infty} A_n$$
$$u_x v_x = \sum_{n=0}^{\infty} B_n \quad (19)$$

Where A_n and B_n are the Adomian polynomials that can be generated for any form of nonlinearity. Substituting (18) and (19) into (17) gives

$$\sum_{n=0}^{\infty} u_n\,(x,t) = x + 2t - L_t^{-1}(\sum_{n=0}^{\infty} A_n),$$
$$\sum_{n=0}^{\infty} v_n\,(x,t) = x - L_t^{-1}(\sum_{n=0}^{\infty} B_n) \quad (20)$$

To accelerate the convergence of the solution, the modified decomposition method will be applied here. The modified decomposition method defines the recursive relations in the form

$$u_0(x,t) = x,$$
$$u_1(x,t) = 2t - L_t^{-1}(A_0),$$
$$u_{k+1}(x,t) = -L_t^{-1}(A_k),\ \ k \geq 1. \quad (21)$$

And

$$v_0(x,t) = x,$$
$$v_{k+1}(x,t) = -L_t^{-1}(B_k),\ \ k \geq 0. \quad (22)$$

The Adomian polynomials for the nonlinear term $u_x v_x$ are given by

$$A_0 = u_{0_x} v_{0_x},$$
$$A_1 = u_{1_x} v_{0_x} + u_{0_x} v_{1_x},$$

And for the nonlinear term $u_x v_x$ by

$$B_0 = u_{0_x} v_{0_x},,$$
$$B_1 = u_{1_x} v_{0_x} + u_{0_x} v_{1_x},$$

Using the derived Adomian polynomials into (21) and (22), we obtain the following pairs of components

$$(u_0, v_0) = (x, x),$$
$$(u_1, v_1) = (t, -t),$$
$$(u_2, v_2) = (0,0),$$

Accordingly, the solution of the system in a series form is given by

$(u,v) = (x+t+0, x-t+0)$ (23)
And in a closed form by $(u,v) = (x+t, x-t)$. (24)

Example 2. Consider the following nonlinear system:
$u_t - vu_x - u = 1,$
$v_t + uv_x + v = 1,$(25)
With the initial conditions
$u(x,0) = e^{-x}, \quad v(x,0) = e^{x}$(26)
Solution.
Following the analysis presented above we obtain

$$u(x,t) = e^{-x} + t + L_t^{-1}(vu_x) + L_t^{-1}(u),$$
$$v(x,t) = e^{x} + t - L_t^{-1}(uv_x) - L_t^{-1}(v). \quad (27)$$

Substituting the decomposition representations for linear and nonlinear into (27) yields

$$\sum_{n=0}^{\infty} u_n\,(x,t) = e^{-x} + t + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} u_n\right),$$
$$\textstyle\sum_{n=0}^{\infty} v_n\,(x,t) = e^{x} + t - L_t^{-1}(\sum_{n=0}^{\infty} B_n) - L_t^{-1}(\sum_{n=0}^{\infty} v_n), \quad (28)$$

Where A_n, B_n, are Adomian polynomials for the nonlinear terms vu_x, uv_x respectively.
To accelerate the convergence of the solution, the modified decomposition method will be applied here. The modified decomposition method defines the recursive relations in the form

$$u_0(x,t) = e^{-x},$$
$$u_1(x,t) = t + L_t^{-1}(A_0 + u_0),$$
$$u_{k+1}(x,t) = L_t^{-1}(A_k + u_k), \;\; k \geq 1. \,(29)$$

And

$$v_0(x,t) = e^{x},$$
$$v_1(x,t) = t - L_t^{-1}(B_0 + v_0),$$
$$v_{k+1}(x,t) = -L_t^{-1}(B_k + v_k), \;\; k \geq 1. \quad (30)$$

For brevity, we list the first three Adomian polynomials for A_n, B_n, as follows:
For vu_x we find

$$A_0 = v_0 u_{0_x},$$
$$A_1 = v_1 u_{0_x} + v_0 u_{1_x},$$
$$A_2 = v_2 u_{0_x} + v_1 u_{1_x} + v_0 u_{2_x},$$

And for uv_x we find

$$B_0 = u_0 v_{0_x},$$
$$B_1 = u_1 v_{0_x} + u_0 v_{1_x},$$
$$B_2 = u_2 v_{0_x} + u_1 v_{1_x} + u_0 v_{2_x},$$

Using the derived Adomian polynomials into equations (29) and (30), we obtain:

$$u_0(x,t) = e^{-x}, v_0(x,t) = e^{x}$$
$$u_1(x,t) = t + L_t^{-1}(-1 + e^{-x}) = te^{-x}$$
$$v_1(x,t) = t - L_t^{-1}(1 + e^{x}) = -te^{x}$$
$$u_2(x,t) = L_t^{-1}(A_1 + u_1) = \tfrac{1}{2}t^2 e^{-x}$$
$$v_2(x,t) = -L_t^{-1}(B_1 + v_1) = \frac{1}{2}t^2 e^{x}$$
$$u_3(x,t) = L_t^{-1}\left(\frac{1}{2}t^2 e^{-x}\right) = \frac{1}{3!}t^3 e^{-x}$$
$$v_3(x,t) = -L_t^{-1}\left(\frac{1}{2}t^2 e^{x}\right) = -\frac{1}{3!}t^3 e^{x}$$

The solutions $u(x,t), v(x,t)$ in a series form are given by :

$$u(x,t) = e^{-x} + te^{-x} + \frac{1}{2!}t^2 e^{-x} + \frac{1}{3!}t^3 e^{-x} + \cdots$$
$$= e^{-x}\left(1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 + \cdots\right) = e^{-x+t}$$
$$v(x,t) = e^{x} - te^{-x} + \frac{1}{2!}t^2 e^{-x} - \frac{1}{3!}t^3 e^{-x} + \cdots$$

$$= e^{-x}\left(1 - t + \frac{1}{2!}t^2 - \frac{1}{3!}t^3 + \cdots\right) = e^{x-t}$$

And in a closed form by:

$$u(x,t) = e^{-x+t}$$

$v(x,t) = e^{x-t}$(31)

Which are the exact solutions.

Example 3. Consider the following nonlinear system:

$u_t + 2vu_x - u = 2,$

$v_t - 3uv_x + v = 3,$ (32)

With the initial conditions

$u(x,0) = e^x, \quad v(x,0) = e^{-x}$ (33)

Solution.

Operating with L_t^{-1},we obtain

$$u(x,t) = e^x + 2t - 2L_t^{-1}(vu_x) + L_t^{-1}(u),$$

$v(x,t) = e^{-x} + 3t + 3L_t^{-1}(uv_x) - L_t^{-1}(v)$.(34)

Substituting the decomposition representations for linear and nonlinear into (34) yields

$$\sum_{n=0}^{\infty} u_n\,(x,t) = e^x + 2t - 2L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} u_n\right),$$

$\sum_{n=0}^{\infty} v_n\,(x,t) = e^{-x} + 3t + 3L_t^{-1}(\sum_{n=0}^{\infty} B_n) - L_t^{-1}(\sum_{n=0}^{\infty} v_n),$ (35)

Where A_n, B_n, are Adomian polynomials for the nonlinear terms vu_x, uv_x respectively.

The modified decomposition method defines the recursive relations in the form

$$u_0(x,t) = e^x,$$

$$u_1(x,t) = 2t - 2L_t^{-1}(A_0 + u_0),$$

$u_{k+1}(x,t) = -2L_t^{-1}(A_k + u_k), \;\; k \geq 1$.(36)

And

$$v_0(x,t) = e^{-x},$$

$$v_1(x,t) = 3t + 3L_t^{-1}(B_0 - v_0),$$

$v_{k+1}(x,t) = 3L_t^{-1}(B_k - v_k), \;\; k \geq 1.$ (37)

For brevity, we list the first three Adomian polynomials for A_n, B_n, as follows:

For vu_x we find

$$A_0 = v_0 u_{0_x},$$

$$A_1 = v_1 u_{0_x} + v_0 u_{1_x},$$

$$A_2 = v_2 u_{0_x} + v_1 u_{1_x} + v_0 u_{2_x},$$

And for uv_x we find

$$B_0 = u_0 v_{0_x},$$

$$B_1 = u_1 v_{0_x} + u_0 v_{1_x},$$

$$B_2 = u_2 v_{0_x} + u_1 v_{1_x} + u_0 v_{2_x},$$

Using the derived Adomian polynomials into equations (36) and (37), we obtain:

$(u_0, v_0) = (e^x, e^{-x})$

$u_1(x,t) = 2t - L_t^{-1}(1 + e^x) = te^x$

$v_1(x,t) = 3t + 3L_t^{-1}(-1 - e^{-x}) = -te^{-x}$

$u_2(x,t) = -2L_t^{-1}(A_1 + u_1) = \frac{1}{2!}t^2 e^x$

$v_2(x,t) = 3L_t^{-1}(B_1 - v_1) = \frac{1}{2!}t^2 e^{-x}$

$A_n = 0, n \geq 1$

$u_n(x,t) = \frac{1}{n!}t^n e^x, n \geq 2$

$B_n = 0, n \geq 1$

$v_n(x,t) = \frac{(-t^n)}{n!}e^{-x}, n \geq 2$

And in a closed form by:

$u(x,t) = e^{x+t}$

$v(x,t) = e^{-x-t}$ (38)

Example 4. Consider the following nonlinear system:
$u_t + vu_x - 3u = 2,$
$v_t - uv_x + 3v = 2,$(39)
With the initial conditions
$u(x,0) = e^{2x}, \quad v(x,0) = e^{-2x}$(40)
Solution.
Operating with L_t^{-1},we obtain

$$u(x,t) = e^{2x} + 2t - L_t^{-1}(vu_x) + L_t^{-1}(3u),$$

$v(x,t) = e^{-2x} + 2t + L_t^{-1}(uv_x) - L_t^{-1}(3v)$.(41)
Substituting the decomposition representations for linear and nonlinear into
(41) yields

$$\sum_{n=0}^{\infty} u_n\,(x,t) = e^{2x} + 2t - L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} 3u_n\right),$$

$\sum_{n=0}^{\infty} v_n\,(x,t) = e^{-2x} + 2t + L_t^{-1}(\sum_{n=0}^{\infty} B_n) - L_t^{-1}(\sum_{n=0}^{\infty} 3v_n),$ (42)

Where A_n, B_n, are Adomian polynomials for the nonlinear terms vu_x, uv_x respectively.
The modified decomposition method defines the recursive relations in the form

$$u_0(x,t) = e^{2x},$$
$$u_1(x,t) = 2t + L_t^{-1}(-A_0 + 3u_0),$$

$u_{k+1}(x,t) = L_t^{-1}(-A_k + 3u_k), \;\; k \geq 1$.(43)
And

$$v_0(x,t) = e^{-2x},$$
$$v_1(x,t) = 2t + L_t^{-1}(B_0 - 3v_0),$$

$v_{k+1}(x,t) = L_t^{-1}(B_k - 3v_k), \;\; k \geq 1$.(44)
We list the first four Adomian polynomials for A_n, B_n, as follows
For vu_x we find

$$A_0 = v_0 u_{0_x},$$
$$A_1 = v_1 u_{0_x} + v_0 u_{1_x},$$
$$A_2 = v_2 u_{0_x} + v_1 u_{1_x} + v_0 u_{2_x},$$
$$A_3 = v_3 u_{0_x} + v_2 u_{1_x} + v_1 u_{2_x} + v_0 u_{3_x},$$

And for uv_x we find

$$B_0 = u_0 v_{0_x},$$
$$B_1 = u_1 v_{0_x} + u_0 v_{1_x},$$
$$B_2 = u_2 v_{0_x} + u_1 v_{1_x} + u_0 v_{2_x},$$
$$B_3 = u_3 v_{0_x} + u_2 v_{1_x} + u_1 v_{2_x} + u_0 v_{3_x},$$

Using the derived Adomian polynomials into equations (43) and (44), we obtain:

$$(u_0, v_0) = (e^{2x}, e^{-2x})$$
$$u_1(x,t) = 2t + L_t^{-1}(3e^{2x} - 2) = 3te^{2x}$$

$$v_1(x,t) = 2t + L_t^{-1}(-2 - 3e^{-2x}) = -3te^{-2x}$$
$$u_2(x,t) = -L_t^{-1}(A_1) + L_t^{-1}(3u_1) = \frac{9}{2!}t^2 e^{2x} = \frac{(3t)^2}{2!}t^2 e^{2x}$$
$$v_2(x,t) = L_t^{-1}(B_1 - 3v_1) = \; 9\frac{1}{2!}t^2 e^{-2x} = \frac{(3t)^2}{2!}t^2 e^{-2x}$$
$$u_3(x,t) = -L_t^{-1}(A_2) + L_t^{-1}(3u_2) = \frac{27}{3!}t^3 e^{2x} = \frac{(3t)^3}{3!}t^3 e^{2x}$$
$$v_3(x,t) = L_t^{-1}\left(0 - \frac{27}{3!}t^2 e^{-2x}\right) = -\frac{27}{3!}t^3 e^{-2x} = -\frac{(3t)^3}{3!}t^3 e^{-2x}$$
$$u_4(x,t) = L_t^{-1}\left(0 + L_t^{-1}(3\frac{(3t)^3}{3!}t^3 e^{2x}\right) = \frac{(3t)^4}{4!}t^4 e^{2x}$$
$$v_4(x,t) = L_t^{-1}\left(0 - 3\frac{(3t)^3}{3!}t^3 e^{-2x}\right) = \frac{(3t)^4}{4!}t^4 e^{2x}$$

The solutions $u(x,t), v(x,t)$ in a series form are given by:

$$u(x,t) = e^{2x} + 3te^{2x} + \frac{(3t)^2}{2!}t^2 e^{2x} + \frac{(3t)^3}{3!}t^3 e^{2x} + \frac{(3t)^4}{4!}t^4 e^{2x} + \cdots$$

$v(x,t) = e^{-2x} - 3te^{-2x} + \frac{(3t)^2}{2!}t^2e^{-2x} - \frac{(3t)^3}{3!}t^3e^{-2x} + \frac{(3t)^4}{4!}t^4e^{2x} - \cdots$ (45)

And in a closed form by:

$u(x,t) = e^{2x+3t}$

$v(x,t) = e^{-2x-3t}$ (46)

V. Conclusion

The nonlinear partial differential equations have been solved byAdomian decomposition methods and Modified decomposition methods these method are very effective and accelerate the convergent of solution , the study showed that these methods are easy to apply and they are more accurate and effective.

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