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New Class of Generalized Closed Sets in Vague Topological Spaces

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Abstract: *In this paper, the basic concepts of vague sets are reviewed and the concepts of vague regular alpha generalized closed sets in vague topological spaces are introduced. The basic properties of vague regular alpha generalized closed sets and their relation with other sets are discussed. Also some absorbing results are established with relevant examples.*

Keywords: *vague topology, vague regular α generalized closed set, vague regular α generalized open set.*

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I. Introduction

In recent years, many researches on vague set theory have been active and great progress has been achieved. Which are an extension of fuzzy set theory and the idea of vague set is welcome because it handles uncertainty and vagueness. In 1994, Gaw and Buchere [5] firstly introduce some definitions of related operations on vague sets and the idea of vague sets is that the membership of every element can be split into two aspects including true membership and false membership. Mariapresenti and Arockiarani[8] further investigated some new operations for vague sets is a vague generalized alpha closed sets in topological spaces and also the concept of vague topological sets and vague topological additive groups introduced by Amarendra Babu, Ahmed Allam, Anitha, Rama Rao [1]. Here we introduce a new class of vague generalized closed set namely, vague regular α generalized closed set and some of their properties are obtained, which lies between vague regular generalized closed set and vague generalized alpha closed set.

II. Preliminaries

Definition 2.1: [8] Let X be the universe of discourse. A vague set is an object having the form A= {<x, [$t_A(x)$, 1-$f_A(x)$] >/x ϵ X} is represented by a true membership function t_A and false membership function f_A. Where $t_A(x)$ is the lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by a sub interval [$t_A(x)$, 1-$f_A(x)$] of [0, 1]. This expresses that if the actual grade of membership μ(x), then $t_A(x) \leq \mu(x) \leq f_A(x)$.

Definition 2.2: [10] Consider a two vague sets A and B of the form A = {< x, [$t_A(x)$, 1-$f_A(x)$] > / x ϵ X} and B = {< x, [$t_B(x)$, 1-$f_B(x)$] > / x ϵ X}

Then the following properties are given below

i) $t_A(x) \leq t_B(x)$ and $1- f_A(x) \leq 1-f_B(x)$ for all x ϵ X $\longleftrightarrow$ A ⊆ B
ii) A = B $\longleftrightarrow$ A ⊆ B and B ⊆ A
iii) A^C = {<x, $f_A(x)$,1-,$t_A(x)$>/x ϵ X}
iv) A ∩ B={<x, min($t_A(x)$, $t_B(x)$), min(1- $f_A(x)$, 1-$f_B(x)$)>/x ϵ X}
v) A ∪ B={<x, max($t_A(x)$,$t_B(x)$), max(1-$f_A(x)$,1-$f_B(x)$)>/x ϵ X}

Definition 2.3: [8] A vague topology on X satisfies the following axioms

i) 0,1ϵ τ
ii) $G_1 \cap G_2$ ϵ τ any G_1, G_2 ϵ τ
iii) $\cup G_i$ ϵ τ for any {G_i : i ϵ J} ϵ τ

In this case the pair (X, τ) is a vague topological space and any vague set A in τ is known as a vague open set (VOS) in X. A vague set A is a vague closed set (VCS) iff its compliment is a vague open set in X.

Definition 2.4 : [8] The vague interior and the vague closure of A are defined by Vint(A) = ∪ {G: G is an VOS and G ⊆ A }
Vcl(A) = ∩{K: K is an VCS in X and A ⊆ K }

Definition 2.5 : [10]

i) A vague regular closed set (VrCS) if A = Vcl(Vint(A)).
ii) A vague α closed set (VαCS) if Vcl(Vint(Vcl(A))) ⊆ A.
iii) A vague generalized closed set (VGCS) if Vcl(A) ⊆ U whenever A ⊆ U.
iv) A vague generalized semi closed set (VGSCS) if Vscl(A) ⊆U whenever A ⊆ U.
v) A vague generalized pre closed set (VGPCS) if Vpcl(A) ⊆ U whenever A ⊆ U.

Definition 2.6:

i) A vague generalized α closed set (VGαCS) [8] if Vαcl(A) ⊆ U whenever A⊆ U.
ii) A vague regular generalized closed set (VrGCS) [1] if Vcl(A) ⊆U whenever A⊆ U.

Definition 2.7: [10] The vague α interior and the vague α closure of A are defined by

i) Vαcl(A) = A ∪Vcl(Vint(Vcl(A))).
ii) Vαint(A) =A∩Vint(Vcl(Vint(A))).

Result 2.8: [11]

Every CS, GCS, GαCS is an gαr closed set but not conversely in general.

III. VAGUE REGULAR α GENERALIZED CLOSED SET

Definition 3.1: A vague set A in a vague topological space is said to be vague regular α generalized closed set (VrαGCS), if Vαcl(A) ⊆ U whenever A ⊆ U and U is a vague regular open set in X.

Example 3.2 : Let X = {a, b} and τ ={0,G,1} where G ={<x,[0.2,0.7],[0.4,0.6]>}. Here the vague set A = {< x, [0.3,0.7],[0.4,0.6]>} is a vague regular α generalized closed set in X. Since A⊆ G and U is VrOS. We have Vαcl(A) = A ∪ Vcl(Vint(Vcl(A)) ⊆ G.

Theorem 3.3:

(i) Every VCS is a VrαGCS in X but not conversely.
(ii) Every VrCS is a VrαGCS in X but not conversely.
(iii) Every VαCS is a VrαGCS in X but not conversely.
(iv) Every VGCS is a VrαGCS in X but not conversely.
(v) Every VrGCS is a VrαGCS in X but not conversely.
(vi) Every VGαCS is a VrαGCS in X but not conversely.
(vii) Every VGPCS is a VrαGCS in X but not conversely.
(viii) Every VGSCS is a VrαGCS in X but not conversely.
(ix) Every VGbCS is a VrαGCS in X but not conversely.

Proof :

(i) Let U be a VrOS in X such that A ⊆ U. Since A is a VCS,
Vcl(A) =A. By hypothesis, Vαcl(A) = A∪Vcl(Vint(Vcl(A))) =A∪Vcl(Vint(A))
⊆A∪Vcl(A)=A∪A=A⊆ U .Thus A is a VrαGCS in X.

(ii) Let U be a VrOS in X such that A ⊆U. Since every VrCS is a
VCS, Vcl(A) = A. By hypothesis, Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ U. Hence A is a VrαGCS in X.

(iii) Let U be a VrOS in X such that A ⊆ U. Since A is a VαCS in X, Vcl(Vint(Vcl(A))) ⊆ A. By hypothesis, Vαcl(A) = A∪Vcl(Vint(Vcl(A)))) ⊆ U. Hence Vαcl(A) ⊆ U and A is a VrαGCS in X.

(iv) Let U be a VrOS in X such that A ⊆ U. Since A is a VGCS in X, vcl(A)) ⊆ U,
Whenever A ⊆ U. By hypothesis, Vαcl(A) = A∪Vcl(Vint(Vcl(A)))) ⊆U. Hence A is aVrαGCS in X.

(v), (vi), (vii), (viii) and (ix) are obvious.

It can be shown by the following examples.

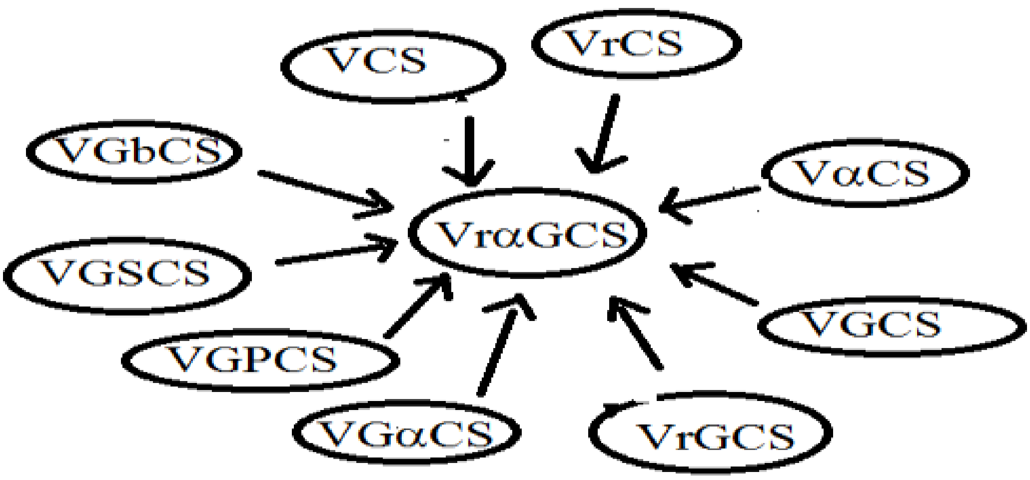
Example 3.4: Let X = {a,b} and τ = {0,1,G} where G ={<x,(0.3,0.5),(0.6,0.7)>}. Then the vague set A = {<x,(0.2,0.3),(0.3,0.5)>} is a VrαGCS. We have Vαcl(A) =
A∪Vcl(Vint(Vcl(A))) ⊆ G. But since Vcl(A) = 1 ≠ A, A is not a vague closed set in X.

Example 3.5: Let X = {a,b} and τ = {0,1,G} where G ={<x,(0.4,0.6),(0.5,0.7)>}. Then the vague set A = {<x,(0.2,0.2),(0.1,0.1)>} is a VrαGCS. We have Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ G. But since Vcl(Vint(A) = G^C ≠ A, A is not a vague regular closed set in X.

Example 3.6:Let X = {a,b} and τ = {0,1,G} where G ={<x,(0.2,0.3),(0.4,0.5)>}. Then the vague set A = {<x,(0.6,0.7),(0.5,0.6)>} is a VrαGCS. We have Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ G whenever A ⊆ G. But since Vcl(Vint(Vcl(A))) = 1 ≠ A, A is not a vague α closed set in X.

Example 3.7: Let X= {a,b} and τ = {0,1,G} where G ={<x,(0.4,0.2),(0.5,0.8)>}.Then the vague set A={<x,(0.3,0.6),(0.4,0.5)>} is a VrαGCS. We have Vαcl(A)=A∪Vcl(Vint(Vcl(A))) ⊆G. But since Vcl(A) = 1 ≠ A, A is not a vague generalized closed set in X.
Example 3.8: Let X= {a, b} and τ = {0,1,G} where G ={<x,(0.3,0.5),(0.4,0.6)>}. Then the vague set A = {<x,(0.5,0.8),(0.4,0.7)>} is a VrαGCS. We have Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ G. But since Vcl(A) = 1 ≠ A, A is not a vague regular generalized closed set in X.
Example 3.9: Let X= {a, b} and τ = {0,1,G} where G ={<x,(0.6,0.5),(0.4,0.2)>}.Then the vague set A = {<x,(0.5,0.8),(0.7,0.8)>} is a VrαGCS. We have Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ G. But since Vαcl(A) ≠ A, A is not a vague generalized α closed set in X.
Example 3.10: Let X= {a, b} and τ = {0,1,G} where G ={<x,(0.3,0.6),(0.4,0.6)>}.Then the vague set A = {<x,(0.3,0.5),(0.2,0.8)>} is a VrαGCS. We have Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ G. But since Vpcl(A) = 1 ≠ A, A is not a vague generalized pre closed set in X.
Example 3.11: Let X= {a, b} and τ = {0,1,G} where G ={<x,(0.2,0.4),(0.2,0.8)>}.Then the vague set A = {<x,(0.2,0.4),(0.3,0.7)>} is a VrαGCS. We have Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ G. But since Vscl(A) = 1 ≠ A, A is not a vague generalized semi closed set in X.
Example 3.12 : Let X= {a, b} and τ = {0,1,G} where G ={<x,(0.2,0.4),(0.6,0.8)>}.Then the vague set A = {<x,(0.5,0.6),(0.3,0.4)>} is a VrαGCS. We have Vαcl(A) = A∪Vcl(Vint(Vcl(A))) ⊆ G. But since Vbcl(A) = 1 ≠ A, A is not a vague generalized b closed set in X.
Remark 3.13: The following diagrammatic representation gives a relation between various types of vague closed sets.
Here A→B means A implies B but not conversely.
VCS – vague closed set, VrCS – vague regular closed set, VαCS – vague α closed set, VGCS – vague generalized closed set, VrGCS – vague regular generalized closed set, VGαCS – vague generalized α closed set, VGPCS – vague generalized pre closed set, VGSCS – vague generalized semi closed set, VGbCS – vague generalized b closed set.

Figure 1- Vage regular α generalized closed set



Theorem 3.14: The union of two vague regular α generalized closed set is a vague regular α generalized closed set in X.
Proof: Let A and B be a vague regular α generalized closed set in X. Let A∪B ⊆ U and U is a vague regular open set in X, where A ⊆ U and B ⊆ U. Then Vαcl(A∪B) =(A∪B)∪(Vcl(Vint(VclA∪B)))⊆(A∪B)∪(Vcl((A∪B))⊆Vcl((A∪B)=Vcl(A)∪Vcl(B)⊆U. Hence A∪B is also a vague regular α generalized closed set in X.
Theorem 3.15: If A is both a vague regular open set and vague regular α generalized closed set in X. Then A is a vague regular generalized closed set in X.
Proof: Let A ⊆ U and U be a vague regular open set in X. By hypothesis we have Vαcl(A)⊆U and Vcl(A)⊆ Vcl(Vint(Vcl(A)))⊆ A∪Vcl(Vint(Vcl(A)))= Vαcl(A)⊆U. Hence A is a vague regular generalized closed set in X.
Theorem 3.16: If A is both a vague pre open set and vague regular α generalized closed set in X. Then A is a vague regular generalized closed set in X.

Proof: Let A ⊆ U and U be a vague regular open set in X. By hypothesis we have Vαcl(A)⊆U and Vcl(A)⊆ Vcl(Vint(Vcl(A)))⊆ A∪Vcl(Vint(Vcl(A)))= Vαcl(A)⊆U. Hence A is a vague regular generalized closed set in X.

Theorem 3.17: If A is both a vague regular open set and vague regular α generalized closed set in X. Then A is a vague α closed set in X.

Proof: As A ⊆A, by hypothesis Vαcl(A)⊆A. But we have A⊆Vαcl(A). This implies A= Vαcl(A). Hence A is a vague α closed set in X.

Theorem 3.18: Let A be a vague regular α generalized closed set in X and A ⊆B⊆ Vαcl(A). Then B is a vague regular α generalized closed set in X.

Proof: Let B⊆U and U is a vague regular open set in X. Then A ⊆ U, since A ⊆ B. As A is a vague regular α generalized closed set in X, Vαcl(A)⊆U and by hypothesis B⊆ Vαcl(A). This implies Vαcl(B)⊆ Vαcl(A) ⊆U. Therefore Vαcl(B)⊆U and hence B is a vague regular α generalized closed set in X.

Theorem 3.19: Let A be a vague regular generalized closed set in X and A ⊆ B ⊆ Vcl(A). Then B is a vague regular α generalized closed set in X.

Proof: Let B ⊆ U and U is a vague regular open set in X. Then A ⊆ U, since A ⊆ B. As A is a vague regular generalized closed set in X, Vcl(A) ⊆ U and by hypothesis B ⊆ Vcl(A). This implies Vαcl(B) ⊆ Vcl(B) ⊆ Vcl(A) ⊆ U. Therefore Vαcl(B) ⊆ U and hence B is a vague regular α generalized closed set in X.

IV. VAGUE REGULAR ALPHA GENERALIZED OPEN SET

Definition 4.1: A vague set A in a vague topological space (X, τ) is said to be vague regular α generalized open set(VrαGOS), if Vαint(A) ⊇U whenever A ⊇ U and U is a vague regular closed set in X. The family of all vague regular α generalized open set of a vague topological space is denoted by VrαGO(X).

Example 4.2: Let X={a, b}and τ = {0,G,1}, where G= {< x,(0.4,0.1),(0.1,0.5)>}. Then the vague set A ={< τ,(0.1,0.2),(0.1,0.1)>} is called VrαGOS in X. Since A⊇G^C and G^C is a vague regular closed set. We have Vαint(A) = A ∩ Vint(Vcl(Vint(A))) = G ⊇G^C .

Theorem 4.3:

(i) Every VOS is a VrαGOS in X but not conversely.

(ii) Every VrOS is a VrαGOS in X but not conversely.

(iii) Every VαOS is a VrαGOS in X but not conversely.

(iv) Every VGOS is a VrαGOS in X but not conversely.

Proof: (i) Let U be a VrCS in X such that A ⊇ U. Since A is a VOS, Vint(A) = A. By hypothesis, Vαint(A) = A∩Vint(Vcl(Vint(A))) = A∩Vint(Vcl(A)) ⊇ A∩Vint(A) = A∩A = A ⊇ U. Thus A is a VrαGOS in X.

(ii), (iii) and (iv) are obvious.

It can be shown by the following examples

Example 4.4: Let X={a, b}and τ= {0,G,1},where G= {< x,(0.5,0.4),(0.1,0.1)>} then the vague set A = {< τ,(0.6,0.7),(0.1,0.1)>) is a VrαGOS in X. Since A ⊇ G^C and we have vαint(A) = A∩Vint(Vcl(Vint(A))) = G ⊇G^C. But since Vint(A) = G ≠A, A is not a vague open set in X.

Example 4.5: Let X={a, b}and τ={0,G,1},where G={< x,(0.5,0.4)(0.1,0.1)>} then the vague set A= {< τ,(0.6,0.7),(0.1,0.1 0.3)>} is a VrαGOS. We have vαint(A) = A∩Vint(Vcl(Vint(A))) = G ⊇ G^C , whenever A ⊇ G^C . But since Vint(Vcl(A)) = 1 ≠ A, A is not a vague regular open set in X.

Example 4.6: Let X={a, b}and τ={0,G,1},where G={< x,(0.2,0.4)(0.2,0.1)>} then the vague set A = {< τ,(0.4,0.7), (0.3,0.1)>} is a VrαGOS. We have vαint(A) = A∩Vint(Vcl(Vint(A)))=G⊇G^C, whenever A ⊇ G^C. But since Vint(Vcl(Vint(A))) = G ≠ A, A is not a vague α open set in X.

Example 4.7: Let X={a, b}and τ={0,G,1},where G= {< x,(0.1,0.4),(0.2,0.1)>} then the vague set A= {< τ,(0.2,0.7), (0.3,0.7)>} is a VrαGOS, we have vαint(A) = A∩Vint(Vcl(Vint(A))) = G ⊇ G^C , whenever A ⊇ G^C. But since Vint(A) = G ≠ A, A is not a vague generalized open set in X.

Theorem 4.8: The intersection of two vague regular α generalized open set is a vague regular α generalized open set in X.

Proof: Let A and B be a vague regular α generalized open sets in X. Let A ∩ B ⊇ U and U is a vague regular closed set in X, where A ⊇ U and B ⊇ U. Then Vαint(A ∩ B) = (A ∩ B) ∩ (Vint(Vcl(Vint(A ∩ B))) ⊇ (A∩B) ∩ (Vint((A ∩ B)) ⊇ (A ∩ B) ∩ Vint(A) ∩ Vint(B) ⊇ U. Hence A ∩ B is also a vague regular α generalized open set in X.

Theorem 4.9: If A is both a vague regular closed set and vague regular α generalized open set in X. Then A is a vague regular generalized open set in X.

Proof: Let A ⊇ U and U be a vague regular closed set in X. By hypothesis we have Vαint(A) ⊇ U and Vint(A) = Vint(Vcl(Vint(A))) ⊇ A ∩ Vint(Vcl(Vint(A)))= Vαint(A) ⊇ U. Hence A is a vague regular generalized open set in X.

Theorem 4.10: If A is both a vague pre closed set and vague regular α generalized open set in X. Then A is a vague regular generalized open set in X.

Proof: Let A ⊇ U and U be a vague regular closed set in X. By hypothesis we have Vαint(A) ⊇ U and Vint(A) ⊇ Vint(Vcl(Vint(A))) ⊇ A ∩ Vint(Vcl(Vint(A)))= Vαint(A) ⊇ U. Hence A is a vague regular generalized open set in X.

Theorem 4.11: If A is both a vague regular closed set and vague regular α generalized open set in X. Then A is a vague α open set in X.

Proof: As A ⊇ A, by hypothesis Vαint(A) ⊇ A. But we have A ⊇ Vαint(A). This implies A = Vαint(A). Hence A is a vague α open set in X.

Theorem 4.12: Let A be a vague regular α generalized open set in X and A ⊇ B ⊇ Vαint(A). Then B is a vague regular α generalized open set in X.

Proof: Let A be a vague regular α generalized open set in X and B be a vague set in X. Let A ⊇ B ⊇ Vαint(A). Then A^C is a vague regular α generalized closed set in X and $A^C \supseteq B^C \subseteq$ Vαcl(A^C). Then B^C is a vague regular α generalized closed set in X. Hence B is a vague regular α generalized open set in X.

Theorem 4.13: Let B be a vague regular closed set in X, B ⊆ A ⊆ Vint(Vcl(B)). Then A is a vague regular α generalized open set in X.

Proof: Let B be a vague regular closed set in X. Then B = Vcl(Vint(B)). By hypothesis, A ⊆ Vint(Vcl(B)) ⊆ Vint(Vcl(Vcl(Vint(B)))) ⊆ Vint(Vcl(Vint(B))) ⊆ Vint(Vcl(Vint(A))). Therefore A is α open set. Since every vague α open α generalized set, A is a vague regular α generalized open set in X.

V. CONCLUSION

We introduced the vague regular α generalized closed sets and vague regular α generalized open sets over vague topological spaces. These vague regular α generalized closed sets are established to show how far vague topological structures are preserved. Further, these basic concepts will be helpful to carry out more absorbing research work.

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