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Sensitivity Analysis in Fuzzy Transportation Problems with Trapezoidal Fuzzy Numbers

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Abstract:
In-depth study of the parameters involved in every decision taken by decision-makers is very essential. Sensitivity generally concerns with inquiring the relationships between the independent and dependent variables in mathematical modeling problems. The motivation towards searching for such information can be adverse, depending on the situations. Sensitivity Analysis in transportation problems study when the predictions to be are far more reliable and in turn it allows the decision-makers to identify and take optimal decisions where the performance can be improved in the future. In this paper, the concept of sensitivity analysis on Fuzzy Transportation Problems (FTP) is carried out by applying transportation algorithms and using the ranking techniques on fuzzy numbers , optimum solutions is obtained and is related with the original fuzzy transportation problem. This study is used to identify how much variations in the input values for a given variable impact the results for a mathematical model.

Key Words: *Sensitivity Analysis, Optimal Decisions, Fuzzy Transportation Problems, Trapezoidal fuzzy numbers.*

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I. Introduction

Imprecise information in making decisions which are quantitatively dealt was first pioneered by Zadeh in the notion of Fuzziness[1]. Transportation Problems are particular cases of Linear Programming Problems (LPPs) where certain commodities or goods are transported from sources to destinations subjected to the availabilities and requirements at the sources and destinations respectively with the total transportation cost is minimized[2]. Route planning in logistics and supply chain is one of the practical applications for transportation problems and many algorithms have been developed which are cost efficient when the supply, demand and cost coefficients quantities are exactly known[3]. The occurrence of fuzziness in real world, is predictable due to situations wherein we may encounter which are not so easy to deal with, and there is no definite answer to them[4]. There are situations where the quantities of a transportation problem may not be certain with uncontrollable factors. The mathematical model of a fuzzy transportation problem can be formulated in where uncertainty is occurred with fact that, the transportation matrix for cost per a unit takes one of the several feasible values in implementing the solution[5]. Uncertainties play a dominant role in every process and hence if such a data is used to process, then always it will results incorrectly[6,7]. In Decision-making algorithms, defuzzification is realized and from the fuzzy set, the best crisp values are taken[8,9]. With simple fuzzy arithmetic operations, crisp numbers are determined corresponding to that fuzzy number and simple algorithms are used for ordering the fuzzy numbers with their corresponding crisp real values and in their domains[10]. By Defuzzification, these fuzzy quantities are altered into crisp quantities with the Robust Ranking Technique, and then by Vogels Approximation Method(VAM), initial basic feasible solution can be obtained and optimum solution by Modified method[11,12,13]. Fully Fuzzy Zero Suffix Method (FFZSM) is used for trapezoidal fuzzy numbers[13,22]. In many systems, the parameters or independent variables are imprecisely known and knowledge of how this imprecision affects outputs is of considerable interest[14,15,16]. Even in cases where the input is precisely known, there will be no clear knowledge on which portion of input is observable[,16,17]. Sensitivity analysis plays an important role in becoming an integral part of any mathematical model, and these kind of analysis shows the impact in changing one data influencing the changing in other[18,19]. Thus, it is very useful to handle such uncertainties in the data to achieve the desired results[20,21,22].

In this paper, the concept of sensitivity analysis on Fuzzy Transportation Problems (FTP) is carried out by applying transportation algorithms and using the ranking techniques on fuzzy numbers optimum solution is obtained and is related with the original fuzzy transportation problem.. Introduction and literature survey are given Section 1. In section 2, preliminary introduction and algorithms are given. In section 3, the optimal solution for a numerical example is given. Sensitivity Analysis of cost coefficient is given in section 4. In Section 5 presents conclusions.

II. Preliminaries

Definition-1: Transportation Problem

A Transportation Problem is a particular of Linear Programming Problem (LPP), where the objective is to transport various quantities that are primarily stored at various sources to similar destinations with an objective that the total cost of transportation is minimum.

Let m be the number of sources, the sources $S_i (i = 1,2,...,m)$ having availabilities $a_i (i = 1,2,...,m)$ units at the i^{th} source and n be the number of destinations, the destinations $D_j (j = 1,2,...,n)$ with the requirements $b_j (j = 1,2,...,n)$ units at the j^{th} destination, C_{ij}, the transporting cost of one unit commodity from source i to destination j, and x_{ij}, quantity transported from source i to destination j.

Mathematically, problem takes the form of

$$Min \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} c_{ij} \qquad \text{…(1)}$$

subjected to the constraints

$$\sum_{i=1}^{m} x_{ij} \le a_i , \ \ i = 1,2,...,m$$
$$\sum_{j=1}^{n} x_{ij} \ge b_j , \ \ j = 1,2,...,n \qquad \text{…(2)}$$

and $x_{ij} \ge 0$, for every i and j.

…(3)

Definition-3: Fuzzy Set

The Fuzzy Set A on real line R with membership function $\mu_A(x) : R \to [0,1]$ is a fuzzy number if a) A is a normal and convex fuzzy set b) the support of A, must be bounded and c) $\alpha.A$ is closed for each α in $[0,1]$.

Definition – 4: Membership Function

The fuzzy number A is a fuzzy set, with membership function $\mu_A(x)$ must satisfy

i. $\mu_A(x)$ is piecewise continuous.
ii. $\mu_A(x)$ is convex.
iii. $\mu_A(x)$ is normal i.e., $\mu_A(x) = 1$

Definition – 5: **Trapezoidal Fuzzy Number**

A Trapezoidal Fuzzy Number (TrFN) can be represented by $A(a,b,c,d;1)$ with its membership function $\mu_A(x)$ represented by

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & otherwise \end{cases} \qquad \text{,,,(4)}$$

Definition – 6: Operations of a Trapezoidal Fuzzy Numbers
Let $A=(a_1,b_1,c_1,d_1)$ and $B=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers then the arithmetic operations A and B are as follows:

Addition: $A+B=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$

Subtraction: $A-B=(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2)$

Definition – 7: Defuzzification
Defuzzification is a technique of finding a single value known as a crisp value which is the middling value of the Trapezoidal Fuzzy Number. Robust Ranking Technique is used to defuzzify the TrFNs as far its accuracy and simplicity.

Definition – 8: Robust Ranking Technique
Robust Ranking Technique is a technique which satisfies linearity, compensation, and additivity which provides the results that consists of human intuition. For $\bar{a}$ to a fuzzy number, its Robust ranking is defined by

$$R(\bar{a}) = \int_0^1 (0.5)\left(a_\alpha^L, a_\alpha^U\right) d\alpha, \quad \left(a_\alpha^L , a_\alpha^U\right) = \{((b-a)\alpha + a), (d-c)\alpha))\}$$

where, $\left(a_\alpha^L, a_\alpha^U\right)$ is α level cut of the given fuzzy number.

…(5)

Definition – 9: Algorithm for solving Transportation Problem using Vogel's Approximation Method
Step 1: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.
Step 2: Identify the row or column with the maximum penalty and assign the corresponding cell's min(supply, demand). If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.
Step 3: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.
Step 4: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.

Definition – 10: Sensitivity Analysis
The sensitivity analysis in the linear programming is a process of studying the degree of sensitivity of the optimum solution. The study may include in different categories of parameter changes in the original problem with the following Changing the objective function coefficient of a non-basic variable and basic variable.

a) Availability of resources.
b) Requirement of destinations.
c) Availability of new resources and destinations.

Definition-11: Discrete change in the objective function co-efficient of a basic variable
The range of variation of C_{kr} from C_{kr} to $C_{kr}+\Delta C_{kr}$ which satisfies the optimality condition can be determined by solving the system of linear inequalities as

$$Min\{C_{ij}-(U_i+V_j)\} \geq \Delta C_{kr} \geq Max\{C_{ij}-(U_i+V_j)\}$$

…(6)

III. Result Numerical Example and its Solution

The company with four sources S_1, S_2, S_3 and S_4, four destinations D_1, D_2, D_3 and D_4 , the fuzzy transportation cost in transporting a unit quantity of the product from i^{th} source j^{th} to destination is C_{ij} given by,

$$[C_{ij}]_{3\times 4} = \begin{pmatrix} (0,1,2,4) & (0,1,2,3) & (1,2,3,4) & (-1,0,1,2) \\ (1,3,4,6) & (5,6,7,8) & (7,9,10,12) & (9,11,12,14) \\ (5,7,8,11) & (3,5,6,8) & (5,8,9,12) & (12,15,16,19) \end{pmatrix}$$

And fuzzy availability at the sources is $((0,1,2,3),(5,10,12,17),(1,6,7,12))$ and the fuzzy demands at the destinations are $((1,2,3,4),(5,7,8,10),(1,5,6,10),(1,3,4,6))$ respectively. Thus the fuzzy transportation problem is given by the following

Table 1 – Representation of Fuzzy Transportation

	Destin 1	Destin 2	Destin 3	Destin 4	Availabilities
Source 1	(0,1,2,4)	(0,1,2,3)	(1,2,3,4)	(-1,0,1,2)	(0,1,2,3)
Source 2	(1,3,4,6)	(5,6,7,8)	(7,9,10,12)	(9,11,12,14)	(5,10,12,17)
Source 3	(5,7,8,11)	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(1,6,7,12)
Requirements	(1,2,3,4)	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	

Solution: The fuzzy transportation problem is formulated mathematically in the form of

$$Min \quad Z = R(0,1,2,4)X_{11} + R(0,1,2,3)X_{12} + R(1,2,3,4)X_{13} + R(-1,0,1,2)X_{14}$$
$$+ R(1,3,4,6)X_{21} + R(5,6,7,8)X_{22} + R(7,9,10,12)X_{23} + R(9,11,12,14)X_{24}$$
$$+ R(5,7,8,11)X_{31} + R(3,5,6,8)X_{32} + R(5,8,9,12)X_{33} + R(12,15,16,19)X_{34}$$

Subjected to the constraints

$$X_{11} + X_{12} + X_{13} + X_{14} = (0,1,2,3)$$
$$X_{21} + X_{22} + X_{23} + X_{24} = (5,10,12,17)$$
$$X_{31} + X_{32} + X_{33} + X_{34} = (1,6,7,12)$$
$$X_{11} + X_{21} + X_{31} = (1,2,3,4)$$
$$X_{12} + X_{22} + X_{32} = (5,7,8,10)$$
$$X_{13} + X_{23} + X_{33} = (1,5,6,10)$$
$$X_{14} + X_{24} + X_{34} = (1,3,4,6)$$

$$where \quad X_{ij} \geq 0 \quad i = 1,2,3 \; and \; j = 1,2,3,4.$$

Using the Robust Ranking Technique, the ranks of all quantities, availability at each source and requirements at each destination are calculated as $R(\bar{a}) = \int_0^1 (0.5)\left(a_\alpha^L, a_\alpha^U\right)d\alpha$ and $\left(a_\alpha^L, a_\alpha^U\right) = \{((b-a)\alpha + a), d-(d-c)\alpha)\}$

Ranks are obtained by using the above relation for each cell, for example the rank for the cell C_{11} is $R(C_{11}) = \int_0^1 (0.5)\left(a_\alpha^L, a_\alpha^U\right)d\alpha = \int_0^1 (0.5)(\alpha, 4-2\alpha)d\alpha = 1.7$ The transportation problem is represented by

	Destin 1	Destin 2	Destin 3	Destin 4	Availabilities
Source 1	1.7	1.5	2.5	0.5	1.5
Source 2	3.5	6.5	9.5	11.5	11.5
Source 3	7.7	5.5	8.5	15.5	6.5
Requirements	2.5	7.5	5.5	3.5	

Table - 2 – Ranks of the quantities in Fuzzy Transportation

Using Vogel's Approximation Method (VAM), the initial basic feasible solution is determined and by the Modified Method, Optimum solution is obtained

	Destin 1	**Destin 2**	**Destin 3**	**Destin 4**	**Availabilities**
Source 1	1.7	1.5	2.5	**[1.5]** **0.5**	6.5
Source 2	**[2.5]** **3.5**	**[1]** **6.5**	**[5.5]** **9.5**	**[2]** **11.5**	1.5
Source 3	7.7	**[6.5]** **5.5**	8.5	15.5	11
Requirements	7.5	5.5	3.5	2.5	

Table 3 – Initial Basic Feasible Solution by VAM

The Initial Basic Feasible Solution (IBFS) by VAM is

$$(1.5)\ (0.5)+(2.5)\ (3.5)+(9.5)\ (5.5)+(6.5)\ (1)+(11.5)\ (2)+(5.5)\ (6.5)=127.$$

Now using Modified Method in transportation problems, the optimum solution as given

	Destin 1	**Destin 2**	**Destin 3**	**Destin 4**	$u_i^{'}s$
Source 1	1.7	1.5	2.5	**[1.5]** **0.5**	u_1=-11
Source 2	**[2.5]** **3.5**	**[1]** **6.5**	**[5.5]** **9.5**	**[2]** **11.5**	$u_2 = 0$
Source 3	7.7	**[6.5]** **5.5**	8.5	15.5	$u_3 = -1$
$v_j^{'}s$	$v_1 = 3.5$	$v_2 = 6.5$	$v_3 = 9.5$	$v_4 = 11.5$	

Table 4 – Modified Transportation Table using MODI Method

The optimum solution is

$$(1.5)\ (0.5)+(2.5)\ (3.5)+(9.5)\ (5.5)+(6.5)\ (1)+(11.5)\ (2)+(5.5)\ (6.5)=127$$

Applying the Fully Fuzzy Zero Suffix Method (FFZSM) for trapezoidal fuzzy numbers, the transportation table is

	Destin 1	**Destin 2**	**Destin 3**	**Destin 4**	**Availabilities**
Source 1	(0,1,2,4)	(0,1,2,3)	(1,2,3,4)	**(-1,0,1,2)** **(0,1,2,3)**	(0,1,2,3)
Source 2	**(1,3,4,6)** **(1,2,3,4)**	**(5,6,7,8)** **(-13,-2,4,15)**	(7,9,10,12)	**(9,11,12,14)** **(-2,1,3,6)**	(5,10,12,17)
Source 3	(5,7,8,11)	**(3,5,6,8)** **(1,6,7,12)**	**(5,8,9,12)** **(1,5,6,10)**	(12,15,16,19)	(1,6,7,12)
Demand	(1,2,3,4)	(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	

Table 5 – Optimum Table with Trapezoidal Fuzzy Numbers

The optimum solution is

$$
\begin{aligned}
&= (0,1,2,3,) \times (-1,0,1,2) + (1,3,4,6) \times (1,2,3,4) + (5,6,7,8) \times (-13,-2,4,15) \\
&+ (9,11,12,14) \times (-2,1,3,6) + (3,5,6,8) \times (1,6,7,12) + (1,5,6,10) \times (5,8,9,12) \\
&= (1.5)(0.5) + (3.5)(2.5) + (6.5)(1) + (5.5)(9.5) + (11.5)(2) + (6.5)(5.5) \\
&= 127
\end{aligned}
$$

IV. Sensitivity Analysis

Now changing the objective function coefficients of basic variables, for example, with respect to the basic variable cell $(1,1)$ the fuzzy trapezoidal problem is

	Destin 1	**Destin 2**	**Destin 3**	**Destin 4**	$u_i^{'}s$
Source 1	**2.5+Δ (1.7)**	**3.5 (1.5)**	2.5	0.5	0
Source 2	3.5	6.5	**6.5 (9.5)**	11.5	-6-Δ
Source 3	**5.5 (7.7)**	5.5	**15.5 (8.5)**	**9.5 (15.5)**	3-Δ
$v_j^{'}s$	2.5+Δ	3.5	12.5+Δ	6.5+Δ	

Table 6 – Changing the coefficient of Basic Cell C_{11}

To determine the limits of Non-Basic Variables we have,

$$
\begin{aligned}
&\overline{C}_{11} = u_1 + v_1 - 1.7 = -11 + \Delta + 3.5 - 1.7 = -9.2 + \Delta \leq 0 \ \ for\ \Delta \leq 9.2 \\
&\overline{C}_{12} = u_1 + v_2 - 1.5 = -11 + \Delta + 6.5 - 1.7 \leq 0 = -6 + \Delta \leq 0 \ \ for\ \Delta \leq 6 \\
&\overline{C}_{13} = u_1 + v_3 - C_{13} = -11 + \Delta + 9.5 - 2.5 \leq 0 = -4 + \Delta \leq 0 \ \ for\ \Delta \leq 4 \\
&\overline{C}_{31} = u_3 + v_1 - C_{31} \leq 0 = 3.5 - 1 - 7.7 \leq 0 = -5.2 \\
&\overline{C}_{33} = u_3 + v_3 - C_{33} = -1 + 9.5 - 8.5 = 0 \\
&\overline{C}_{34} = u_3 + v_4 - C_{34} = -1 + 11.5 - 15.5 = -5
\end{aligned}
$$

The current solution is optimal for $0 \leq \Delta \leq 9.2$ and the optimal value is $127 + \Delta(c_{14})$, and thus the corresponds bounds for C_{14} are $0.5 \leq C_{14} \leq 9.7$

Similar calculations are carried out for the other Basic Cells and the limits are found and the optimal solution is obtained. The Limits are

For the cell C_{21}, Δ is $0 \leq \Delta \leq 9.2$ and $3.5 \leq C_{12} \leq 12.7$

For the cell C_{22}, Δ is $0 \leq \Delta \leq 6$ and $6.5 \leq C_{22} \leq 12.5$

For the cell C_{23}, Δ is $0 \leq \Delta \leq 5$ and $9.5 \leq C_{23} \geq 14.5$

For the cell C_{24}, Δ is $0 \leq \Delta \leq 5$ and $11.5 \leq C_{24} \leq 16.5$

For the cell C_{32}, Δ is $0 \leq \Delta \leq 5.2$ and $5.5 \leq C_{12} \leq 10.7$

V. Conclusion

A LPP of a specific structure is a fuzzy transportation problem(FTP) with all the variables and parameters are fuzzy. With the values obtained by the weighted function of model, new cost coefficients are achieved and using Vogel's Approximation Method and MODI Method, Optimum Solution is obtained. Later, Sensitivity Analysis with varying cost matrix, availabilities and requirements, optimum solutions are studied and **related with the original fuzzy transportation problem and conclude that the changes made in original transportation problem does not affect the fuzzy transportation problem.** Decision and policy making under uncertainty can significantly benefit formally and informally from the advancements in

Sensitivity Analysis. It is observed that preserving the optimality of the transportation problem, the limits are determined. Although, predicting future is uncertain, the results guarantee that the variations in some quantities is not influencing the optimality and the decision makers always can address the answers to where and how does uncertainty matters and have the impacts of all important assumptions been treated.

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