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A New Approach to the Branch and Bound Method for Solving Assignment Problems

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Abstract: *This article addresses an assignment problem that employs a branching approach and a lower bound. To address the assignment problem, we provide a novel approach. The approach uses branch and bound decisional problem versions to solve the problem using column creation.First, a generalized assignment problem that is currently solved by a Hungarian approach using row generation is solved in this study by using column generation, and the best solution is produced, which is quite comparable to the original Hungarian solution.For the purpose of determining the lower bound, bound tree column generation is carried out at each node of the branch.*

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I. Introduction

Thischapter will cover how the branch and bound approach and assignment problem can work together.

The Generalized Assignment Problem (GAP) investigates the greatest profit assignment of n jobs to m agents such that each job is allocated to exactly one agent under the condition that the agents' capacity is limited. Its main significance comes from the fact that it appears as a substructure in many models created to solve real-world problems in fields like vehicle routing, plant location, resource scheduling, and flexible manufacturing systems. This substructure is not only interesting and useful in and of itself but also plays an important role in many other models.

It is simple to demonstrate that the GAP is NP-hard, and there is a substantial body of work on the quest for efficient enumeration algorithms to solve problems of a reasonable size to optimality[Ross and Soland 1975,Martello and Toth 1981, Fisher, Jaikumar and Van Wassenhove 1986, Guignard and Rosenwein 1989, Karabakal, Bean and Lohmann 1992]. The majority of these techniques are thoroughly discussed in a recent survey by Cattrysse and Van Wassenhove [1992].

In this study, we offer a GAP algorithm that combines column generation with branch and bind to produce the best possible integer answers to the set partitioning formulation of the issue. We describe various branching techniques that permit column formation at each branch node.

II. Definition

2.1 Combinatorial Optimization

Combinatorial problems are those involving the minimization and maximization of a function of variables subject to inequality or equality constraints as well as integrality limits on all or part of the restrictions.

2.2 Assignment Problem

The assignment problem is a fundamental combinatorial optimization issue. This particular sort of linear programming problem involves allocating different resources to different objects while minimizing the cost.

2.3 Lower Bound

The lowest cost of distributing work in assignment problems is known as the lower bound. The ultimate or ideal outcome cannot be less than the lower bound.

2.4 Branch and Bound

A technique called the branch and bound method is based on listing all potential answers to a combinatorial optimization problem. It is employed to locate the best solution in general, discrete, and combinatorial mathematics. To tackle optimization problems, the concepts of trees, logic trees, and boundaries are used.

III. Assignment Problem Using Branch and Bound Method

The Assignment Problem of $n \times n$ cost matrix of real numbers is as follows.
Consequently, the general assignment problem has a mathematical form.

Minimize

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_i = 1$$

$$\sum_{j=1}^{n} x_j = 1$$

$$x_{ij} = \begin{Bmatrix} 1 \; if \; jth \; job \; is \; assigned \; to \; the \; ith \; person \\ 0 \; otherwise \end{Bmatrix}$$

x_{ij} is the decision variable, denotes the assignment of jthjob to the ithobject and a_{ij} denotes the cost of assigning. The Assignment problem has the general form where the objective is to assigning jthjob to ith person exactly one job to one person and also the company should have its minimum cost for assigning jobs to earn more profits.

Table 1. *Cost matrix of job assignment problem*

	Job1	*Job2*	*Job3*	*Job4*	*Job j*	*Job N*
Person 1	a_{11}	a_{12}	a_{13}	a_{14}	$\ldots a_{1j}$	a_{1n}
Person 2	a_{21}	a_{22}	a_{23}	a_{24}	$\ldots a_{2j}$	a_{2n}
Person 3	a_{31}	a_{32}	a_{33}	a_{34}	$\ldots a_{3j}$	a_{3n}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
Person i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	$\ldots a_{ij}$	a_{in}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
Person N	a_{n1}	a_{n2}	a_{n3}	a_{n4}	$\ldots a_{nj}$	$\ldots a_{nn}$

3.1 Algorithm

1. By selecting the row minimum and adding up all the values, first find the lowest bound for allocating the jobs. This is the bare minimum for the issue.
2. Then, at random, assign a task to one person or machine.
3. After stepping away from the corresponding row and column where the chosen job was at random we have $(n-1)$ rows and $(n-1)$ columns. So $(n-1)*(n-1)$ matrix was created, with each column being left blank and the very lowest cost being selected from the matrix.
4. To determine the cost for each job to be assigned, repeat this process and add up the results.

3.2 Branching Guidelines

1. Assigning a task to a single person or machine, we completed one branch from this branching tree, and the lowest possible cost must be taken into account.
2. The terminal node at the lowest point should be taken into consideration for further branching if there is a tie on the lower bound.
3. If any of the terminal nodes at the $(n-1)$ th level has the lowest lower bound happiness, the optimality is achieved [16]. The assignments on the node's path to that node are then inserted, along with the two missing row-column combinations from the ideal solution.

Numerical Examples:

Example 1

A corporation has four projects that need to be completed on four different machines; any job can be completed on any machines. The time in hours that the machines require to complete certain job is listed below. To reduce the overall machine hours, assign the machines to the appropriate jobs.

9	2	7	8
6	4	3	7
5	8	1	8
7	6	2	4

Figure 1. $\mathbf{4 \times 4}$*Cost matrix for job assignment problem*

First, we need to determine the job assignment problems lowest bound. It's the minimum cost. To accomplish this, we will select one candidate for each position before eliminating the matching row and column from consideration. Then adding lowest value of each column of remaining $(n-1) \times (n-1)$ matrix. For example, if candidate 1 is given job 1, the first row and column will not be taken into account.Then pick the lowest value of 2^{nd} column excepting 1^{st} row. Then determine the lowest cost from the matrix form, eliminate the associated row and column once more, and repeat this process.

$$P_{11}^1 = 9+4+1+4 = 18$$
$$P_{12}^1 = 2+5+1+4 = 12$$
$$P_{13}^1 = 7+5+4+4 = 20$$
$$P_{14}^1 = 8+5+4+1 = 18$$

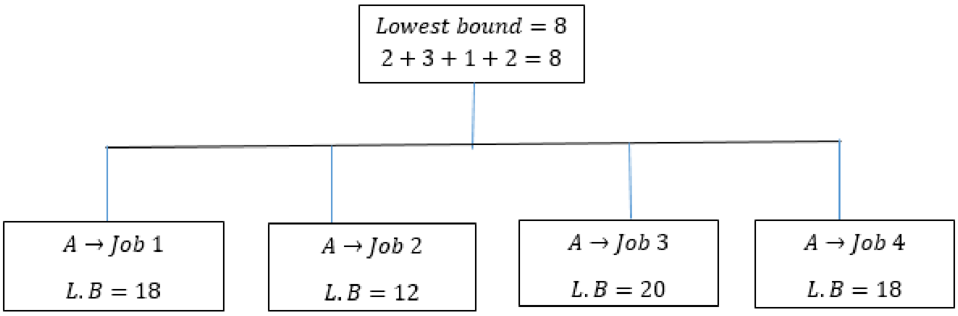

Figure 2. *Lower bound cost for assigning job to candidate A.*

Candidate A has a lower bound of 12. As a result, A should be assigned to $job2$, and further branching should begin from here. A has been assigned to job 2.Next check B for $job1$or $job3$ or $job4$ as A has been assigned to $job2$. $Job2's$ corresponding row and column would be omitted from consideration when we check for B. Following the initial branch, the following positions were left for candidates A, C, and D.

9	2	7	8
6	4	3	7
5	8	1	8
7	6	2	4

Figure 3. *Assigning job2 to candidate A.*

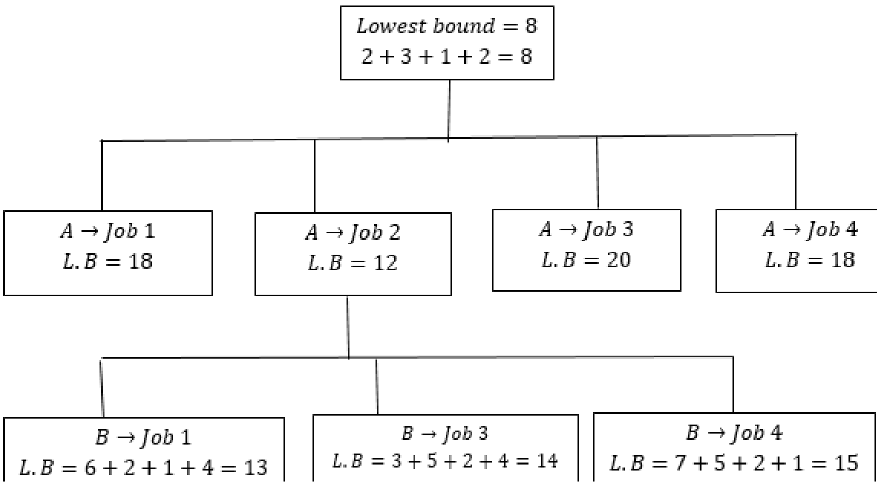

Figure 4. *Lower bound cost for assigning job to candidate B.*

Candidate B has a lower bound of 13. As a result, B should be assigned to $job1$, and further branching should begin from here. B has been assigned to $job1$.Following that, additional branching is required for candidates C and D. These jobs are still open for C and D after our second branching.

If we assign A to $job2$, B to $job1$, C to $job3$, D to $job4$ would be the job assigning for the problem for minimum cost, Minimum cost is 13. Solving it, we get the solution as with the optimal objective value 13 which represents the optimal total cost. Which is the solution of the problem using proposed branch and bound techniques.

9	2	7	8
6	4	3	7
5	8	1	8
7	6	2	4

Figure 5. *Assigning job1 to candidate B*

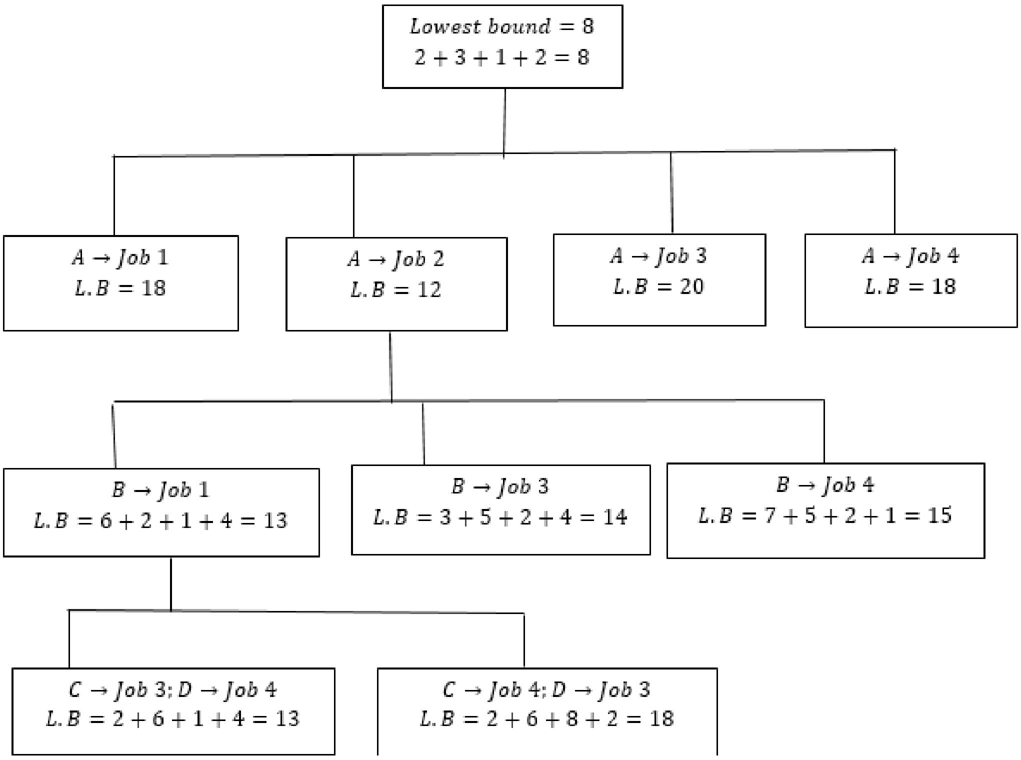

Figure 6. *Lower bound cost for assigning jobs to candidate C and D.*

IV. Comparison

The implementation of these agents allocation problem in firm was time consuming. Though the computational perception is limited, our algorithm appears to be fairly effective. Despite its simplicity, the proposed solution algorithm gives us the same solution obtained by the established Hungarian method.

V. Conclusion

In this study, a mathematically based approach is proposed to assign the agents to the organization or firm, although column operation is employed instead of the well-established Hungarian method.We obtain the same optimal value as determined by the well-established Hungarian method after performing column operations using the branch and bound technique followed by integer programming.

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