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Heat Transfer of Nanofluid Flow over a Stretching Sheet of Nanoparticle Volume Fraction

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Abstract
The present paper deals numerical analysis of heat transfer of nanofluid flow over a flat stretching sheet. Two set of boundary conditions, a constant and a linear stream wise variation of nano particle volume fraction and wall temperature were analyzed. The governing equations were reduced to a set of nonlinear ordinary differential equations, ODE's,. The dependencies of solutions on Prandtl number Pr, Lewis number Le, Brownian motion number Nb and thermophoresis number Nt were studied in detail. The results showed that the reduced Nusselt number and the reduced Sherwood number increased for the of compared to. The increase of the N_b, N_t and Le numbers caused decrease of the reduced Nusselt number; while the reduced Sherwood number increased with increase of the N_b and Le numbers. For low $Prandt_l$ numbers, increase of the N_t number caused to decrease the reduced Sherwood number; while it increased for high Prandtl numbers.

Key words *stretching sheet, nanofluid, laminar boundary layer, Brownian motion, Thermophoresis, partial differential equations, numerical solution*
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I. Introduction

The stretching sheets moving in nanofluid flows found useful applications in a wide range of manufacturing processes such as hot rolling, glass fiber production, melt-pinning, extrusion, manufacture of rubbers and plastics, cooling of large metallic plates in bathes, etc. Crane [1] investigated the boundary layer flow over an elastic flat sheet. He achieved a precise solutionfor the 2D Navier Stokes equations.

II. Review of literature

- Choi et al. [8] showed the thermal conductivity of conventional liquids increases by adding the nano-particles of metals
- Khan and Pop [9] studied the heat transfer phenomena in the steady boundary layer nanofluid flow of a stretching sheet as the surface temperature was constant.
- Bachok et al [10].Steady flow of a nanofluid developed over a flat stretching sheet while it was moving in the uniform free stream Khan and Pop's [9]
- Hassani et al. [11] studied analytically the boundary layer of flow over a long flat stretching sheet by applying the Homotopy method.
- Makinde and Aziz [12] analyzed the problem subjected to a convective boundary condition instead of an isothermal condition.
- , Oztop and Nada [13], industries such as extrusion, melt-spring, the hot rolling,
- Kuznetsov and Nield [14] have examined the influence of wire drawing, glass-fiber production, manufacture of nano particles on natural convection boundary- layer flow plastic and rubber sheets, polymer sheet and filaments are past a vertical plate.
- Hamad et al. [18] used the a viscelastic fluid. studied the application of a one-parameter group to present similarity stretching problem of an incompressible fluid over a reductions for problems of magnetic field effects on freepermeable wall.

❖ Vajravelu [6] studied flow and heat convection flow of a nanofluid past a semi-infinite vertical transfer in a viscous fluid over a nonlinear stretching flat plate following a nanofluid model proposed by sheet without viscous dissipation.

❖ Buongiorno [19]. The same nanofluid model has been Nanotechnology has been widely used in industry

III. Mathematical Method

2D boundary layer flow of a nanofluid past a flat stretching sheet investigated numerically. The stretching sheet velocity equaled $u_{w(x)}$ $=ax$, where a has a constant value and x is the coordinate surface. Two Boundary conditions were assumed for this problem. One includes a constant temperature and a constant nanoparticle volume fraction, called as and the other includes the variable temperature and nanoparticle volume fraction on the stretching surface, called as

The continuity and transport equations of momentum, thermal energy and nanoparticle volume fraction of nanofluids:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial x} u + v\frac{\partial v}{\partial y} = \vartheta \frac{\partial^2 u}{\partial x^2} \qquad (2)$$

$$\frac{\partial u}{\partial x} u + v\frac{\partial v}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{V}{c_p}\left(\frac{\partial T}{\partial x}\right)^2 + \tau\{D_B[(\frac{\partial T}{\partial x})^2 + (\frac{\partial T}{\partial y})^2\} \qquad (3)$$

$$\frac{\partial C}{\partial x} u + v\frac{dC}{dx} = D_B\left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_B}{T_\infty}\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

U= $u_w(x) = cx^n$, v=0 , T= $T_w(x) = T_\infty(x) + bx^{2n}$, C= $C_w(x)$

Ar y=o u $\rightarrow 0$, $T \rightarrow T_\infty(x)$, C= $C_\infty(x)$as y $\rightarrow \infty$

The boundary conditions for the first case were considered as:

$$\eta = y\sqrt{c(n+1)/2v_f} \qquad (6)$$

U= $cx^n F(\eta)$

V= $-\sqrt{\frac{(n+1)cv_f)}{2}}\ x^{\frac{n-1}{2}[F(\eta)+\eta F'(\eta)\frac{n-1}{n+1}}$

while, for the second case the boundary conditions were as follows:

$$\theta(\eta) = \frac{(T-T_\infty)}{(T_w-T_\infty)} \quad \phi(\eta) = \frac{(C-C_\infty)}{(C_w-C_\infty)} \qquad (7)$$

$$f''' + FF'' - \frac{2n}{n+1}f'^2 = 0 \qquad (8)$$

$$\frac{1}{P_r}\theta'' + F\theta' - \frac{4n}{n+1}F'\theta + Nb\theta'\phi' + Nt\theta'^2 + E_cF''2 = 0 \qquad (8)$$

$$\theta'' + Le\,F\theta' + \frac{Nt}{Nb}\theta'' = 0 \qquad (9)$$

F=0,F'=1 $\theta = 1, \phi = 1 \quad at \quad \eta = 0$

F'$\rightarrow 0$, $\theta \rightarrow 0, \emptyset = 0 as\eta \rightarrow \infty$, $P_r = \frac{v}{k}$, Le= $\frac{v}{D_B}$, Nb = $\tau D_B \ \frac{(C-C_\infty)}{v}$, NT= $\tau D_\tau \ \frac{(T_w-T)}{vT}$:$E_c = \frac{u_w^2}{C_p((T_w-T_\infty)}$

$$Nu_x = \frac{-x}{(T_w-T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}\left(\frac{\partial T}{\partial y}\right)y = o$$

$$Sh_x = \frac{x}{(C_w-C_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}\left(\frac{\partial c}{\partial y}\right)y = o$$

$$\text{Re}x^{1/2} N_u = -\left(\frac{n+1}{2}\right)^{1/2}\theta'(0)$$

$$\text{Re}x^{1/2} S_h = \left(\frac{n+1}{2}\right)^{1/2}\theta'(0)$$

Where $Re_x = xu_x/v_f$

For simplifying the mathematical complexity of the problem, similar to the procedure thatused for transforming the basic governing equations of fluid flow and heat transfer to a set of coupled non-linear ordinary differential equations, a similarity solution of Eqs. (1) - (5) subjected to the above boundary conditions

can be defined in the following form: (8) where the stream function ψ is defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. For extracting the similarity solution, it was considered that in the outer flow (inviscid), the pressure was $P = P_0$ (constant). By substituting the variables defined as Eq. (8) by Eqs. (2)-(5) and taking into account two different boundary conditions, the following ordinary differential equations were observed for both cases: The equations considering the first boundary conditions (6) were: while the equations considering the second case of boundary conditions (7) were:Both sets of these equations were subjected to the following forms of boundary conditions: So, using Eq. (8), two different types of equations with the same boundary condition mentioned above were derived. In the Eq. (15), primes denoted a differentiation respect to η.So the fluid flow parameters were defined by:

Based on the above quantities, the Sherwood number and Nusselt number were defined as: where q_w and q_m are the heat and mass fluxes of the wall, respectively. Using variables defined in Eq. (8), the reduced Sherwood number and Nusselt number equations were obtained as:

IV. Methodology And Solution Of The Problem:

We have applied free parameter method to solve governing ODE equations to find similarity solution. The free parameter method is the way of finding "Similarity Solution" of ODE by assuming that the dependent variables in the equations are to express in terms "similarity parameters" known as similarity variables, finally a single variable. Similarity variables must be constructed in such way that the number of independent variables that occur in the equations reduced by one (at least) from the total number of the independent variables. The governing boundary layer equations (2) - (3) subject to boundary conditions (4) and (5) are solved numerically by using shooting method. First of all higher order non-linear differential equations (2) -(3) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem. The corresponding velocity and temperature profiles

IV. Result And Discussion

In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in Figs., to illustrate the influence of physical parameters viz., magnetic parameterMp , Prandtl number Pr Eckert number Ec ,Unsteadiness parameter and variation (exponent) τ on the velocity f' and temperature θ. The profiles for velocity and temperature are shown in fig.

Effect for velocity profile:

4.1 **Effect of the Prandtl number:** From the fig.1 it is observed that the velocity $f'(0)$ decreases as the power-law index of the surface temperature variation (exponent) τ and the magnetic parameter(Mp) increases with Prandtl number Pr = 0.05

4.2 **Effect of the viscosity Parameter:.** It is that the velocity decreases as the power-law index of the surface temperature variation τ and the magnetic parameter (Mp) increases with the variable viscosity parameter $\theta = 3.0$

4.3 **Effect of the Eckert number:**, it is observed that the velocity profiles are almost identical for different values of temperature variation τ and the magnetic parameter (Mp) with Eckert number Ec = 0.0 andEc = 0.1

4.4 Effect of the Magnetic Parameter: Iit is observed that the velocity decreases as magnetic parameter (Mp) increases. Effect for temperature profile:

4.5 **Effect of variation (exponent):** it is observed that as variation (exponent) τ increases the temperature decreases for fixed value of magnetic parameter (Mp).

4.6 Effect of the Magnetic Parameter: that the temperature decreases as magnetic parameter Mp increases.

4.7 Effect of the Prandtl and Eckert number: it is observed that the temperature decreases as the magnetic parameter increases with Pr=6.8, $\tau = 0.3$ and Ec = 0.1

4.8 Effect of Heat transfer: that as Mp increases the heat transfer rate $-\theta'(0)$, decreases but as τ increases the heat transfer rate increases.That is why, the parameters τ and Mp have considerable influence on the heat transfer rate $-\theta'(0)$.

4.9 Effect of the Unsteadiness Parameter: it is observed that the velocity profiles are approximately symmetrical for different values of unsteadiness parameter$A1$, temperature variation τ and the magnetic parameter (Mp) with Pr = 6.8, Eckert number Ec = 0.1,$A1 = 0.5$. $A1 = 0.5$ and $\theta = 3.0$

Table 1: Comparison of results for

n	Present results	Cortell [27]
0.0	0.628316	0.6276
0.2	0.767489	0.7668
0.5	0.890103	0.8895
1	1.000484	1.0000
3.0	1.148986	1.1486
10	1.235220	1.2349
20	1.257758	1.2574

Table 2: Comparison of results for $-\theta'(0)$ when $Nb = 0$, $Nt = 0$ and $Pr = 5$

	$E_c = 0$		$E_c = 0.1$	
n	Present results	Cortell [27]	Present results	Cortell [27]
0.75	3.125152	3.1250	3.016658	3.0170
1.5	3.567797	3.5677	3.455594	3.4557
7.0	4.185335	4.1854	4.065541	4.0657
10	4.255853	4.2560	4.135057	4.1353

Table 3: Values of $-\theta'(0)$ and $\phi'(0)$ when Pr=10, n=10, Le=10 and Nt=0.3

	$-\theta'(0)$		$\lvert\phi'(0)\rvert$	
Nb	$E_c = 0$	$E_c = 0.1$	$E_c = 0$	$E_c = 0.1$
0.1	3.771628	3.658830	5.621979	5.345776
0.2	3.251502	3.149618	0.998139	0.875024
0.3	2.827929	2.735630	0.451703	0.525092

Table 4: Values of $-\theta'(0)$ and $\phi'(0)$ when Pr=10, n=10, Nb=0.3 and Nt=0.3

	$-\theta'(0)$		$-\phi'(0)$	
Le	$E_c = 0$	$E_c = 0.1$	$E_c = 0$	$E_c = 0.1$
10	2.827929	2.735630	0.451703	0.525092
20	2.311415	2.234830	2.159347	2.213024
30	2.039608	1.971317	3.303876	3.347141

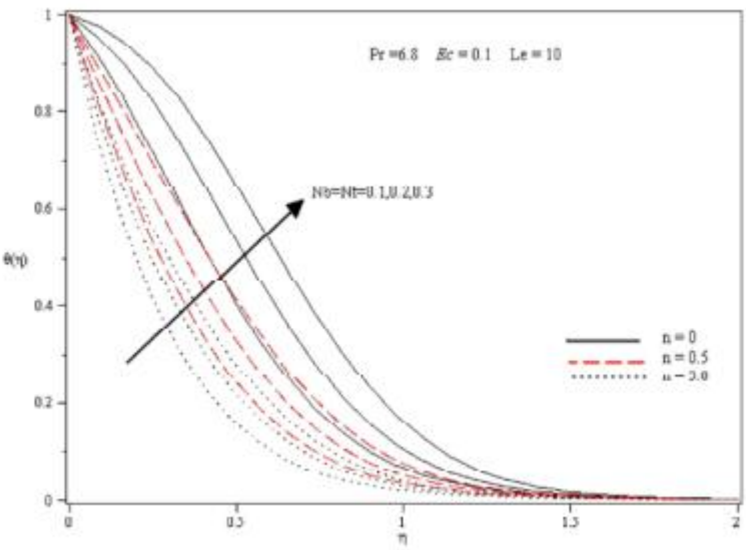


Fig 1 :Effect of Ntand Nb on Temperature Distribution forvirous n=0.0.0.5.1.3.0.10.20

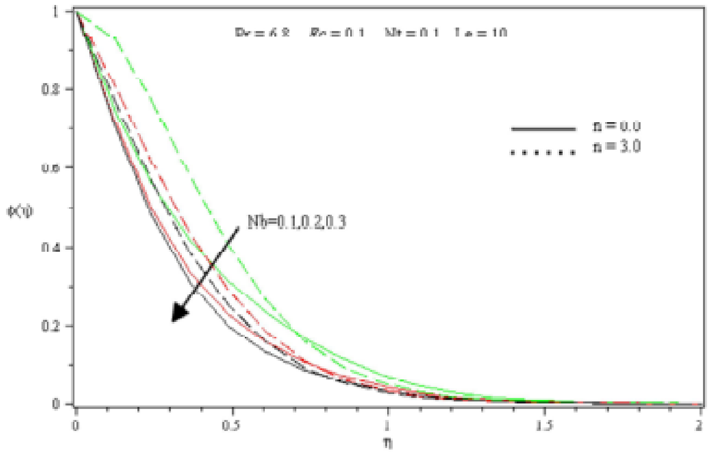
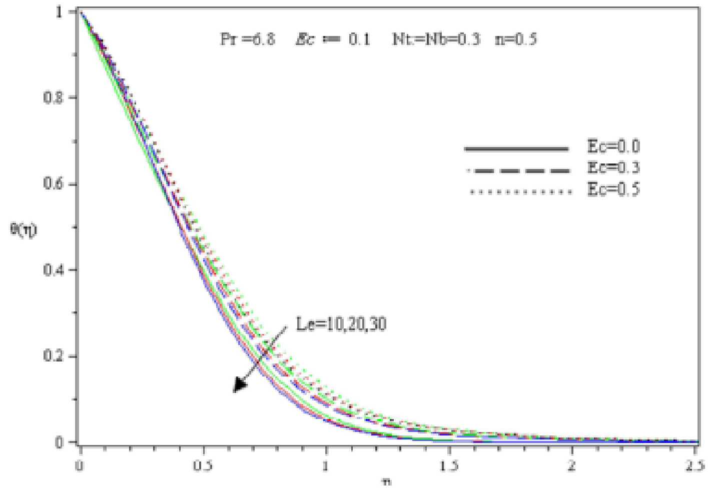


Fig. 2: Effect of Nb on concentration distribution for virous n when n =0 and n=0.3.



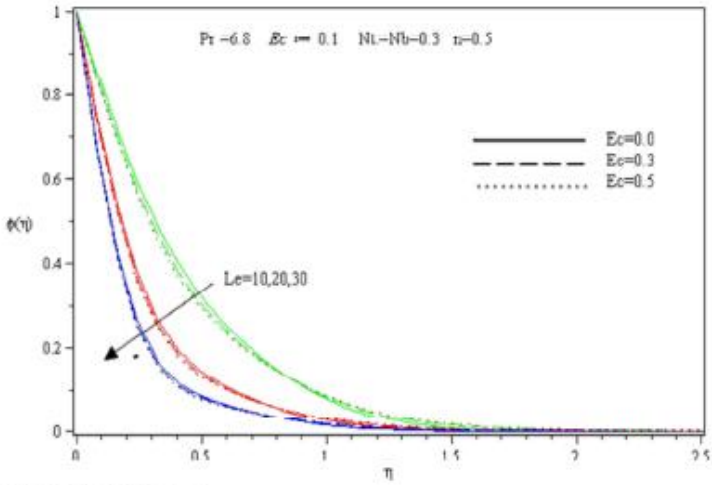


Fig. 4: Effect of Le and Ec on concentration.

V. Conclusions

1. In this work the heat transfer of boundary layer nanofluid flow developed over a continuous flat stretching sheet moving in a quiescent flow has been investigated numerically.
2. The governing equations were transformed to a boundary value problem in similarity variables and then the new set of equations was solved numerically.

3. Two different cases were investigated which included stretching sheet with constant temperature and nanoparticle volume fraction and variable temperature and nanoparticle volume fraction.
4. The novelty of this work is to use a linear variation of surface temperature and nanoparticle volume fraction in respect to previous works.
5. The reduced Sherwood number and the reduced Nusselt number depended on the Prandtl number Pr, Lewis number Le, Brownian motion number Nb and thermophoresis number Nt.
6. Thickness of thermal boundary layer was an increasing function of Nb and Nt; however, it was a decreasing function for Pr number. The thickness of boundary layer of the nanoparticle volume fraction was a decreased as the Nb or Le increased.
7. Both boundary layer thicknesses of thermal and volume fraction were lower as temperature and nanoparticle volume fraction varied at the stretching sheet compared . So, higher values were caused for the reduced Nusselt and Sherwood numbers.
8. Reduced Nusselt number was decreased as the Nb, Nt and Le numbers increased. However, there were three distinct regions as Pr number increased based on the amount of Nb. For Nb=0.1, reduced Nusselt number increased with the increase of Pr number. But, for Nb=0.5, reduced Nusselt number increased at first and then decreased. In contrast, for Nb=1, reduced Nusselt number decreased with the increase of Pr number.
9. Reduced Sherwood number increased with the increase of Nb. However, for Nb=0.1, reduced Sherwood number decreased as Nt increased. But, for Nt>0.1, as Nt increased, reduced Sherwood number was constant.
10. Reduced Sherwood number increased with the increase of Pr number. For Pr=1, reduced Sherwood number decreased as Nt increased and, for Pr >1, it increased with the increase of Nt.
11. Reduced Sherwood number increased as Le number increased. When Le number was constant, reduced Sherwood number was constant with the increase of Nt.

Nomenclature

C volume fraction of nanoparticles T_w temperature of the stretching sheet
$C\infty$ volume fraction of nanoparticles at ambient fluid T_∞ temperature of the ambient fluid
Cw volume fraction of nanoparticles at the stretching surface u, v velocity components along x- and yaxes
D_B Brownian diffusion coefficient u_w stretching sheet velocity
D_T Thermophoresis diffusion coefficient x, y Cartesian coordinate
f(η) dimensionless stream function

Greek Symbols

k thermal conductivity α thermal diffusivity
Le Lewis number *(η)* rescaled nanoparticle volume fraction
Nb Brownian motion number *η* similarity variable
Nt thermophoresis number *θ(η)* dimensionless temperature
Nu Nusselt number *υ* fluid kinematic viscosity
T local temperature *ρf* density of base fluid
P local pressure *ρp* mass density of nanoparticles
Pr Prandtl number $(\rho C)_f$ heat capacity of base fluid
qm mass flux of the wall $(\rho C)_p$ heat capacity of nanoparticles
q_w heat flux of the wall ψ stream function
Rex Reynolds number, locally τ ratio between the heat capacity of the nanoparticles and heat capacity of the base fluid
Sh_x Sherwood number, locally
$\mathrm{Re}x^{1/2}\, N_u \;=\; -\,\theta'(0)$ Reduced Nusselt number
$\mathrm{Re}x^{1/2}\, S_h \;=\; -\,\theta'(0)$ Reduced Sherwood number

References

[1]. Crane, L. J. Flow past a stretching plate, *ZAMP- J. Appl. Mathematics and Physics*, **21**(4), 645–647, (1970)

[2]. Wang, C.Y. The three-dimensional flow due to a stretching flat surface, *Physics of Fluids*, **27**, 1915–1917 (1984)

[3]. Lakshmisha, K.N., Venkateswaran, S., and Nath G. Three-dimensional unsteady flow with heat and mass transfer over a continuous stretching surface, *ASME J. Heat Transfer*, **110**(3), 590–595 (1988)

[4]. Andersson H. I., Bech, K. H. and Dandapat B.S. Magnetohydrodynamic Flow of a power-law fluid over a stretching sheet, *Int. J. Non-Linear Mechanics*, **27**(6), 929–936 (1992)

[5]. Magyari E., and Keller, B. Exact solutions for self-similar boundary-layer flows induced by permeable stretching walls, *Eur. J. Mechanics-B/Fluids*, **19**(1), 109–122 (2000)

[6]. Abraham, J. P., and Sparrow, E. M. Friction drag resulting from the simultaneous imposed motions of a freestream and its bounding surface, *Int. J. Heat Fluid Flow*, **26**(2), 289–295 (2005).

[7]. Sparrow, E. M., and Abraham, J. P., Universal solutions for the streamwise variation of the temperature of a moving sheet in the presence of a moving fluid, *Int. J. Heat Mass Transfer*, **48**(15), 3047–3056 (2005)

[8]. Choi, S. U. S., Zhang, Z. G., Yu, W., Lockwood, F. E., and Grulke, E.A. Anomalously thermal conductivity enhancement in nanotube suspensions, *Applied Physics Letters*, **79**(14), 2252–2254 (2001)

[9]. Khan, W. A., and Pob, I. Boundary- layer flow of a nanofluid past a stretching sheet, *Int. J. Heat and Mass Transfer*, **53**(11), 2477-2483 (2010)

[10]. Bachok, N., Ishak, A., and Pob, I. Boundary- layer flow of nanofluids over a moving surface in a flowing fluid, *Int. J. Thermal Sciences*, **49**(9), 1663-1668 (2010)

[11]. Hassani, M., Mohammad Tabar, M., Nemati, H., Domairry, G. , Noori, F., An analytical solution for boundary layer flow of a nanofluid past a stretching sheet, *Int. J. Thermal Sciences*, **50**(11), 2256–2263 (2011)

[12]. Makinde, O. D., Aziz, A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition, *Int. J. Thermal Sciences*, **50**(7), 1326-1332 (2011)

[13]. Gireesha, B. J., Roopa, G. S., Bagewadi, C. S., Effect of viscous dissipation and heat source on flow and heat transfer of dusty fluid over unsteady stretching sheet, *Appl. Mathematics and Mechanics* (*Engl. Ed.*),, **33**(8), 1001-1014, (2012)

[14]. Hamad, M. A. A., Ferdows, M., Similarity solutions to viscous flow and heat transfer of nanofluid over nonlinearly stretching sheet, *Appl. Mathematics and Mechanics* (*Engl. Ed.*), **33**(7), 923-930, (2012)

[15]. Wang, C. Y. Free convection on a vertical stretching surface, *ZAMM- J. Appl. Mathematics and Mechanics*, **69**(11), 418–420 (1989)

[16]. Gorla, R. S. R., Sidawi, I. Free convection on a vertical stretching surface with suction and blowing, *Applied Scientific Research*, **52**(3), 247–257 (1994)

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