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Comparative Analysis of the Effects of Moving Loads Travelling On Highly Prestressed Orthotropic Rectangular Plate Resting On Pasternak Foundation

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Summary
The comparative analysis of the effects of moving loads travelling on highly prestressed orthotropic rectangular plate resting on Pasternak foundation is investigated. When moving load traversing a structure is considered as a mass body, its effects are twofold: the gravitational effect of the moving load and the inertial effects of the load mass of the deformed structure. In this work, the two cases are presented. The motion of the plate is governed by a fourth-order partial differential equation for which the highest derivatives is multiplied by a small parameter. Such problem is amenable to singular perturbation. Of particular interest is the method of matched asymptotic expansions (MMAE). The basic idea underlying this method is the representation of the solution to the dynamical problem by two expansions, each of which is valid in part of the domain after which matching of the different expansions is done in order to obtain a composite solution which is uniformly valid in the entire domain of definition of the problem. The result is then shown in plotted curves. These reveal that the critical speed is higher in moving force problem than in the moving mass for the various values of shear modulus and rotatory inertia. So this is a clear indication that resonance is reached earlier with moving force than with moving mass.

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I. Introduction

The dynamics of elastic structures, beams and plates, under moving loads is nowadays under intensive study due to the development of high speed transportation system and its importance in engineering designs and constructions. Moving loads such as trucks, cars, trains, bikes and so on, have a great effects on the dynamic stresses in the elastic media they traverse. They cause such bodies to vibrate even very intensively especially at high speeds. These may have devastating effects on such engineering structures on which they move. According to [1] dynamical problems involving moving loads can be generally grouped into the following three classes:

(a) In the first case, the mass of the moving load is considered much smaller than the mass of the structure it is traversing,

(b) The second class comprises of the system for which the mass of the structure is assumed to be much smaller than that of the moving load and thirdly is

(c) The case in which both the mass of the structure and that of the moving load are of comparable magnitude.

The first case is much simpler than the second and the third. In fact, the first is the commonest problem treated in literature. In this problem, the inertial effects of the moving load are assumed negligible and only the force effects of the moving load are taken into consideration. Thus, this type of problem is termed the **"moving force"** problem. Though, the problem, on the assumption, has been greatly simplified, the following question arises: how safe is a design based on this assumption. The justification of this assumption would have been established had the solution of this approximate model been proved to be an upper bound for the actual deflection of the dynamical system. This approximation model in which the vehicle-track interaction is completely neglected has been described as the crudest approximation known in the literature of assessing the dynamic response of an elastic system which supports moving concentrated masses. The most difficult of all the three types of the problem is the third, while both the second and third problems involve not only the consideration of the force effects of the moving load but also its inertial effects, the moving load in the formal does not have mass commensurable with the mass of the structure. The third type of problem may be termed **"moving mass"** problem. This is an improvement over existing works in literature that determine the displacement response of plate-structures under the action of moving loads. In this work, the third case is presented. When the inertia effect of the moving load is so considered, the governing partial differential equation of motion becomes complex and laborious and no longer has constant coefficients. In particular, the coefficients become variable and singular. More complicatedly is the situation whereby the plate-structure is highly prestressed and orthotropic in which bending effects at the boundaries are considered. In particular when a plate-structure is highly prestressed a small parameter multiplies the highest derivative in the governing differential equation. This class of dynamical plate problem in which a small parameter multiplies the highest derivatives in the governing differential equation is not common in literature. However, this class of plate problems have been solved when the plate is executing free vibration or when a static load is acting on such plates, Hutter and Olunloyo [2, 3]. Instances in which plate-structures are subjected to moving concentrated forces are attempted by [4], with the discovery that it may not be safe to rely on the approximate solution produced by the problem of dynamic response of plates to moving forces (as opposed to moving masses). The problem of highly prestressed orthotropic plate-structures subjected to moving masses is rear in literature. Notwithstanding it is being treated by the work of Tolorunshagba and Oni [5], Tolorunshagba [6]. Outside the aforementioned researchers who worked on orthotropic structures, several authors have treated moving mass problems. Foremost amongst researchers of a moving load on a plate is the work of Willis [7] who investigated the effects of weights travelling over bars with different velocities. Others are Stokes [8], Timoshenko [9], Lowan [10], Bondar [11], Reissmann [12, 13], Holl [14] and the monograph of Fryba [1] to mention but few. Some of these renown researchers who have worked on moving mass beam problems are Stanisic et al. [15, 16, 17], Milormir et al. [18], Sadiku and Leipholz [19], Oni [20], Gbadeyan and Oni [21] to mention just very few. Generally speaking, structures may be seen as subjected to two loadings, namely the applied load and inertial forces at every instant time.

Keywords: gravitational effect, inertial effect, singular perturbation,

II. Problem Formulation

In this study, the effects of moving loads travelling on highly prestressed orthotropic rectangular plate resting on Pasternak foundation is investigated. When the effects of shear prestress and shear deformation is neglected, the transverse displacement response of a highly prestressed orthotropic rectangular plate occupying the domain and traversed by moving concentrated loads, when the rotatory inertia correction factor is incorporated, is governed by the fourth-order partial differential equation of the form

$$D_{\bar{x}\bar{x}}\frac{\partial^4 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{x}^4} + 2D_{\bar{x}\bar{y}}\frac{\partial^4 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{x}^2\partial \bar{y}^2} + D_{\bar{y}\bar{y}}\frac{\partial^4 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{y}^4} - N_{\bar{x}\bar{x}}\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{x}^2} - N_{\bar{y}\bar{y}}\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{y}^2}$$
$$- \mu \mathrm{R}_{o\bar{t}}\frac{\partial^2}{\partial \bar{t}^2}\left(\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{x}^2} + \frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{y}^2}\right) + \mu\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{t}^2} - G\left(\frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{x}^2} + \frac{\partial^2 V(\bar{x},\bar{y},\bar{t})}{\partial \bar{y}^2}\right) + KV(\bar{x},\bar{y},\bar{t})$$
$$= P(\bar{x},\bar{y},\bar{t}) \quad (1)$$

Where $D_{\bar{x}\bar{x}}$ and $D_{\bar{y}\bar{y}}$ are respectively the flexural rigidities in $\bar{x}$- and $\bar{y}$- directions, $D_{\bar{x}\bar{y}}$ is the effective torsional rigidity, $N_{\bar{x}\bar{x}}$ and $N_{\bar{y}\bar{y}}$ are respectively the axial prestress in $\bar{x}$- and $\bar{y}$- directions, $\bar{x}, \bar{y}$ are the spatial coordinates, $\bar{t}$ is the time coordinate, $\mathrm{R}_{o\bar{t}}$ is the rotatory inertia correction factor, μ is the mass of the plate per unit area, $V(\bar{x},\bar{y},\bar{t})$ is the displacement response of the plate, $P(\bar{x},\bar{y},\bar{t})$ is the applied external moving load, $\nabla^2 V(\bar{x},\bar{y},\bar{t})$ is the Laplacian operator on $V(\bar{x},\bar{y},\bar{t})$, G and K are respectively the shear and foundation moduli.

Since the plate is assumed to be fully clamped, the boundary conditions are

$$\left.\begin{array}{ll}\bar{x}=0, & 0\le \bar{y} \le b\\ \bar{x}=1 & 0\le \bar{y} \le b\end{array}\right\} V(\bar{x},\bar{y},\bar{t}) = 0, \quad \frac{\partial V(\bar{x},\bar{y},\bar{t})}{\partial \bar{x}} = 0 \quad (2.1)$$

$$\left.\begin{array}{ll}\bar{y}=0, & 0\le \bar{x} \le 1\\ \bar{y}=b & 0\le \bar{x} \le 1\end{array}\right\} V(\bar{x},\bar{y},\bar{t}) = 0, \quad \frac{\partial V(\bar{x},\bar{y},\bar{t})}{\partial \bar{y}} = 0 \quad (2.2)$$

For simplicity, the plate is assumed to be at rest prior to the arrival of the load, and so the initial conditions are

$$V(\bar{x},\bar{y},0) = 0, \quad \frac{\partial V(\bar{x},\bar{y},0)}{\partial \bar{t}} = 0 \quad (3)$$

The concentrated load travelling on the rectangular plate has mass commensurable with the mass of the plate. Thus, the external load takes on the complicated form

$$P(\bar{x},\bar{y},\bar{t}) = P_f(\bar{x},\bar{y},\bar{t})\left(1 - \frac{1}{g}\frac{d^2V(\bar{x},\bar{y},\bar{t})}{d\tau^2}\right) \quad (4)$$

in which $P_f(\bar{x},\bar{y},\bar{t})$, the continuous moving force acting on the rectangular plate, is denoted as

$$P_f(\bar{x},\bar{y},\bar{t}) = Mg\delta(\bar{x},-c^*\bar{t},)\delta(\bar{y},-\bar{y},_0) \tag{4.1}$$

where $\delta(\bar{x}-a)$ is the unit concentrated force, acting at a point $\bar{x}=a$, called the Dirac delta function, M is the mass of the rectangular plate, g is the acceleration due to gravity and $\frac{d^2V(\bar{x},\bar{y},\bar{t})}{d\bar{t}^2}$ is a convective acceleration operator on $V(\bar{x},\bar{y},\bar{t})$ defined in Fryba [1] as

$$\frac{d^2}{d\bar{t}^2} = \frac{\partial^2}{\partial\bar{t}^2} + 2c^*\frac{\partial^2}{\partial\bar{x}\partial\bar{t}} + c^{*2}\frac{\partial^2}{\partial\bar{x}^2} \tag{5}$$

Here physical meanings can be given for the terms involved in the right hand side of equation (5). The first term measures the effect of acceleration in the direction of deflection, the second term measures the effect of complementary acceleration (i.e. Coriolis force) and the third term measures the effect of the part of curvature (i.e. centrifugal acceleration) of the plate induced by the mass of speed c^* at the position of action.

In view of equations (4) and (5) equation (1) becomes

$$\begin{aligned} D_{\bar{x}\bar{x}}\frac{\partial^4V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^4} &+ 2D_{\bar{x}\bar{y}}\frac{\partial^4V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^2\partial\bar{y}^2} + D_{\bar{y}\bar{y}}\frac{\partial^4V(\bar{x},\bar{y},\bar{t})}{\partial\,\bar{y}^4} - N_{\bar{x}\bar{x}}\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^2} - N_{\bar{y}\bar{y}}\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\,\bar{y}^2} \\ &- \mu\mathrm{R}_{o\bar{t}}\frac{\partial^2}{\partial\bar{t}^2}\left(\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^2} + \frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\,\bar{y}^2}\right) + \mu\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\bar{t}^2} - G\left(\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^2} + \frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\,\bar{y}^2}\right) + KV(\bar{x},\bar{y},\bar{t}) \\ &- Mg\delta(\bar{x},-c^*\bar{t},)\delta(\bar{y},-\bar{y},_0)\left(\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\bar{t}^2} + 2c^*\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}\partial\bar{t}} + c^{*2}\frac{\partial^2V(\bar{x},\bar{y},\bar{t})}{\partial\bar{x}^2}\right) \\ &= Mg\delta(\bar{x},-c^*\bar{t},)\delta(\bar{y},-\bar{y},_0) \end{aligned} \tag{6}$$

5.2 NON-DIMENSIONALIZATION

For the purpose of approximate solution, it is pertinent to present equations (6), (2) and (3) in dimensionless form. This act brings out the important dimensionless parameters that govern the behaviour of the dynamical system. With the introduction of the dimensionless quantities $L, N_0, D_{xx}, and\ t$, into equation (6) results in

$$\begin{aligned} \varepsilon^2&\left[\frac{\partial^4W(x,y,t)}{\partial x^4} + 2\alpha_1^2\frac{\partial^4W(x,y,t)}{\partial x^2\partial y^2} + \alpha_2^2\frac{\partial^4W(x,y,t)}{\partial y^4}\right] - \beta_1^2\frac{\partial^2W(x,y,t)}{\partial x^2} - \beta_2^2\frac{\partial^2W(x,y,t)}{\partial y^2} + \frac{\partial^2W(x,y,t)}{\partial\bar{t}^2} \\ &- \alpha_{0t}\left[\frac{\partial^4W(x,y,t)}{\partial t^2\partial x^2} + \frac{\partial^4W(x,y,t)}{\partial t^2\partial y^2}\right] - G\left(\frac{\partial^2V(x,y,t)}{\partial x^2} + \frac{\partial^2V(x,y,t)}{\partial y^2}\right) + KV(x,y,t) \\ &+ \Gamma_0\delta(x-ct)\delta(y-y_0)\left[\frac{\partial^2W(x,y,t)}{dt^2} + 2c\frac{\partial^2W(x,y,t)}{\partial t\partial x} + c^2\frac{\partial^2W(x,y,t)}{\partial x^2}\right] \\ &= M_0g\delta(x-ut)\delta(y-y_0) \end{aligned} \tag{7}$$

where it is assumed that $0<\varepsilon\ll 1$, as will happen if the flexural rigidity is weak compared to the high prestress, and is defined as

$$\varepsilon^2 = \frac{D_{xx}}{N_0L^2} \tag{8.1}$$

where as usual D_{xx} denote the plate's bending rigidity in the x-direction, N_0 a reference prestress and L is a characteristic length with respect to which the deflection and the two coordinates are normalized viz: $V=WL,\ u=xL,\ \ v=yL$. On the other hand time is assumed to be normalized with respect to a characteristic frequency ω such that

$$\bar{t} = \omega t \quad \text{and} \quad \frac{\mu L^2}{N_0\omega^2} = 1. \tag{8.2}$$

The coefficients α_1^2 and α_2^2 measure material orthotropy such that

$$\alpha_1^2 = \frac{D_{xy}}{D_{xx}},\quad \alpha_2^2 = \frac{D_{yy}}{D_{xx}} \tag{8.3}$$

While β_1^2 and β_2^2 measure the prestress ratio and are defined as

$$\beta_1^2 = \frac{N_{xx}}{N_0},\quad \beta_2^2 = \frac{N_{yy}}{N_0} \tag{8.4}$$

respectively.
Also

$$\alpha_{0t} = \frac{R_{0\tau}}{L^3} \tag{8.5}$$

is the coefficient of rotatory inertia.

$$\Gamma_0 = \frac{\omega}{L^3}\frac{M}{\mu} \tag{8.6}$$

is a measure of the mass ratio and

$$c = \frac{c^* L}{\omega}, \quad y_0 = \frac{v_0}{L} \tag{8.7}$$

subject to the boundary conditions (in non-dimensional form).

$$\left.\begin{matrix} x = 0, & 0 \le y \le b \\ x = 1 & 0 \le y \le b \end{matrix}\right\} \quad W(x,y,t) = 0 = \frac{\partial W(x,y,t)}{\partial x} \tag{9.1}$$

$$\left.\begin{matrix} y = 0, & 0 \le x \le 1 \\ y = b & 0 \le x \le 1 \end{matrix}\right\} \quad W(x,y,t) = 0 = \frac{\partial W(x,y,t)}{\partial y} \tag{9.2}$$

together with the initial conditions

$$W(x,y,0) = 0 = \frac{\partial W(x,y,0)}{\partial t} \tag{10}$$

OPERATIONAL SIMPLIFICATION

It is observed that a small parameter multiplies the highest derivatives in equation (7). For such problems a regular perturbation lowers the order of the differential equation –except in this regions of rapid change(often called boundary layer) where the high value of the derivative cancels the effects of the multiplying small parameter- which in turn means the solution cannot satisfy all the boundary conditions. A special treatment is therefore needed in the region near as well as at the boundary where its boundary condition is yet to be satisfied. And as such, the problem is only amenable to singular perturbations, in particular the method of matched asymptotic expansions (MMAE). However, equation (7) is considerably simplified by introducing the Laplace integral transformation defined by

$$W(x,y,s) = \int_0^\infty W(x,y,t)\, e^{-st} dt, \qquad s > 0, \qquad t \ge 0 \tag{11}$$

with the properties

$$\int_0^\infty \delta(t - t_0)\, e^{-st} dt = e^{-st_0} \tag{12}$$

Taking t as the principal variable will make equation (7) to become

$$\begin{aligned} &\varepsilon^2 \left[\frac{\partial^4 w(x,y,s)}{\partial x^4} + 2\alpha_1^2 \frac{\partial^4 w(x,y,s)}{\partial x^2 \partial y^2} + \alpha_2^2 \frac{\partial^4 w(x,y,s)}{\partial y^4}\right] - \beta_1^2 \frac{\partial^2 w(x,y,s)}{\partial x^2} - \beta_2^2 \frac{\partial^2 w(x,y,s)}{\partial y^2} + S^2 W(x,y,s) \\ &\quad - \alpha_{0t} S^2 \left[\frac{\partial^2 w(x,y,s)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2}\right] - G \left[\frac{\partial^2 w(x,y,s)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2}\right] + KW(x,y,s) \\ &\quad + \Gamma_0 \delta(y - y_0)\{I_a + 2c^* I_b + c^{*^2} I_c\} = M_0 g \delta(y - y_0) I_d \end{aligned} \tag{13}$$

where

$$I_a = \int_0^\infty \delta(x - ct) \frac{\partial^2 w(x,y,t)}{\partial t^2} e^{-st} dt \tag{14.1}$$

$$I_b = \int_0^\infty \delta(x - ct) \frac{\partial^2 w(x,y,t)}{\partial t \partial x} e^{-st} dt \tag{14.2}$$

$$I_c = \int_0^\infty \delta(x - ct) \frac{\partial^2 w(x,y,t)}{\partial x^2} e^{-st} dt \tag{14.3}$$

$$I_d = \int_0^\infty \delta(x - ct)\, e^{-st} dt \tag{14.4}$$

The integrals (14) cannot be easily evaluated and so use is made of trigonometrical series representation of the Dirac delta function obtained from the Fourier series expansion of the function as

$$\delta(x - ct) = 1 + 2\sum_{r=1}^{\infty} [\cos 2\pi rct \cos 2\pi rcx + \sin 2\pi rct\, sin 2\pi rcx] \tag{15}$$

In view of equation (15) the complete Laplace transformation of equation (13) is

$$\begin{aligned} &\varepsilon^2 \left[\frac{\partial^4 w(x,y,s)}{\partial x^4} + 2\alpha_1^2 \frac{\partial^4 w(x,y,s)}{\partial x^2 \partial y^2} + \alpha_2^2 \frac{\partial^4 w(x,y,s)}{\partial y^4}\right] - \beta_1^2 \frac{\partial^2 w(x,y,s)}{\partial x^2} - \beta_2^2 \frac{\partial^2 w(x,y,s)}{\partial y^2} + S^2 W(x,y,s) \\ &\quad - \alpha_{0t} S^2 \left[\frac{\partial^2 w(x,y,s)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2}\right] - G \left[\frac{\partial^2 w(x,y,s)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2}\right] + KW(x,y,s) \\ &\quad + \Gamma_0 \delta(y - y_0)[s^2 W(x,y,s) + 2c^* s W_x(x,y,s) + c^{*^2} W_{xx}(x,y,s)] = \frac{M_0}{c} g \delta(y - y_0) e^{-s\frac{x}{c}} \end{aligned} \tag{16}$$

subject to the boundary conditions

$$\left.\begin{array}{l} x=0,\ \ 0\le y\le b \\ x=1\ \ 0\le y\le b \end{array}\right\}\quad W(x,y,s)=0=\frac{\partial w(x,y,s)}{\partial x} \tag{17.1}$$

$$\left.\begin{array}{l} y=0,\ \ 0\le x\le 1 \\ y=b\ \ 0\le x\le 1 \end{array}\right\}\quad W(x,y,s)=0=\frac{\partial w(x,y,s)}{\partial y} \tag{17.2}$$

together with the initial conditions

$$W(x,y,0)=0=\frac{\partial w(x,y,0)}{\partial t} \tag{18}$$

6.2 SOLUTION PROCEDURE

In equation (16), an exact uniformly valid solution in the entire domain is not possible since it is observed that a small parameter multiplies the highest derivative in the governing differential equation. In accordance with the informal principle that the behaviour of solution is governed primarily by the highest order terms [37]. This is due to the bending effects at the boundaries. Consequently, solution valid away from the boundaries breaks down near as well as at the boundaries. Thus only approximate solutions are possible. The two but equivalent approaches that could be used to tackle this type of problem are the method of composite expansions (MCE)and the method of matched asymptotic expansions (MMAE). In this research work, MMAE is used. This technique provides an approximate solution to the given problem in terms of two separate expansions which are valid in a closed interval $\Omega[0\le x\le 1, 0\le y\le b]$. The two expansions called inner and outer, neither of which is uniformly valid but whose domain of validity together cover the interval Ω. To define "domain of validity" one needs to consider intervals whose endpoints depend on the small parameter, say ε. In the construction of inner and outer expansions, constants and functions of ε occur which are determined by comparison of the two expansions that is "matching". The comparison is possible only in the domain of overlap of their regions of validity. Once overlap is established, matching is easily carried out. The method of matched asymptotic expansions (MMAE) developed by Bretheton [12] required that the asymptotic solution of equation (16) be of the form

$$W_0(x,y,t)+\varepsilon W_1(x,y,t)+O(\varepsilon^2) \tag{19}$$

Substitution of equation (19) into equation (16) produces, after rearranging and equating coefficients of the powers of ε, the following recurrence relations

Substitution of equation (19) into equation (16) produces, after rearranging and equating coefficients of the powers of ε, the following recurrence relations

$$H_v^*(x,y,s) = \begin{cases} \dfrac{M_0 g}{c}\delta(y-y_0)e^{-\frac{sx}{c}}, & v=0 \\ 0, & v=1 \\ D\nabla^4 W_{v-2}(x,y,s), & v\ge 2 \end{cases} \tag{20}$$

where

$$\begin{aligned} H_v^*(x,y,s) = & -\beta_1^2\frac{\partial^2 W_v(x,y,s)}{\partial x^2}-\beta_2^2\frac{\partial^2 W_v(x,y,s)}{\partial y^2}+s^2W_v(x,y,s)-\alpha_{0t}s^2\left[\frac{\partial^2 W_v(x,y,s)}{\partial x^2}+\frac{\partial^2 W_v(x,y,s)}{\partial y^2}\right] \\ & -G\left[\frac{\partial^2 W_v(x,y,s)}{\partial x^2}+\frac{\partial^2 W_v(x,y,s)}{\partial y^2}\right]+KW_v(x,y,s) \\ & +\Gamma_0\delta(y-y_0)\left(s^2W_v(x,y,s)+2cs\frac{\partial W_v(x,y,s)}{\partial x}+c^2\frac{\partial^2 W_v(x,y,s)}{\partial x^2}\right) \end{aligned} \tag{21}$$

and

$$D\nabla^4 W_{v-2}(x,y,s) = -\left(\frac{\partial^4 W_{v-2}(x,y,s)}{\partial x^4}+2\alpha_1^2\frac{\partial^4 W_{v-2}(x,y,s)}{\partial x^2\partial y^2}+\alpha_2^2\frac{\partial^4 W_{v-2}(x,y,s)}{\partial y^4}\right) \tag{22}$$

Here the subscript of $W(x,y,s)$ denote the order of ε and $\nabla^4=\nabla^2\cdot\nabla^2$ (∇^2 is the Laplacian operator) while D is as earlier defined.

Subject to the transformed conditions at the boundaries

$$W_j(x,y,s)\big|_{x=0,1}=0,\quad j=0,1,2,\dots \tag{23.1}$$

$$\left.\frac{\partial W_j(x,y,s)}{\partial x}\right|_{x=0,1}=0,\ \ j=0,1,2,\dots \tag{23.2}$$

$$W_j(x,y,s)\big|_{y=0,b}=0,\quad j=0,1,2,\dots \tag{23.3}$$

$$\left.\frac{\partial W_j(x,y,s)}{\partial x}\right|_{y=0,b}=0,\ \ j=0,1,2,\dots \tag{23.4}$$

To obtain expression valid at the boundary, say near $x=0$, the inner variable is set as $X=\frac{x}{\varepsilon}$, to yield a solution valid at the edge $x=0$ as

$$W^i(X,y,s) = W_0^i(X,y,s) + \varepsilon W_1^i(X,y,s) + O(\varepsilon^2) \tag{24}$$

where superscript i refer to inner solution. Equation (24) is also valid near $x = 1$, where the inner variable is set as $X = \frac{1-x}{\varepsilon}$. Expressions similar to (24) can be written down for the solutions near $y = 0$ and $y = b$, where the inner variables are set respectively as $Y = \frac{y}{\varepsilon}$ and $Y = \frac{b-y}{\varepsilon}$, thus

$$W^i(x,Y,s) = W_0^i(x,Y,s) + \varepsilon W_1^i(x,Y,s) + O(\varepsilon^2) \tag{25}$$

Substitution of equation (24) into equation (16) near either $x = 0$ or $x = 1$ produces respectively

$$\begin{aligned}\frac{\partial^4 W_v^i(X,y,s)}{\partial X^4} - \left[\beta_1^2 + \alpha_{ot}s^2 + G_0 - c^2\Gamma_0\,\delta(y-y_o)\right]\frac{\partial^2 W^i_{v(X,y,s)}}{\partial X^2} \\ = \left[\beta_2^2 + \alpha_{ot}s^2 + G_0\right]\frac{\partial^2 W^i_{v-2(X,y,s)}}{\partial y^2} - 2\alpha_2^2\frac{\partial^4 W^i_{v-2(X,y,s)}}{\partial X^2\partial y^2} - \left[K_0 + s^2 + s^2\Gamma_{0\delta(y-y_0)}\right]W^i_{v-2(X,y,s)} \\ - 2cs\Gamma_0\,\delta(y-y_o)\frac{\partial W^i_{v-1}(X,y,s)}{\partial X} + \begin{cases}0, & v = 0,1,3,4,\ldots \\ M_0 g\delta(y-y_0)e^{\frac{-s}{c}x}, & v = 2\end{cases}\end{aligned} \tag{26}$$

Alternatively, the expression of equation (26) can be re-written as

$$\begin{aligned}\frac{\partial^4 W_v^i(X,y,s)}{\partial x^4} - \beta_1^2\frac{\partial^2 W_v^i(X,y,s)}{\partial x^2} - \alpha_{ot}s^2\frac{\partial^2 W_v^i(X,y,s)}{\partial x^2} + c^2\Gamma_0\delta(y-y_0)\frac{\partial^2 W_v^i(X,y,s)}{\partial x^2} + G_0\frac{\partial^2 W_v^i(X,y,s)}{\partial X^2} \\ = 2cs\Gamma_0\,\delta(y-y_0)\frac{\partial^2 W_{v-1}^i(X,y,s)}{\partial x^2} - 2\alpha_1^2\frac{\partial^4 W_{v-2}^i(X,y,s)}{\partial x^2\partial y^2} + \beta_2^2\frac{\partial^4 W_{v-2}^i(X,y,s)}{\partial y^2} + s^2 W_{v-2}^i(X,y,s) \\ + \alpha_{0t}s^2\frac{\partial^2 W_{v-2}^i(X,y,s)}{\partial y^2} - s^2\Gamma_0\,\delta(y-y_0)W_{v-2}^i(X,y,s) + \frac{M_0 g}{u}e^{sx/u}, \quad v = 2.\end{aligned} \tag{27}$$

Subject to boundary conditions

$$W_v^i(X,y,s) = 0 = \frac{\partial W_v^i(X,y,s)}{\partial X}, \qquad v = 0,1,3,4,\ldots \tag{28}$$

Similarly, near $y = 0$ or $y = b$, one obtains the differential equations

$$\begin{aligned}\alpha_2^2\frac{\partial^4 W_{v(x,Y,s)}}{\partial y^4} - (\beta_2^2 + \alpha_{0t}s^2 + G_0)\frac{\partial^2 W_{v(x,Y,s)}}{\partial y^2} \\ = \left[\beta_1^2 + \alpha_{0t}s^2 + G_0 - c^2\Gamma_0\delta(y-y_0)\right]\frac{\partial^2 W_{v-2(x,Y,s)}}{\partial x^2} - \frac{\partial^4 W_{v-4}(x,Y,s)}{\partial x^4} - 2\alpha_1^2\frac{\partial^4 W_{v-2(x,Y,s)}}{\partial x^2\partial y^2} \\ - 2sc\Gamma_0\delta(y-y_0)\frac{\partial W_{v-2}(x,Y,s)}{\partial x} - \left(s^2 + K_0 + +s^2\Gamma_0\delta(y-y_0)\right)W_{v-2(x,Y,s)}, \quad v = 0,1,3,4,\ldots\end{aligned} \tag{29}$$

$$\begin{aligned}\alpha_2^2\frac{\partial^4 W_{v(x,Y,s)}}{\partial y^4} - (\beta_2^2 + \alpha_{0t}s^2 + G_0)\frac{\partial^2 W_{v(x,Y,s)}}{\partial y^2} \\ = \left[\beta_1^2 + \alpha_{0t}s^2 + G_0 - c^2\Gamma_0\delta(y-y_0)\right]\frac{\partial^2 W_{v-2(x,Y,s)}}{\partial x^2} - \frac{\partial^4 W_{v-4}(x,Y,s)}{\partial x^4} - 2\alpha_1^2\frac{\partial^4 W_{v-2(x,Y,s)}}{\partial x^2\partial y^2} \\ - 2sc\Gamma_0\delta(y-y_0)\frac{\partial W_{v-2}(x,Y,s)}{\partial x} - \begin{bmatrix}s^2 + K_0 \\ +s^2\Gamma_0\delta(y-y_0)\end{bmatrix}W_{v-2(x,Y,s)} + M_0 g\,\delta(y-y_0)e^{\frac{-s}{c}x}, \quad v = 2\end{aligned} \tag{30}$$

Subject to the boundary conditions

$$W_v^i(x,Y,s) = 0 = \frac{\partial W_v^i(x,Y,s)}{\partial Y}, \qquad v = 0,1,2,3,\ldots \tag{31}$$

SOLUTION PROCESS

The solutions of equations (20) for the function $W_v(x,y,s)$ and equations (2), (27), (29) and (30) for the functions $W_v(X,y,s)$ and $W_v(x,Y,s)$ subject to the respective boundary conditions (28) and (31) are sought using finite Fourier sine integral transformation method.

LEADING ORDER SOLUTION

Here the solutions of $W_0^o(x,y,s)$ and $W_0^i(x,y,s)$ are sought.

SOLUTION FOR $W_0^o(x,y,s)$

Substitute $v = 0$ in the recurrence equation (20), the governing differential equation for $W_0^o(x,y,s)$ is given as

$$\begin{aligned}\beta_1^2\frac{\partial^2 W_{0(x,y,s)}}{\partial x^2} + \beta_2^2\frac{\partial^2 W_{0(x,y,s)}}{\partial y^2} - s^2V_0 + \alpha_{ot}s^2\left[\frac{\partial^2 W_{0(x,y,s)}}{\partial x^2} + \frac{\partial^2 W_{0(x,y,s)}}{\partial y^2}\right] - K_0 W_0(x,y,s) \\ + G_0\left[\frac{\partial^2 W_0(x,y,s)}{\partial x^2} + \frac{\partial^2 W_{0(x,y,s)}}{\partial y^2}\right] - \Gamma_0\delta(y-y_0)\left[s^2 W_{0(x,y,s)} + 2sc\frac{\partial W_{0(x,y,s)}}{\partial x} + c^2\frac{\partial^2 W_{0(x,y,s)}}{\partial x^2}\right] \\ = -M_0 g\,\delta(y-y_0)e^{\frac{-s}{c}x}\end{aligned} \tag{32}$$

Now, one attempts equation (32) for the solution of $W_0^o(x,y,s)$ by introducing the finite Fourier sine transform defined as

$$W(m,y,s) = \int_0^1 W(x,y,s)\sin m\pi x \; dx \qquad (33$$

With the inverse

$$W(x,y,s) = 2\sum_{m=1}^{\infty} W(m,y,s)\sin m\pi x \qquad (34)$$

and

$$W(x,n,s) = \int_0^b W(x,y,s)\sin\frac{n\pi y}{b}\; dy \qquad (35)$$

with the inverse

$$W(x,y,s) = \frac{2}{b}\sum_{m=1}^{\infty} W(x,n,s)\sin\frac{n\pi y}{b} \qquad (36)$$

Thus, the transformation of (32) with respect to x is

$$\frac{\partial^2 W_0(m,y,s)}{\partial y^2} + \varphi_1^2 W_0^0(m,y,s) = \tau_1\delta(y-y_0) \qquad (37)$$

where

$$\varphi_1^2 = \frac{\beta_1^2 m^2\pi^2 - s^2 + m^2\pi^2\alpha_{0t}s^2 - K_0 + m^2\pi^2 G_0 - \Gamma_0\delta(y-y_0) + (s^2 + c^2m^2\pi^2)}{[\beta_2^2 + \alpha_{0t}s^2 + G_0]} \qquad (37.1)$$

$$\tau_1 = \frac{M_0 g m\pi c^2\left[1-(-1)^m e^{\frac{-s}{c}}\right]}{(\beta_2^2 + \alpha_{0t}s^2 + G_0)(s^2 + c^2m^2\pi^2)} \qquad (37.2)$$

The general solution of (34) is

$$W_0^0(m,y,s) = G_1 cos\varphi_1 y + G_2 sin\varphi_1 y + \frac{\tau_1}{\varphi_1} sin\varphi_1(y-y_0) \qquad (38)$$

While the transformation with respect to y is

$$\frac{\partial^2 W_0^o(x,n,s)}{\partial x^2} + \varphi_2\frac{\partial W_0^o(x,n,s)}{\partial x} + \varphi_3 W_0^o(x,n,s) = \tau_2 e^{-sx/u} \qquad (39)$$

where

$$\eta_1 = \beta_1^2 + \alpha_{0t}s^2 + G_0 - \frac{\Gamma_0 c^2}{b} \qquad (40.1)$$

$$\varphi_2 = \frac{2\Gamma_0 sc}{b}\Big/\eta_1 \qquad (40.2)$$

$$\varphi_3 = \frac{\eta^2\pi^2}{b^2}\beta_2^2 - s^2 + \frac{\eta^2\pi^2}{b^2}\alpha_{0t}s^2 - K_0 + \frac{\eta^2\pi^2}{b^2}G_0 - \frac{s^2\Gamma_0}{b}\Big/\eta_1 \qquad (40.3)$$

$$\tau_2 = \frac{M_0}{c}\,\mathrm{g}\sin\frac{n\pi y_0}{b}\Big/\eta_1 \qquad (40.4)$$

The complimentary solution of (39) is

$$W_{0\,c}(x,n,s) = G_3 e^{\gamma_1 x} + G_4 e^{\gamma_2 x} \qquad (41)$$

where

$$\gamma_1 = \varphi_2 + \sqrt{\varphi_2^2 - 4\varphi_3} \qquad (42.1)$$

$$\gamma_2 = \varphi_2 - \sqrt{\varphi_2^2 - 4\varphi_3} \qquad (42.2)$$

Making use of the method of variation of parameters, the particular solution of (39) can be shown to be

$$W_{0\,p}(x,n,s) = \frac{\tau_2}{(\gamma_1-\gamma_2)}\left(\frac{c}{c\gamma_1+s}\right)\left[1-e^{-\left(\frac{c\gamma_1+s}{c}\right)}\right]e^{\gamma_1 x} + \frac{\tau_2}{(\gamma_2-\gamma_1)}\left(\frac{c}{c\gamma_2+s}\right)\left[1-e^{-\left(\frac{c\gamma_2+s}{c}\right)}\right]e^{\gamma_2 x} \qquad (43)$$

Consequently, the general solution of the ordinary differential equation (39) is

$$W_0^0\,(x,n,s) = G_3 e^{\gamma_1 x} + G_4 e^{\gamma_1 x} + \frac{\tau_2}{(\gamma_1-\gamma_2)}\left(\frac{c}{c\gamma_1+s}\right)\left[1-e^{-\left(\frac{c\gamma_1+s}{c}\right)}\right]e^{\gamma_1 x} + \frac{\tau_2}{(\gamma_2-\gamma_1)}\left(\frac{c}{c\gamma_2+s}\right)\left[1-e^{-\left(\frac{c\gamma_2+s}{c}\right)}\right]e^{\gamma_2 x} \tag{44}$$

The inversion of (38) and (44) gives the general solution of the equation (31) as

$$W_o^o(x,y,s) = 2\left[G_1 cos\varphi_1 y + G_2 sin\varphi_1 y + \frac{\tau_1}{\varphi_1}sin\varphi_1(y-y_0)\right]\sin m\pi x + \frac{2}{b}\Big[G_3 e^{\gamma_1 x} += G_4 e^{\gamma_1 x} + \frac{\tau_2}{(\gamma_1-\gamma_2)}\left(\frac{c}{c\gamma_1+s}\right)\left[1-e^{-\left(\frac{c\gamma_1+s}{c}\right)}\right]e^{\gamma_1 x} + \frac{\tau_2}{(\gamma_2-\gamma_1)}\left(\frac{c}{c\gamma_2+s}\right)\left[1-e^{-\left(\frac{c\gamma_2+s}{c}\right)}\right]e^{\gamma_2 x}\Big]\sin\frac{n\pi y}{b} \tag{45}$$

where G_1, G_2, G_3 and G_4 are arbitrary constants yet to be determined by matching.

LEADING ORDER SOLUTION (INNER PROBLEM)

The differential equation governing the inner solution $(near\ x = 0,1,)$ in equation (27) where one neglects the terms with negative subscripts, one obtains for the leading order problem

$$\frac{\partial^4 w_0^i(X,y,s)}{\partial X^4} - [\beta_1^2 + \alpha_{0t}s^2 + G_0 - c^2\Gamma_0\delta(y-y_0)]\frac{\partial^2 w(X,y,s)}{\partial X^2} = 0 \tag{46}$$

subject to

$$w_0^i(X,y,s) = 0 = \frac{\partial w_0^i(X,y,s)}{\partial X} \tag{47}$$

Solving equation (46) subject to equation (47) produces

$$W_0^i(X,y,s) = \begin{cases} \breve{b}_0(y)\left[X + \frac{1}{\theta_1}e^{-\theta_1 X} - \frac{1}{\theta_1}\right], & near\ x = 0 \\ \breve{b}_0(y)\left[X + \frac{1}{\theta_1}e^{-\theta_1 X} - \frac{1}{\theta_1}\right], & near\ x = 1 \end{cases} \tag{48}$$

where

$$\theta_1^2 = \beta_1^2 + \alpha_{0t}s^2 + G_0 - c^2\Gamma_0\delta(y-y_0)$$

Similarly, the differential equation governing the inner solution (near $y = 0, b$) in equation (29), when one neglects the terms with negative subscript, one obtains for the leading order problem

$$\frac{\partial^4 W_0^i(x,Y,s)}{\partial Y^4} - \theta_2^2\frac{\partial^2 W(x,Y,s)}{\partial Y^2} = 0 \tag{49}$$

Subject to

$$W_0^i(x,Y,s) = \frac{\partial W_0^i(x,Y,s)}{\partial Y} = 0 \tag{50}$$

where

$$\theta_2^2 = \frac{(\beta_1^2 + \alpha_{0t}s^2 + G_0)}{\alpha_2^2} \tag{51}$$

Solving equation (49) subject to equation (50) produces

$$W_0^i(x,Y,s) = \begin{cases} \breve{f}_0(x)\left[Y + \frac{1}{\theta_2}e^{-\theta_2 Y} - \frac{1}{\theta_2}\right], & near\ y = 0 \\ \breve{\breve{f}}_0(x)\left[Y + \frac{1}{\theta_2}e^{-\theta_2 Y} - \frac{1}{\theta_2}\right], & near\ y = b \end{cases} \tag{52}$$

Thus, the leading order solution of the inner problem $(27-31)$ can be written down as

$$\begin{matrix} \tilde{b}_0(y)\left[X + \frac{1}{\theta_1}e^{-\theta_1 X} - \frac{1}{\theta_1}\right] & near\ x = 0 \\ \tilde{\tilde{b}}_0(y)\left[X + \frac{1}{\theta_1}e^{-\theta_1 X} - \frac{1}{\theta_1}\right] & near\ x = 1 \end{matrix} \tag{53}$$

$$\tilde{f}_0(x)\left[Y + \frac{1}{\theta_2}e^{-\theta_2 Y} - \frac{1}{\theta_2}\right] \qquad near\ y = 0$$

$$\tilde{\tilde{f}}_0(x)\left[Y + \frac{1}{\theta_2}e^{-\theta_2 Y} - \frac{1}{\theta_2}\right] \qquad near\ y = b$$

where exponentially growing terms have been discarded as unmatchable. The functions $\tilde{b}_0, \tilde{\tilde{b}}_0, \tilde{f}_0\ and\ \tilde{\tilde{f}}_0$ are yet to be determined. The unknowns in (45) and (53) will be determined by matching inner and outer solutions. To this end, Van Dyke's matching principle which requires m-term inner expansion of (the n-term outer expansion) equals the n-term outer expansion of (the m-term inner expansion) is adopted. Thus matching one-term outer expansion written in inner variable (45) with one term inner expansion written in outer variable (53) (1-1 matching) immediately produces

$$G_1 = \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{\frac{-s}{c}}\right]}{(\beta_2^2 - \alpha_{0t}s^2 + G_0)(s^2 + c^2m^2\pi^2)} sin\varphi_1 y_0 \tag{54.1}$$

$$G_2 = \frac{\tau_1}{\varphi_1} sin\varphi_1 y_0 cot\varphi_1 b - \frac{\tau_1}{\varphi_1} cos\varphi_1 y_0 \ - \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right] sin\varphi_1 y_0 cot\varphi_1 b}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\ (s^2 + c^2m^2\pi^2)} \tag{54.2}$$

$$G_3 = \frac{\tau_2}{(\gamma_1-\gamma_2)}\left(\frac{c}{c\gamma_1+s}\right)\left[1 - e^{-\left(\frac{c\gamma_1+s}{c}\right)}\right] \tag{54.3}$$

$$G_4 = \frac{\tau_2}{(\gamma_1-\gamma_2)}\left(\frac{c}{c\gamma_2+s}\right)\left[1 - e^{-\left(\frac{c\gamma_2+s}{c}\right)}\right] \tag{54.4}$$

In view of equations (54) equation (45) becomes

$$\begin{aligned} W_0(x,y,s) = &\frac{2M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\ (s^2 + c^2m^2\pi^2)} sin\varphi_1 y_0 cos\varphi_1 y sinm\pi x \\ &+ 2\left[\frac{\tau_1}{\varphi_1} sin\varphi_1 y_0 \frac{cos\varphi_1 b sin\varphi_1 y}{sin\varphi_1} - \frac{\tau_1}{\varphi_1} cos\varphi_1 y_0 sin\varphi_1 y \right. \\ &\left. - \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\ (s^2 + c^2m^2\pi^2)} sin\varphi_1 y_0 cos\varphi_1 y sin\varphi_1 y\right] sinm\pi x + 2\frac{\tau_1}{\varphi_1} sin\varphi_1(y - y_0) sinm\pi x \\ &+ \left\{\frac{2}{b}e^{\gamma_2 x}\frac{\tau_2}{(\gamma_2-\gamma_1)}\left(\frac{c}{c\gamma_1+s}\right)\left[1 - e^{-\left(\frac{c\gamma_1+s}{c}\right)}\right] + \frac{2}{b}e^{\gamma_1 x}\frac{\tau_2}{(\gamma_1-\gamma_2)}\left(\frac{c}{c\gamma_1+s}\right)\left[1 - e^{-\left(\frac{c\gamma_2+s}{c}\right)}\right]\right. \\ &\left. + \frac{2}{b}e^{\gamma_2 x}\frac{\tau_2}{(\gamma_2-\gamma_1)}\left(\frac{c}{c\gamma_2+s}\right)\left[1 - e^{-\left(\frac{c\gamma_2+s}{c}\right)}\right]\right\} sin\frac{n\pi y}{b} \end{aligned} \tag{55}$$

where

$$\varphi_1^2 = \frac{\left[\beta_1^2 m^2\pi^2 - s^2 + m^2\pi^2\alpha_{0t}s^2 - K_0 + m^2\pi^2 G_0 - \Gamma_0\delta(y - y_0)(s^2 + c^2m^2\pi^2)\right]}{\beta_2^2 + \alpha_{0t}s^2 + G_0}$$

$$\varphi_2 = \frac{2\Gamma_0 sc}{\beta_2^2 + \alpha_{0t}s^2 + G_0 - \frac{\Gamma_0 c^2}{b}}$$

$$\varphi_3 = \frac{n^2\pi^2\beta_2^2 - b^2 s^2 + n^2\pi^2\alpha_{0t}s^2 - b^2K_0 + n^2\pi^2 G_0 - bs^2\Gamma_0}{b^2\beta_1^2 + b^2\alpha_{0t}s^2 + b^2 G_0 - b\Gamma_0 c^2}$$

$$\tau_1 = \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{(\beta_1^2 + \alpha_{0t}s^2 + G_0)\ (s^2 + c^2m^2\pi^2)}$$

$$\tau_2 = \frac{M_0 g e^{-sx/c} sin\frac{n\pi y_0}{b}}{\beta_1^2 + \alpha_{0t}s^2 + G_0 - \frac{\Gamma_0 c^2}{b}}$$

$$\gamma_1 = \varphi_2 + \sqrt{\varphi_2^2 - 4\varphi_3}$$

$$\gamma_2 = \varphi_2 - \sqrt{\varphi_2^2 - 4\varphi_3}$$

$$\beta_1^2 - \alpha_{0t}s^2 + G_0 = -\alpha_{0t}\left[s^2 - \frac{\beta_1^2}{\alpha_{0t}} - \frac{G_0}{\alpha_{0t}}\right] - \alpha_{0t}[s^2 - w_1^2] \qquad (56)$$

Futher simplification of equation (55) produces

$$W(x,y,s) = 2M_\circ g m\pi c^2 \sin m\pi x \left(\frac{-[1-(-1)^m e^{-s/c}]}{\alpha_{ot}(s^2+\Lambda_5)(s^2+\Lambda_2)} \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y_o \cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) y \right.$$
$$+ \frac{[1-(-1)^m e^{-s/c}] \cos\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] b \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y}{\Lambda_3(s^2+\Lambda_4)(s^2+\Lambda_2)\sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] b}$$
$$+ \frac{-[1-(-1)^m e^{-s/c}]}{\alpha_{ot}(s^2+\Lambda_5)(s^2+\Lambda_2)} \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y_o \cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) y \sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) y$$
$$+ \frac{[1-(-1)^m e^{-s/c}]}{2\Lambda_3(s^2+\Lambda_4)(s^2+\Lambda_2)} \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y \cos\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y_0$$
$$\left. - \frac{[1-(-1)^m e^{-s/c}]}{\Lambda_3(s^2+\Lambda_4)(s^2+\Lambda_2)} \cos\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y_\circ \right)$$
$$+ \frac{2M_\circ g m\pi c^2 \sin\frac{n\pi y}{b}}{b\alpha_{ot}} \left(\left(\frac{c}{cr_1+s}\right) \frac{[1-e^{-(cr_1+s/c)}] e^{r_1 x}}{r_1 - r_2} \left(\frac{1}{s^2+\Lambda_6} - \frac{1}{s^2+\Lambda_1}\right) \right) \qquad (57)$$

The Laplace inversion of (57) is defined as

$$W_0^o(x,y,t) = P_{b_1}\big(F_1(x,y,t) + F_2(x,y,t) - F_3(x,y,t) + F_4(x,y,t) - F_5(x,y,t)\big) + P_{b_2}\big(F_6(x,y,t) - F_7(x,y,t)\big) \qquad (58)$$

where

$$P_{b_1} = 2M_\circ g m\pi c^2 \sin m\pi x \qquad (59.1)$$

$$P_{b_2} = \frac{2M_\circ g m\pi c^2}{b\alpha_{ot}} \sin\frac{n\pi y}{b} \qquad (59.2)$$

$$F_1(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} \frac{[1-(-1)^m e^{-s/c}]}{\alpha_{ot}(s^2+\Lambda_5)(s^2+\Lambda_2)} \sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) y_o \cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) y \; ds \quad (60.1)$$

$$F_2(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} \frac{[1-(-1)^m e^{-s/c}]}{\Lambda_3(s^2+\Lambda_4)(s^2+\Lambda_2)} \frac{\cos\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] b \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y}{\sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] b} ds \quad (60.2)$$

$$F_3(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} \frac{[1-(-1)^m e^{-s/c}]}{\alpha_{ot}(s^2+\Lambda_5)(s^2+\Lambda_2)} \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y \cos\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y \; ds (60.3)$$

$$F_4(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} \frac{[1-(-1)^m e^{-s/c}]}{\Lambda_3(s^2+\Lambda_4)(s^2+\Lambda_2)} \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y \cos\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y_o \, ds \quad (60.4)$$

$$F_5(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} \frac{[1-(-1)^m e^{-s/c}]}{\Lambda_3(s^2+\Lambda_4)(s^2+\Lambda_2)} \cos\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y \sin\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right] y_o ds \quad (60.5)$$

$$F_6(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} \frac{c\,[1-e^{-(cr_1+s/c)}]\, e^{r_1 x}}{(cr_1+s)(r_1-r_2)\,(s^2+\Lambda_6)} ds \qquad (60.6)$$

$$F_7(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\frac{c\,[1-e^{-(cr_1+s/c]}\,e^{r_1x}}{(cr_1+s)(r_1-r_2)\,(s^2+\Lambda_1)}\,ds \qquad (60.7)$$

In order to evaluate the integrals in (60), the Cauchy residue theorem is employed. The singularities in the integrals are poles. In particular the denominators of the integrands of $F_1(x,y,t)$, $F_2(x,y,t)$,$F_3(x,y,t)$, $F_4(x,y,t)$, and $F_5(x,y,t)$ have simple poles at at $S = \pm i\Lambda_5$,$S = \pm i\Lambda_2$, $S = \pm i\Lambda_4$, $S = \pm i\Lambda_{21}$It is straightforward to show that

$$\begin{aligned}
F_1\,(x,y,t) &= \frac{\text{Sin}\left[\frac{\Lambda_3(\Lambda_4-\Lambda_5^2)}{\alpha_{ot}(\Lambda_1-\Lambda_5^2)}\right]y_o\,\text{Cos}[\frac{\Lambda_3(\Lambda_4-\Lambda_5^2)}{\alpha_{ot}(\Lambda_1-\Lambda_5^2)}]\,\text{y}}{2\text{i}\Lambda_5(\Lambda_2-\Lambda_5^2)\,\alpha_{ot}}\left(e^{i\Lambda_5t}\left[1-(-1)^m e^{-i\Lambda_5/c}\right]-e^{-i\Lambda_5t}\left[1-(-1)^m e^{i\Lambda_5/c}\right]\right)\\
&+\frac{\text{Sin}\,[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}]\,y_o\ \text{Cos}[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}]\,\text{y}}{2\text{i}\Lambda_2(\Lambda_5-\Lambda_2^2)\,\alpha_{ot}}\left(e^{i\Lambda_2t}\left[1-(-1)^m e^{-i\Lambda_2/c}\right]\right.\\
&\left.-e^{-i\Lambda_2t}\left[1-(-1)^m e^{i\Lambda_2/c}\right]\right)
\end{aligned} \qquad (61.1)$$

$$\begin{aligned}
F_2\,(x,y,t) &= \frac{\text{Cos}[\frac{\Lambda_3(\Lambda_4-\Lambda_4^2)}{\alpha_{ot}(\Lambda_1-\Lambda_4^2)}]\ \text{b Sin}\,[\frac{\Lambda_3(\Lambda_4-\Lambda_4^2)}{\alpha_{ot}(\Lambda_1-\Lambda_4^2)}]\,\text{y}}{2\,\Lambda_3\,\Lambda_4\,(\Lambda_2-\Lambda_4^2)(\Lambda_{21}^2+\Lambda_4^2)}\left(ie^{i\Lambda_4t}\left[1-(-1)^m e^{-i\Lambda_4/c}\right]-ie^{\Lambda_4t}\left[1-(-1)^m e^{i\Lambda_4/c}\right]\right)\\
&+\frac{\text{Cos}\left[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right]b\ \text{Sin}\,[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}]\ y}{2\,\Lambda_2\,\Lambda_3\,(\Lambda_2-\Lambda_2^2)(\Lambda_{21}^2-\Lambda_2^2)}\left(ie^{-i\Lambda_2t}\left[1-(-1)^m e^{i\Lambda_2/c}\right]\right.\\
&\left.-\ ie^{i\Lambda_2t}\left[1-(-1)^m e^{-i\Lambda_2/c}\right]\right)\\
&+\frac{\text{Cos}[\frac{\Lambda_3(\Lambda_{21}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{21}^2+\Lambda_1)}]\ \text{b Sin}\left[\frac{\Lambda_3(\Lambda_{21}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{21}^2+\Lambda_1)}\right]y}{2\,\Lambda_{21}\,\Lambda_3\,(\Lambda_{21}^2+\Lambda_4)(\Lambda_{21}^2+\Lambda_2)}\left(e^{\Lambda_{21}t}\left[1-(-1)^m e^{-\Lambda_{21}/c}\right]\right.\\
&\left.+\ e^{-\Lambda_{21}t}\left[1-(-1)^m e^{\Lambda_{21}/c}\right]\right)
\end{aligned} \qquad (61.2)$$

$$\begin{aligned}
F_3\,(x,y,t) &= \frac{\text{Sin}\,[\frac{\Lambda_3(\Lambda_4-\Lambda_5^2)}{\alpha_{ot}(\Lambda_1-\Lambda_5^2)}]\ y_o\ \text{Cos}\,[\frac{\Lambda_3(\Lambda_4-\Lambda_5^2)}{\alpha_{ot}(\Lambda_1-\Lambda_5^2)}]\,\text{y Sin}\,[\frac{\Lambda_3(\Lambda_4-\Lambda_5^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}]\,\text{y}}{2i\alpha_{ot}\Lambda_5\,(\Lambda_2-\Lambda_5^2)}\left(e^{i\Lambda_5t}\left[1-(-1)^m e^{-i\Lambda_5/c}\right]e^{-i\Lambda_5t}\right[1\\
&\left.-(-1)^m e^{i\Lambda_5/c}]\right)+\sin\left[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right]\ y_o\ \cos\left[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right]\text{y Sin}\left[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right]\text{y}\\
&\times\left(e^{i\Lambda_2t}\left[1-(-1)^m e^{-i\Lambda_2/c}\right]-e^{-i\Lambda_2t}\left[1-(-1)^m e^{i\Lambda_2/c}\right]\right)
\end{aligned} \qquad (61.3)$$

$$\begin{aligned}
F_4\,(x,y,t) &= \frac{\sin\,[\frac{\Lambda_3(\Lambda_4-\Lambda_4^2)}{\alpha_{ot}(\Lambda_1-\Lambda_4^2)}]\ y\ \cos\,[\frac{\Lambda_3(\Lambda_4-\Lambda_4^2)}{\alpha_{ot}(\Lambda_1-\Lambda_4^2)}]\ y_o}{2\Lambda_3\,\Lambda_4(\Lambda_4^2-\Lambda_4)}\left(ie^{i\Lambda_4t}\left[1-(-1)^m e^{-i\Lambda_4/c}\right]\right)\\
&-\left(ie^{i\Lambda_2t}\left[1-(-1)^m e^{-i\Lambda_2/c}\right]\right)-\left(ie^{-i\Lambda_2t}\left[1-(-1)^m e^{i\Lambda_2/c}\right]\right)
\end{aligned} \qquad (61.4)$$

$$\begin{aligned}
F_5\,(x,y,t) &= \cos\,[\frac{\Lambda_3(\Lambda_4-\Lambda_4^2)}{\alpha_{ot}(\Lambda_1-\Lambda_4^2)}]\ \text{y}\ \sin\ [\frac{\Lambda_3(\Lambda_4-\Lambda_4^2)}{\alpha_{ot}(\Lambda_1-\Lambda_4^2)}]\ y_o\left(ie^{i\Lambda_4t}\left[1-(-1)^m e^{-i\Lambda_4/c}\right]-ie^{-i\Lambda_4t}\left[1-(-1)^m e^{i\Lambda_4/c}\right]\right)\\
&+\cos\,[\frac{\Lambda_3(\Lambda_4-\Lambda_4^2)}{\alpha_{ot}(\Lambda_1-\Lambda_4^2)}]\ \text{y}\ \sin\,[\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}]y_o\\
&\times\left(ie^{i\Lambda_2t}\left[1-(-1)^m e^{-i\Lambda_2/c}\right]-ie^{-i\Lambda_2t}\left[1-(-1)^m e^{i\Lambda_2/c}\right]\right)
\end{aligned} \qquad (61.5)$$

$$\begin{aligned}
f_6(x,y,x) &= \frac{e^{\Lambda_{11}t}[1-e^{-\alpha_1}]e^{k_1x}\sqrt{(\Lambda_{11}^2+\Lambda_1)(\Lambda_{11}^2+\Lambda_9)}}{(\Lambda_{11}-\Lambda_{17})(\Lambda_{11}-\Lambda_{18})(\Lambda_{11}-\Lambda_{19})(\Lambda_{11}-\Lambda_{20})(\Lambda_{11}^2-\Lambda_6)\sqrt{(\Lambda_{11}-\Lambda_{12})(\Lambda_{11}-\Lambda_{13})(\Lambda_{11}-\Lambda_{14})}}\\
&+\frac{e^{\Lambda_{12}t}[1-e^{-\alpha_2}]e^{k_2x}\sqrt{(\Lambda_{12}^2+\Lambda_1)(\Lambda_{12}^2+\Lambda_9)}}{(\Lambda_{12}-\Lambda_{17})(\Lambda_{12}-\Lambda_{18})(\Lambda_{12}-\Lambda_{19})(\Lambda_{12}-\Lambda_{20})(\Lambda_{12}^2-\Lambda_6)\sqrt{(\Lambda_{12}-\Lambda_{11})(\Lambda_{12}-\Lambda_{13})(\Lambda_{12}-\Lambda_{14})}}\\
&+\frac{e^{\Lambda_{13}t}[1-e^{-\alpha_3}]e^{k_3x}\sqrt{(\Lambda_{13}^2+\Lambda_1)(\Lambda_{13}^2+\Lambda_9)}}{(\Lambda_{13}-\Lambda_{17})(\Lambda_{13}-\Lambda_{18})(\Lambda_{13}-\Lambda_{19})(\Lambda_{13}-\Lambda_{20})(\Lambda_{13}^2-\Lambda_6)\sqrt{(\Lambda_{13}-\Lambda_{11})(\Lambda_{13}-\Lambda_{13})(\Lambda_{13}-\Lambda_{14})}}\\
&+\frac{e^{\Lambda_{14}t}[1-e^{-\alpha_4}]e^{k_4x}\sqrt{(\Lambda_{14}^2+\Lambda_1)(\Lambda_{14}^2+\Lambda_9)}}{(\Lambda_{14}-\Lambda_{17})(\Lambda_{14}-\Lambda_{18})(\Lambda_{14}-\Lambda_{19})(\Lambda_{14}-\Lambda_{20})(\Lambda_{14}^2-\Lambda_6)\sqrt{(\Lambda_{14}-\Lambda_{11})(\Lambda_{14}-\Lambda_{13})(\Lambda_{14}-\Lambda_{13})}}
\end{aligned} \qquad (61.6)$$

$$\begin{aligned}
&f_7(x,y,t)\\
&= \frac{e^{\Lambda_{11}t}[1-e^{-\alpha_1}]e^{k_1x}}{2\Lambda_{15}(\Lambda_{11}-\Lambda_{17})(\Lambda_{11}-\Lambda_{18})(\Lambda_{11}-\Lambda_{19})(\Lambda_{11}-\Lambda_{20})(\Lambda_{11}^2-\Lambda_1)\sqrt{(\Lambda_{11}-\Lambda_{12})(\Lambda_{11}-\Lambda_{13})(\Lambda_{11}-\Lambda_{14})}}\\
&+\frac{e^{\Lambda_{12}t}[1-e^{-\alpha_2}]e^{k_2x}}{2\Lambda_{15}(\Lambda_{12}-\Lambda_{17})(\Lambda_{12}-\Lambda_{18})(\Lambda_{12}-\Lambda_{19})(\Lambda_{12}-\Lambda_{20})(\Lambda_{12}^2-\Lambda_1)\sqrt{(\Lambda_{12}-\Lambda_{11})(\Lambda_{12}-\Lambda_{13})(\Lambda_{12}-\Lambda_{14})}}\\
&+\frac{e^{\Lambda_{13}t}[1-e^{-\alpha_3}]e^{k_3x}}{2\Lambda_{15}(\Lambda_{13}-\Lambda_{17})(\Lambda_{13}-\Lambda_{18})(\Lambda_{13}-\Lambda_{19})(\Lambda_{13}-\Lambda_{20})(\Lambda_{13}^2-\Lambda_1)\sqrt{(\Lambda_{13}-\Lambda_{11})(\Lambda_{13}-\Lambda_{12})(\Lambda_{13}-\Lambda_{14})}}\\
&+\frac{e^{\Lambda_{14}t}[1-e^{-\alpha_4}]e^{k_4x}}{2\Lambda_{15}(\Lambda_{14}-\Lambda_{17})(\Lambda_{14}-\Lambda_{18})(\Lambda_{14}-\Lambda_{19})(\Lambda_{14}-\Lambda_{20})(\Lambda_{14}^2-\Lambda_1)\sqrt{(\Lambda_{14}-\Lambda_{11})(\Lambda_{14}-\Lambda_{12})(\Lambda_{14}-\Lambda_{13})}}
\end{aligned} \qquad (61.7)$$

The combination of the results (61.1 – 61.7) substituted into (58) yields the desired leading order solution of (27) which represents the uniformly valid solution of the entire domain of definition of the given plate.

First order correction

Solution for $W_1^o(x,y,t)$

The next corrections in outer solution are obtained by setting $\nu = 1$ in equation (20). The governing equation for $W_1^o(x,y,t)$ is given as

$$-\gamma_1^2\frac{\partial^2 W_1}{\partial x^2}(x,y,s)-\gamma_2^2\frac{\partial^2 W_1}{\partial y^2}(x,y,s)+s^2W_1(x,y,s)-\alpha_{0t}s^2\left[\frac{\partial^2 N_1}{\partial x^2}(x,y,s)+\frac{\partial^2 W}{\partial y^2}(x,y,s)\right]$$
$$+\Gamma_0\delta(y-y_0)\left[s^2W_1(x,y,s)+2us\frac{\partial W_1}{\partial x}(x,y,s)+u^2\frac{\partial^2 W_1}{\partial x^2}(x,y,s)\right]=0 \tag{62}$$

Now one attempts the solution of $W_1(x,y,s)$ by introducing the finite Fourier sine transform of (33) in equation (62) with respect to x produces

$$-\gamma_1^2(m\pi)^2W_1(m,y,s)-\gamma_2^2\ \frac{\partial^2 W_1}{\partial y^2}(m,y,s)\ +\ s^2\ W_1(m,y,s)+\alpha_{0t}s^2\left[(m\pi)^2W_1(m,y,s)+\frac{\partial^2 W_1}{\partial y^2}(m,y,s)\right]$$
$$+\ \Gamma_0\delta(y-y_0)[s^2\ -(m\pi u)^2\ ]W_1(m,y,s)=0 \tag{63}$$

Re-arranging (63) gives

$$W_{1,yy}(m,y,s)+\eta^2W_1(m,y,s)=0 \tag{64}$$

where

$$\eta^2=\left[\frac{[s^2+m^2\pi^2\alpha_{0t}s^2-m^2\pi^2\gamma_1^2+\Gamma_0\delta(y-y_0)(s^2\ -(m\pi u)^2)]}{\alpha_{0t}s^2-\gamma_2^2}\right]$$

The homogeneous solution of (64) gives

$$W_1(m,y,s)=B_1\cos\eta y-B_2\sin\eta y \tag{65}$$

Similarly, if equation (62) is subjected to finite Fourier sine transform (35) with respect to y, one obtains

$$\left[-\gamma_1^2-\alpha_{0t}s^2+\frac{u^2\Gamma_0}{b}\right]\frac{\partial^2 W_1(x,n,s)}{\partial x^2}+\left[\left(\frac{n\pi\gamma_2}{b}\right)^2+s^2-\alpha_{0t}\left(\frac{n\pi}{b}\right)^2s^2+\frac{\Gamma_0}{b}s^2W_1(x,n,s)=0\right. \tag{66}$$

Which when rewritten produces

$$\frac{\partial^2 W_{1(x,n,s)}}{\partial x^2}+\eta_2^2W_1(x,n,s)=0 \tag{67}$$

where

$$\eta_2^2=\frac{\left(\frac{n\pi\gamma_2}{b}\right)+s^2-\alpha_{0t}\left(\frac{n\pi}{b}\right)^2s^2}{-\gamma_1^2-\alpha_{0t}s^2+\frac{u^2\Gamma_0}{b}}$$

The solution of which is

$$W_1(x,n,s)=B_3\cos\eta_2x-B_4\sin\eta_2 x \tag{68}$$

The finite Fourier sine inversion of equation (65) together with equation (68) gives

$$W_1(x,y,s)=2\sum_{m=1}^{\infty}[B_1\cos\eta y-B_2 sin\eta y]\,sin m\pi x+\frac{2}{b}\sum_{n=1}^{\infty}[B_3\cos\eta_2x-B_4\sin\eta_2 x]\sin\frac{n\pi y}{b} \tag{69}$$

where $B_1,\ B_2,\ B_3\ and\ B_4$ are unknown constants to be determined by matching.

FIRST ORDER CORRECTION (INNER PROBLEM)

The first order correction is obtained by setting $\nu = 1$ in the differential equations (27) and (29). Doing this and neglecting terms with negative subscripts yields

$$\frac{\partial^4 \mathrm{W}^{\mathrm{i}}_{1(x,y,s)}}{\partial x^4}-\varphi_1^2\frac{\partial^4 \mathrm{W}^{\mathrm{i}}_{1(x,y,s)}}{\partial x^4}=0 \tag{70}$$

where

$$\varphi_1^2=[\gamma_1^2+\alpha_{0t}s^2-u^2\Gamma_0\delta(y-y_0)]$$

Subject to the boundary conditions

$$W_1^i(x,y,s) = 0 = \frac{\partial W^i_{1(x,y,s)}}{\partial x} \tag{71}$$

Following usual arguments in equations (48) and (52), the first order correction of the inner problem can be written as

$$W_1^i(x,y,s) = \begin{cases} \hat{b}_1(y)\left[x + \frac{1}{\varphi_1}e^{-\varphi_1 x} - \frac{1}{\varphi_1}\right] & \text{near } x = 0 \\ \hat{\hat{b}}_1(y)\left[x + \frac{1}{\varphi_1}e^{-\varphi_1 x} - \frac{1}{\varphi_1}\right] & \text{near } x = 1 \\ \hat{f}_1(x)\left[y + \frac{1}{\varphi_2}e^{-\varphi_2 y} - \frac{1}{\varphi_2}\right] & \text{near } y = 0 \\ \hat{\hat{f}}_1(x)\left[y + \frac{1}{\varphi_2}e^{-\varphi_2 y} - \frac{1}{\varphi_2}\right] & \text{near } y = b \end{cases} \tag{72}$$

Here exponentially growing terms have been neglected as unmatchable. The function $\hat{b}_1(y)$, $\hat{\hat{b}}_1(y)$, $\hat{f}_1(x)$ and $\hat{\hat{f}}_1(x)$ will be determined by matching. By matching one term outer solution with two terms inner solution expansion written in outer variable, produces the following.

$$\hat{b}_1(y) = 2m\pi\left\{\frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]sin\varphi_1 y_0}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\,(s^2 + c^2m^2\pi^2)}[cos\varphi_1 y - cot\varphi_1 b\; sin\varphi_1 y] \right.$$
$$\left. + \frac{\tau_1}{\varphi_1}sin\varphi_1 y_0[cot\varphi_1 b\; sin\varphi_1 y - cos\varphi_1 y]\right\} \tag{73}$$

$$\hat{\hat{b}}_1(y) = -2(-1)^m\, m\pi\left\{\frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\,(s^2 + c^2m^2\pi^2)}sin\varphi_1 y_0(cos\varphi_1 y - cot\varphi_1 b\; sin\varphi_1 y)\right.$$
$$\left. + \frac{\tau_1}{\varphi_1}sin\varphi_1 y_0(cot\varphi_1 b\; sin\varphi_1 y - cos\varphi_1 y) - 2\frac{\tau_1}{\varphi_1}sin\varphi_1 y cos\varphi_1 y_0\right\} \tag{74}$$

$$\hat{f}_1(x) = 2\left\{\tau_1 - \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]\varphi_1}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\,(s^2 + c^2m^2\pi^2)}\right\}sin\varphi_1 y_0 cot\varphi_1 b sinm\pi x \tag{75}$$

$$\hat{\hat{f}}_1(x) = 2\left\{\varphi_1\frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\,(s^2 + c^2m^2\pi^2)}sin\varphi_1 y_0 sin\varphi_1 b\; sinm\pi x - \tau_1 cos\varphi_1 y_0 cos\varphi_1 b\right\} \tag{76}$$

One seeks an asymptotic outer solution of the form

$$W_1^o(x,y,s) = 2[B_1\cos\eta y - B_2 sin\eta y]sinm\pi x + \frac{2}{b}[B_3\cos\eta_2 x - B_4\sin\eta_2 x]\sin\frac{n\pi y}{b} \tag{77}$$

By matching two terms outer solution with two terms inner solution (2-2 matching) of equation (72) as $\varepsilon \to 0$, one obtains

$$B_3 = b\left[2m\pi\left(\frac{T_0}{\eta}\sin y_0\right)\cos\eta y - 2m\pi\left(\frac{T_0}{\eta}\csc\eta b[\sin(b-y_0) - \sin y_0\cos\eta b]\right)\sin\eta y \right.$$
$$\left. + \frac{T_0}{\eta}m\pi\sin(y-y_0)\right] \tag{78}$$

$$B_4 = 2\left(\frac{T_0}{\eta}sin(y_0)\right)(-1)^{m+1}(m\pi) + 2\left(\frac{T_0}{\eta}\csc\eta b[sin(b-y_0) - siny_0 cos\eta b]\right)sin\eta y(-1)^{m+1}(m\pi)$$
$$+\frac{T_0}{\eta}sin(y-y_0)(-1)^{m+1} + \frac{2}{b}B_1 e^{\theta_1}\theta_1\sin\frac{n\pi y}{b} + \frac{2}{b}B_2 e^{\theta_2}\theta_2\sin\frac{n\pi y}{b}$$
$$-\frac{2}{b}\eta_4\frac{\left[e^{-\left(\frac{s}{u}+\theta_1\right)} - 1\right]}{(\theta_1-\theta_2)\left(\frac{s}{u}+\theta_1\right)}e^{\theta_1}\sin\frac{n\pi y}{b}\theta_1 - \frac{2}{b}\eta_4\frac{\left[e^{-\left(\frac{s}{u}+\theta_2\right)} - 1\right]}{(\theta_2-\theta_1)\left(\frac{s}{u}+\theta_2\right)}e^{\theta_2}\theta_2\sin\frac{n\pi y}{b} \tag{79}$$

$$B_1 = \left[\left\{\tau_1 - \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]\varphi_1}{(\beta_1^2 - \alpha_{0t}s^2 + G_0)\,(s^2 + c^2m^2\pi^2)}\right\}sin\varphi_1 y_0 cot\varphi_1 b_0\right]/\theta_2 \tag{80}$$

$$B_2 = \frac{\varphi_1 M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]\varphi_1 sin\varphi_1 y_0}{\theta_2(\beta_1^2 - \alpha_{0t}s^2 + G_0)\,(s^2 + c^2m^2\pi^2)} - \frac{\tau_1 cos\varphi_1 y_0 cos\varphi_1 b}{\theta_2 sin\varphi_1 b sinm\pi x} - \frac{\tau_1 cos\varphi_1 b}{\theta_2 sin\varphi_1 b}$$
$$+\frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]\theta_1 cos^2\varphi_1 b sin\varphi_1 y_0}{\theta_2(\beta_1^2 - \alpha_{0t}s^2 + G_0)\,(s^2 + c^2m^2\pi^2)sin\varphi_1 b} \tag{81}$$

In view of equations (78) – (81) equation (77) becomes

$$
\begin{aligned}
{}_1^0(x,y,s) = \Bigg\{ 2\left[\left\{\tau_1 - \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]\varphi_1}{\theta_2(\beta_1^2-\alpha_{0t}s^2+G_0)\,(s^2+c^2m^2\pi^2)}\right\} sin\varphi_1 y_0 cot\varphi_1 b\right] / \theta_2 \ * cos\varphi_1 y \\
+ 2\left\{\frac{\varphi_1 M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]\varphi_1 sin\varphi_1 y_0}{\theta_2(\beta_1^2-\alpha_{0t}s^2+G_0)\,(s^2+c^2m^2\pi^2)} - \frac{\tau_1 cos\varphi_1 y_0 cos\varphi_1 b}{\theta_2 sin\varphi_1 b sinm\pi x} - \frac{\tau_1 cos\varphi_1 b}{\theta_2 sin\varphi_1 b}\right. \\
\left. + \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]\theta_1 cos^2\varphi_1 b sin\varphi_1 y_0}{\theta_2(\beta_1^2-\alpha_{0t}s^2+G_0)\,(s^2+c^2m^2\pi^2) sin\varphi_1 b}\right\} sin\varphi_1 y \Bigg\} sinm\pi x \\
+ \Bigg[\Bigg\{\frac{2}{b}\frac{(-1)^m m\pi}{\theta_1(e^{\gamma_2}-e^{\gamma_1}) sin\frac{n\pi y}{b}}\left[\frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{(\beta_1^2-\alpha_{0t}s^2+G_0)\,(s^2+c^2m^2\pi^2)}\right] sin\varphi_1 y_0(cos\varphi_1 y - cot\varphi_1 b\, sin\varphi_1 y) \\
+ \frac{\tau_1}{\varphi_1} sin\varphi_1 y_0(cot\varphi_1 b\, sin\varphi_1 y + cos\varphi_1 y) - \frac{2\tau_1}{\varphi_1} cos\varphi_1 y_0 sin\varphi_1 y\Bigg\} e^{\gamma_2}\Bigg] sin\frac{n\pi y}{b} \qquad (82)
\end{aligned}
$$

With

$$\tau_1 = \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{(\beta_2^2+\alpha_{0t}s^2+G_0)\,(s^2+c^2m^2\pi^2)}$$

$$\varphi_1 = \sqrt{\frac{[\beta_1^2 m^2\pi^2 - s^2 + c^2m^2\pi^2\alpha_{0t}s^2 - k_0 + m^2\pi^2 G_0 - \Gamma_0\delta(y-y_0)(s^2+c^2m^2\pi^2)]}{\beta_2^2+\alpha_{0t}s^2+G_0}}$$

$$\Lambda_1 = \frac{\beta_2^2+G_0}{\alpha_{0t}}$$

$$\Lambda_2 = c^2m^2\pi^2$$

$$\Lambda_5 = \frac{(\beta_1^2+G_0)}{\alpha_{0t}}$$

$$\Lambda_3[s^2+\Lambda_4]$$

$$\Lambda_4 = \frac{m^2\pi^2[\beta_1^2+G_0-c^2\Gamma_0\delta(y-y_0)]-k_0}{m^2\pi^2\alpha_{0t}-1-\Gamma_0\delta(y-y_0)}$$

$$\tau_1 = \frac{M_0 gm\pi c^2\left[1-(-1)^m e^{-s/c}\right]}{\alpha_{0t}[s^2+\Lambda_1]}$$

$$\varphi_1 = \frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}[s^2+\Lambda_1]}$$

$$\Lambda_6 = \frac{bG_0+b\beta_1^2-\Gamma_0c^2}{b\alpha_{0t}}$$

$$\Lambda_7 = \frac{n^2\pi^2\alpha_{0t}-b\Gamma_0-b^2}{b^2}$$

$$\Lambda_8 = \frac{n^2\pi^2\beta_2^2 + n^2\pi^2 G_0 - b^2 k_0}{n^2\pi^2\alpha_{0t} - b\Gamma_0 - b^2}$$

$$\Lambda_9 = \frac{bG_0 + b\beta_1^2 - \Gamma_0 c^2}{b\alpha_{0t}}$$

$$\Lambda_{10} = \frac{m^2\pi^2\left(\beta_1^2 + \alpha_{0t} + G_0 - c^2\Gamma_0\delta(y-y_0)\right) - k_0}{1 - \Gamma_0\delta(y-y_0)}$$

$$\Lambda_{15} = n^2\pi^2\alpha_{0t} - b\Gamma_0 - b^2$$

$$\varphi_2 = \frac{2\Gamma_0 sc}{b\left(\beta_1^2 + \alpha_{0t}s^2 + G_0 - \frac{\Gamma_0 c^2}{b}\right)}$$

$$\beta_1^2 + \alpha_{0t}s^2 + G_0 - \frac{\Gamma_0 c^2}{b} = \alpha_{0t}[s^2 + \Lambda_6]$$

$$\tau_2 = \frac{M_0 g e^{-sx/c} sin\frac{n\pi y_0}{b}}{\alpha_{0t}(s^2+\Lambda_6)}$$

Further simplification and rearrangement of equation (82) gives

$$W_1^0(x,y,s) = \Bigg\{\frac{2}{\theta_2}\Bigg[\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]}{\alpha_{0t}(s^2+\Lambda_1)} +$$

$$\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]}{\alpha_{0t}(s^2-\Lambda_{10})(s^2+\Lambda_2)}\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\Bigg] sin\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}y_0\, cot\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{0t}(s^2+\Lambda_1)}b\, cos\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}y +$$

$$2\Bigg[\frac{\Lambda_3(s^2+\Lambda_4)M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]}{\alpha_{0t}(s^2+\Lambda_1)\theta_2\alpha_{0t}(s^2-\Lambda_{10})(s^2+\Lambda_2)} sin\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}y_0 - \frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]}{\theta_2\alpha_{0t}(s^2+\Lambda_1)} cos\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}y_0 cos\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}b_1\frac{1}{sinm\pi x} -$$

$$\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}] sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]b\, cos\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]b}{\theta_2\alpha_{0t}(s^2+\Lambda_1) sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]b} +$$

$$\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}] cos^2\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]b\, sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y_0}{\theta_2\alpha_{0t}(s^2-\Lambda_{10})(s^2+\Lambda_2) sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]b}\Bigg] sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\Bigg\} sinm\pi x +$$

$$\Bigg\{\Bigg[\frac{2}{b}\frac{(-1)^m m\pi e^{\gamma_1 x}}{\theta_1(e^{\gamma_1}-e^{\gamma_2})\sin\frac{n\pi y}{b}}\Bigg[\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]\ sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y_0}{-\alpha_{0t}(s^2-\Lambda_{10})(s^2+\Lambda_2)}\ cos\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y - sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\, cot\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{0t}(s^2+\Lambda_1)}\right]b +$$

$$\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]\ \alpha_{0t}(s^2+\Lambda_1)\ sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y_0}{\alpha_{0t}(s^2+\Lambda_1)\Lambda_3(s^2+\Lambda_4)}\left(sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\, cot\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{0t}(s^2+\Lambda_1)}\right]b + cos\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\right) -$$

$$2\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]\ \alpha_{0t}(s^2+\Lambda_1)\ cos\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y_0\, sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y}{\alpha_{0t}(s^2+\Lambda_1)\Lambda_3(s^2+\Lambda_4)}\Bigg] +$$

$$\left[\frac{2}{b}\frac{(-1)^m m\pi e^{\gamma_1 x}}{\theta_1(e^{\gamma_1}-e^{\gamma_2})\sin\frac{n\pi y}{b}}\right]\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]\ sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y_0}{-\alpha_{0t}(s^2-\Lambda_{10})\Lambda_3(s^2+\Lambda_2)}\left(cos\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y - sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\, cot\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{0t}(s^2+\Lambda_1)}\right]b\right) +$$

$$\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]\ (s^2+\Lambda_1)\ sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y_0}{\Lambda_3(s^2+\Lambda_1)(s^2+\Lambda_4)}\left(sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\, cot\left[\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{0t}(s^2+\Lambda_1)}\right]b + cos\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\right) -$$

$$-2\frac{M_0 gm\pi c^2[1-(-1)^m e^{-s/c}]}{\Lambda_3(s^2+\Lambda_4)(s^2+\Lambda_2)} cos\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y_0\, sin\left[\frac{\Lambda_3[s^2+\Lambda_4]}{\alpha_{0t}(s^2+\Lambda_1)}\right]y\Bigg]\Bigg\}\sin\frac{n\pi y}{b} \quad (83)$$

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The Laplace inversion of (83) is given by

$$V_1(x,y,t) = \left\{\frac{m_0 g m\pi c^2}{\alpha_{0t}}[E_1(x,y,t)+E_2(x,y,t)+E_3(x,y,t)+E_4(x,y,t)+E_5(x,y,t)+E_6(x,y,t)]\right\}\sin m\pi x + \left[\frac{2(-1)^m m\pi}{b\theta_1 \sin\frac{m\pi y}{b}}\left\{-\frac{M_0 g m\pi c^2}{\alpha_{0t}}E_7(x,y,t) - E_8(x,y,t) + M_0 g m\pi c^2[E_9(x,y,t)+E_{10}(x,y,t)] - 2M_0 g m\pi c^2 E_{11}(x,y,t)\right\} - \frac{2(-1)^m m\pi}{b\theta_1 \sin\frac{m\pi y}{b}}\left\{-\frac{M_0 g m\pi c^2}{\alpha_{0t}}E_{12}(x,y,t) - E_{13}(x,y,t) + M_0 g m\pi c^2[E_{14}(x,y,t)+E_{15}(x,y,t)]\right\} - \frac{2M_0 g m\pi c^2}{\Lambda_3}E_{16}(x,y,t)\right] \quad (84)$$

where

$$E_1(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{2\alpha_2 e^{st}[1-(-1)^m e^{-\frac{s}{c}}]}{\sqrt{\alpha_{ot}}(s^2+\Lambda_1)^{\frac{3}{2}} sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b}\, sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o \times \cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) b\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\ ds \quad (85.1)$$

$$E_2(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{2\alpha_2 e^{st}[1-(-1)^m e^{-\frac{s}{c}}]\Lambda_3(s^2+\Lambda_4)}{\sqrt{\alpha_{ot}}(s^2-\Lambda_{10})(s^2+\Lambda_2)(s^2+\Lambda_1)^{\frac{3}{2}} sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b} \times sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) b\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y \quad (85.2)$$

$$E_3(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\frac{2\Lambda_3(s^2+\Lambda_4)[1-(-1)^m e^{-\frac{s}{c}}]}{(s^2+\Lambda_1)(s^2-\Lambda_{10})(s^2+\Lambda_2) sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b} \times\ sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\, ds \quad (85.3)$$

$$E_4(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} 2e^{st}\frac{\left[1-(-1)^m e^{-\frac{s}{c}}\right] cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o}{(s^2+\Lambda_1) sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b\sin m\pi x}\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds} \quad (85.4)$$

$$E_5(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{c}}\right]\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds}}{(s^2+\Lambda_1) sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b} \quad (85.5)$$

$$E_6(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st}\frac{\left[1-(-1)^m e^{-\frac{s}{c}}\right]\theta_1 cos^2\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b}{(s^2-\Lambda_{10})(s^2+\Lambda_2) sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b} \times\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds} \quad (85.6)$$

$$E_7(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}e^{\gamma_1 x}\left[1-(-1)^m e^{-\frac{s}{c}}\right] sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds}}{(e^{\gamma_1}-e^{\gamma_2})(s^2\pm)(s^2+\Lambda_2)} \quad (85.7)$$

$$E_8(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}e^{\gamma_1 x}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(e^{\gamma_1}-e^{\gamma_2})(s^2-\Lambda_{10})(s^2+\Lambda_2) sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b} \times sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)\mathrm{ds} \quad (85.8)$$

$$E_9(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{\Lambda_3(s^2+\Lambda_4)} \times sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)\mathrm{ds} \quad (85.9)$$

$$E_{10}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{\Lambda_3(s^2+\Lambda_4)}\times sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds} \tag{85.10}$$

$$E_{11}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{\Lambda_3(s^2+\Lambda_4)}\times cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds} \tag{85.11}$$

$$E_{12}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}e^{\gamma_2 x}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(e^{\gamma_2}-e^{\gamma_1})(s^2-\Lambda_{10})(s^2+\Lambda_2)}\times sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds} \tag{85.12}$$

$$E_{13}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}e^{\gamma_2 x}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(e^{\gamma_2}-e^{\gamma_1})(s^2-\Lambda_{10})(s^2+\Lambda_2)sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b}\times sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)\mathrm{ds} \tag{85.13}$$

$$E_{14}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{\Lambda_3(s^2+\Lambda_4)sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)b}\times sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)\mathrm{ds} \tag{85.14}$$

$$E_{15}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{\Lambda_3(s^2+\Lambda_4)}\times\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)\mathrm{ds} \tag{85.15}$$

$$E_{16}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(s^2+\Lambda_4)(s^2+\Lambda_2)}\times cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right)y\,\mathrm{ds} \tag{85.16}$$

Now employing the Cauchy's residue theorem, one proceeds to the evaluation of integrals (85.1) – (85.16) to obtain the sum of the residues of $E_1(x,y,t)-E_{16}(x,y,t)$ as

$$\begin{aligned}
&E_1(x,y,t)\\
&=2\alpha_2 e^{i\Lambda_1 t}\frac{\left[1-(-1)^m e^{\frac{-i\Lambda_1}{c}}\right]sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)}{\sqrt{(2\mathrm{i}\Lambda_1)^3}(i\Lambda_1-\Lambda_{22})(i\Lambda_1+\Lambda_{22})\sqrt{\alpha_{ot}}}\\
&+\frac{e^{-i\Lambda_1 t}\left[1-(-1)^m e^{\frac{-i\Lambda_1}{c}}\right]sin\left(\frac{\Lambda_3(\Lambda_1^2+\Lambda_4)}{\alpha_{ot}(\Lambda_1^2-\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_1^2+\Lambda_4)}{\alpha_{ot}(\Lambda_1^2-\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_1^2+\Lambda_4)}{\alpha_{ot}(\Lambda_1^2-\Lambda_1)}\right)}{\sqrt{(2\mathrm{i}\Lambda_1)^3}(\Lambda_{22}+i\Lambda_1)(\Lambda_{22}-i\Lambda_1)}\\
&+\frac{e^{i\Lambda_{22}t}\left[1-(-1)^m e^{\frac{-i\Lambda_{22}}{c}}\right]sin\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y}{2\Lambda_{22}\sqrt{(\Lambda_{22}^2+\Lambda_1)^3}}\\
&-\frac{e^{-i\Lambda_{22}t}\left[1-(-1)^m e^{\frac{\Lambda_{22}}{c}}\right]sin\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y}{2\Lambda_{22}\sqrt{(\Lambda_{22}^2+\Lambda_1)^3}}
\end{aligned} \tag{86.1}$$

$$E_2(x,y,t)=\frac{2\alpha_2 e^{\Lambda_{10}t}\left[1-(-1)^m e^{\frac{-\Lambda_{10}}{c}}\right]\Lambda_3(\Lambda_{10}+\Lambda_4)}{2\sqrt{\alpha_{0t}}\Lambda_{10}(\Lambda_{10}^2+\Lambda_1)(\Lambda_{10}^2+\Lambda_1)^{\frac{3}{2}}(\Lambda_{10}^2+\Lambda_{22})}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_{10}+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}+\Lambda_1)}\right)y_o b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_{10}+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}+\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_{10}+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}+\Lambda_1)}\right)$$
$$+\frac{\alpha_2 e^{\Lambda_{10}t}\left[1-(-1)^m e^{\frac{\Lambda_{10}}{c}}\right]\Lambda_3(\Lambda_{10}+\Lambda_4)}{2\sqrt{\alpha_{0t}}\Lambda_{10}(\Lambda_{10}+\Lambda_2)(\Lambda_{10}^2+\Lambda_1)^{\frac{3}{2}}(\Lambda_{10}^2-\Lambda_{22})}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_{10}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_{10}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}^2+\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_{10}+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}+\Lambda_1)}\right)$$
$$+\frac{2\alpha_2 e^{i\Lambda_2}\left[1-(-1)^m e^{\frac{-i\Lambda_2}{c}}\right]\Lambda_3(\Lambda_4-\Lambda_2^2)}{2\sqrt{\alpha_{0t}}\Lambda_{10}(\Lambda_{10}^2+\Lambda_1)i\Lambda_2(\Lambda_1-\Lambda_2^2)^{\frac{3}{2}}(\Lambda_2^2+\Lambda_{22})}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_4+\Lambda_2^2)}{\alpha_{ot}(\Lambda_1+\Lambda_2^2)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_4+\Lambda_2^2)}{\alpha_{ot}(\Lambda_1+\Lambda_2^2)}\right)$$
$$-\frac{2\alpha_2 e^{-i\Lambda_2}\left[1-(-1)^m e^{\frac{i\Lambda_2}{c}}\right]\Lambda_3(\Lambda_4-\Lambda_2^2)}{2i\alpha_2\sqrt{\alpha_{0t}}\Lambda_{10}(\Lambda_{10}^2+\Lambda_{10})(\Lambda_1-\Lambda_2^2)^{\frac{3}{2}}(\Lambda_2^2+\Lambda_{22})}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)$$
$$+\frac{2\alpha_2 e^{i\Lambda_1 t}\left[1-(-1)^m e^{\frac{-i\Lambda_1}{c}}\right]\Lambda_3(\Lambda_4-\Lambda_1^2)}{2i\alpha_1\sqrt{\alpha_{0t}}(\Lambda_1^2+\Lambda_{10})i\Lambda_2(\Lambda_2-\Lambda_1^2)^{\frac{3}{2}}(\Lambda_1^2+\Lambda_{22})}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y$$
$$-\frac{2\alpha_2 e^{-i\Lambda_1 t}\left[1-(-1)^m e^{\frac{i\Lambda_1}{c}}\right]\Lambda_3(\Lambda_4-\Lambda_1^2)}{2i\alpha_1\sqrt{\alpha_{0t}}(\Lambda_1^2+\Lambda_{10})(\Lambda_2-\Lambda_1^2)^{\frac{3}{2}}(\Lambda_1^2+\Lambda_{22})}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y$$
$$+\frac{2\alpha_2 e^{\Lambda_{22}t}\left[1-(-1)^m e^{\frac{-\Lambda_{22}}{c}}\right]\Lambda_3(\Lambda_2^2+\Lambda_4)}{2\Lambda_{22}\sqrt{\alpha_{0t}}(\Lambda_{22}^2-\Lambda_{10})(\Lambda_{22}^2+\Lambda_2)(\Lambda_{22}^2+\Lambda_1)^{\frac{3}{2}}}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y$$
$$-\frac{2\alpha_2 e^{-\Lambda_{22}t}\left[1-(-1)^m e^{\frac{\Lambda_{22}}{c}}\right]\Lambda_3(\Lambda_2^2+\Lambda_4)}{2\alpha_{22}\sqrt{\alpha_{0t}}(\Lambda_{22}^2-\Lambda_{10})(\Lambda_{22}^2+\Lambda_2)(\Lambda_{22}^2+\Lambda_1)^{\frac{3}{2}}}$$
$$\times sin\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y_o\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)b\cos\left(\frac{\Lambda_3(\Lambda_{22}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{22}^2+\Lambda_1)}\right)y \quad (86.2)$$

$$E_3(x,y,t)=\frac{2e^{i\Lambda_1 t}\Lambda_3(\Lambda_4-\Lambda_1^2)\left[1-(-1)^m e^{\frac{-i\Lambda_1}{c}}\right]}{2i\Lambda_1(\Lambda_1^2+\Lambda_{10})(\Lambda_2+\Lambda_1^2)}sin\left(\frac{\Lambda_3(\Lambda_4+\Lambda_1^2)}{\alpha_{ot}(\Lambda_1+\Lambda_1^2)}\right)y_o\sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y$$
$$+\frac{2e^{i\Lambda_1 t}\Lambda_3(\Lambda_4-\Lambda_1^2)\left[1-(-1)^m e^{\frac{i\Lambda_1}{c}}\right]}{2i\Lambda_1(\Lambda_1^2+\Lambda_{10})(\Lambda_2-\Lambda_1^2)}sin\left(\frac{\Lambda_3(\Lambda_4+\Lambda_1^2)}{\alpha_{ot}(\Lambda_1+\Lambda_1^2)}\right)y_o\sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right)y$$
$$+\frac{2e^{\Lambda_{10}t}\Lambda_3(\Lambda_{10}^2-\Lambda_4)\left[1-(-1)^m e^{\frac{-\Lambda_{10}}{c}}\right]}{2\Lambda_{10}(\Lambda_{10}^2+\Lambda_1)(\Lambda_{10}^2+\Lambda_2)}sin\left(\frac{\Lambda_3(\Lambda_{10}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(\Lambda_{10}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}^2+\Lambda_1)}\right)y$$
$$-\frac{2e^{\Lambda_{10}t}\Lambda_3(\Lambda_{10}^2+\Lambda_4)\left[1-(-1)^m e^{\frac{\Lambda_{10}}{c}}\right]}{2i\Lambda_{10}(\Lambda_{10}^2+\Lambda_1)(\Lambda_{10}^2+\Lambda_2)}sin\left(\frac{\Lambda_3(\Lambda_{10}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}^2+\Lambda_1)}\right)y_o\sin\left(\frac{\Lambda_3(\Lambda_{10}^2+\Lambda_4)}{\alpha_{ot}(\Lambda_{10}^2+\Lambda_1)}\right)y$$
$$-\frac{2e^{i\Lambda_2 t}\Lambda_3(\Lambda_4-\Lambda_2^2)\left[1-(-1)^m e^{\frac{-i\Lambda_2}{c}}\right]}{2i\Lambda_2(\Lambda_1-\Lambda_2^2)(\Lambda_2^2+\Lambda_{10})}sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)y_o\sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)y$$
$$+\frac{2e^{-i\Lambda_2 t}\Lambda_3(\Lambda_4-\Lambda_2^2)\left[1-(-1)^m e^{\frac{i\Lambda_2}{c}}\right]}{2i\Lambda_2(\Lambda_1-\Lambda_2^2)(\Lambda_2^2+\Lambda_{10})}sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)y_o\sin\left(\frac{\Lambda_3(\Lambda_4-\Lambda_2^2)}{\alpha_{ot}(\Lambda_1-\Lambda_2^2)}\right)y \quad (86.3)$$

$$E_4(x,y,t) = -2e^{i\Lambda_1 t}\frac{\left[1-(-1)^m e^{\frac{-i\Lambda_1}{c}}\right]}{2i\Lambda_1(\Lambda_1^2+\Lambda_{22})} \times \cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) y_o \cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) y$$
$$-2e^{i\Lambda_1 t}\frac{\left[1-(-1)^m e^{\frac{i\Lambda_1}{c}}\right]}{2i\Lambda_1(\Lambda_1^2+\Lambda_{22})} \times \cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) y_o \cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) y$$
$$+\lim_{s\to\Lambda_{22}} \frac{2(s-\Lambda_{22})e^{st}\left[1-(-1)^m e^{\frac{-s}{c}}\right]}{(s^2+\Lambda_1)(s-\Lambda_{22})(s+\Lambda_{22})}$$
$$\times \cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) b\cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) y_o \cos\left(\frac{\Lambda_3(s^2+\Lambda_4)}{\alpha_{ot}(s^2+\Lambda_1)}\right) y - \frac{2e^{-\Lambda_{22}}\left[1-(-1)^m e^{\frac{\Lambda_{22}}{c}}\right]}{2\Lambda_{22}(\Lambda_{22}^2+\Lambda_1)}$$
$$\times \cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) b\cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) y_o \cos\left(\frac{\Lambda_3(\Lambda_4-\Lambda_1^2)}{\alpha_{ot}(\Lambda_1-\Lambda_1^2)}\right) y \quad (86.4)$$

Substitution of integrals $E_1(x,y,t) - E_{16}(x,y,t)$ into equation (84) produces the complete inversion of $W_1(x,y,t)$.

From equation (19), the perturbation scheme of a uniformly valid solution in the entire domain of definition of the plate problem is given as

$$W(x,y,t) = W_o(x,y,t) + \varepsilon W_1(x,y,t) \quad (87)$$

where $W_o(x,y,t)$ and $W_1(x,y,t)$ are respectively the leading order solution and the first order correction . These are given as (58) and (84) in that order. In view of equations (58) and (84) equation (87) becomes the required uniformly valid approximate analytical solution of the plate dynamical problem.

HIGHLY PRESTRESSED ORTHOTROPIC RECTANGULAR PLATE TRAVERSED BY MOVING FORCE

By setting $\Gamma_0 = 0$ in equation (16) an approximate model of the differential equation describing the response of a highly prestressed orthotropic rectangular plate resting on a pasternak foundation and traversed by a moving force is obtained.

Thus setting $\Gamma_0 = 0$, equation (16) reduces to

$$\beta_1^2\frac{\partial^2 W_0(x,y.s)}{\partial x^2} + \beta_2^2\frac{\partial^2 W_0(x,y.s)}{\partial y^2} - S^2 W_0(x,y,s) + \alpha_{ot}S^2\left[\frac{\partial^2 W(x,y)}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right] - K_0 W(x,y,s)$$
$$+ G_0\left[\frac{\partial^2 W(x,y)}{\partial x^2} + \frac{\partial^2 W(x,y.s)}{\partial y^2}\right] = -M_0 g\delta(y-y_0)e^{-\frac{s}{c}x} \quad (88)$$

as the classical case of moving force problem associated with our system. This is an approximate model which assumes the inertial effect of the moving mass as negligible

Equation (88) can be rewritten as

$$(\beta_1^2 + \alpha_{ot}S^2 + G_0)\frac{\partial^2 W_0(x,y.s)}{\partial x^2} + (\beta_2^2 + \alpha_{ot}S^2 + G_0)\frac{\partial^2 W_0(x,y.s)}{\partial y^2} - (s^2 + k_0)w_0(x,y,s)$$
$$= -M_0 g\delta(y-y_0)e^{-\frac{s}{c}x} \quad (89)$$

Now, one attempts equation (89) for the solution of $w_0(x,y,s)$ by introducing the finite Fourier sine transform defined in equation (33)-(36). Thus the transform of (89) with respect to x is

$$\frac{\partial^2}{\partial y^2}W_0(m,y.s) + \varphi_3^2 W(m,y.s) = \tau_3\delta(y-y_o) \quad (90)$$

where

$$\varphi_3^2 = \frac{[(1-m^2\pi^2\alpha_{ot})S^2 - m^2\pi^2\beta_1^2 + m^2\pi^2 G_0 + k_0]}{\alpha_{ot}S^2 + \beta_2^2 + G_0} \quad (91)$$

$$\tau_3 = M_o g\left(\frac{m\pi c^2\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(\beta_2^2 + \alpha_{ot}S^2 + G_0)(S^2 + m^2\pi^2 c^2)}\right) \quad (92)$$

Using the method of variation of parameters, one obtains the complementary and particular solutions of the differential equation (90) respectively as

$$W_{oc}(m, y.s) = A_1 \cos\varphi_3 y + A_2 \sin\varphi_3 y \tag{93}$$

and

$$W_{op}(m, y.s) = -\frac{\tau_3}{\varphi_3}[-\sin\varphi_3 y_0 \cos\varphi_3 y - \cos\varphi_3 y_0 \sin\varphi_3 y] \tag{94}$$

The combination of (93) and (94) produces the general solution of (90) as

$$W_o(m, y.s) = A_1 \cos\varphi_3 y + A_2 \sin\varphi_3 y - \frac{\tau_3}{\varphi_3}[\sin\varphi_3 y_0 \cos\varphi_3 y - \cos\varphi_3 y_0 \sin\varphi_3 y] \tag{95}$$

Equation (95) is rewritten as

$$W_o(m, y.s) = A_1 \cos\varphi_3 y + A_2 \sin\varphi_3 y - \frac{\tau_3}{\varphi_3}\sin\varphi_3 (y - y_o) \tag{96}$$

Similarly, the finite Fourier transformation of (89) with respect to y is

$$\frac{\partial^2 W_o(x, n.s)}{\partial x^2} + \varphi_4^2 W_o(x, n.s) = \tau_4 e^{-\frac{s}{c}x} \tag{97}$$

where

$$\varphi_4^2 = \frac{n^2\pi^2(\beta_2^2 + \alpha_{ot}S^2 + G_0)}{b^2(\beta_1^2 + \alpha_{ot}S^2 + G_0)} \tag{98}$$

$$\tau_4 = \frac{M_o g \sin\frac{n\pi y_0}{b}}{\beta_1^2 + \alpha_{ot}S^2 + G_0} \tag{99}$$

The process of obtaining the general solution of the differential equation (97) is analogous to that of (90). Hence, the general solution of (97) is

$$\begin{aligned} W_o(x, n.s) &= A_3 \cos\varphi_4 x + A_4 \sin\varphi_4 x \\ &+ \frac{\tau_4}{\varphi_4}\left(\frac{[cs(1+\cos\varphi_4) - c^2\varphi_4 \sin\varphi_4]e^{-\frac{s}{c}}\sin\varphi_4 x}{(s^2 + c^2\varphi_4^2)}\right. \\ &\left. + \frac{\left[cse^{-\frac{s}{c}}\sin\varphi_4 - c^2\varphi_4 e^{-\frac{s}{c}}\cos\varphi_4 + c^2\varphi_4\right]\cos\varphi_4 x}{(s^2 - c^2\varphi_4^2)}\right) \end{aligned} \tag{100}$$

Consequently, the inversion of the finite Fourier sine transformation in (96) and (100) gives the general solution of the equation (92) as

$$\begin{aligned} W_0^o(x, y, s) &= 2\left[A_1 \cos\varphi_3 y + A_2 \sin\varphi_3 y - \frac{\tau_3}{\varphi_3}\sin(y - y_0)\right]\sin m\pi x + \frac{2}{b}\left[A_3 \cos\varphi_4 x + A_4 \sin\varphi_4 x\right. \\ &+ \frac{\tau_4}{\varphi_4}\left(\frac{[cs(1+\cos\varphi_4) - c^2\varphi_4 \sin\varphi_4]e^{-\frac{s}{c}}\sin\varphi_4 x}{(s^2 + c^2\varphi_4^2)}\right. \\ &\left.\left. + \frac{\left[cse^{-\frac{s}{c}}\sin\varphi_4 - c^2\varphi_4 e^{-\frac{s}{c}}\cos\varphi_4 + c^2\varphi_4\right]\cos\varphi_4 x}{(s^2 - c^2\varphi_4^2)}\right)\sin\frac{n\pi y}{b}\right] \end{aligned} \tag{101}$$

where A_1, A_2, A_3 and A_4 are arbituary constants yet to be determined by matching.
where A_1, A_2, A_3 and A_4 are arbituary constants yet to be determined by matching.

LEADING ORDER SOLUTION (INNER PROBLEM)

The differential equation governing the inner solution (near $x = 0, 1$) in equation (16) where one equates r_o to zero produces the leading order problem as :

$$\frac{\partial^4 W_o^i(x.y.s)}{\partial x^4} - \omega_1^2 \frac{\partial^2 W(x.y.s)}{\partial x^2} = 0 \tag{102}$$

Subject to :

$$W_o^i(x, y, s) = 0 = \frac{\partial W_o^i(x, y, s)}{\partial X} \tag{103}$$

where

$$\omega_1^2 = \beta_1^2 + \alpha_{ot}S^2 + G_o \tag{104}$$

Now the solution of $W_o^i(x, y, s)$ is sought. Solving equation (102) together with (103), and neglecting exponentially growing terms (i.e. when rectangular plate is treated, terms cannot grow exponentially since the plate dimensions are finite) so that the solution does not become unbounded, produces a solution

$$W_o^i(x,y,s) = \begin{cases} \tilde{c_o}(y)\left[X + \frac{1}{w_1}e^{-w_1 x} - \frac{1}{w_1}\right] & near\ x = 0 \\ \tilde{\tilde{c_o}}(y)\left[X + \frac{1}{w_1}e^{-w_1 x} - \frac{1}{w_1}\right] & near\ x = 1 \end{cases} \tag{105}$$

Similarly, the differential equation governing the inner solution (near $y = 0, b$) in (29), when $\Gamma_o = 0$ and one neglects the term with negative subscript in the leading order problem, is

$$\frac{\partial^4 w_o^i}{\partial y^4}(x,y,s) - \omega_2^2 \frac{\partial^2 w_o^i}{\partial y^2}(x,y,s) = 0 \tag{106}$$

Subject to

$$W_o^i(x,y,s) = 0 = \frac{\partial w_o^i}{\partial y}(x,y,s) \tag{107}$$

Where

$$\omega_2^2 = \frac{\beta_1^2 + \alpha_{ot}s^2 + G_o}{\propto_2^2} \tag{108}$$

Equations (106) and (107) are in complete agreement with equations (102) - (103). As a result the inner solution is adopted as:

$$W_o^i(x,y,s) = \begin{cases} \tilde{K}_o(x)\left[y + \frac{1}{w_2}e^{-w_2 y} - \frac{1}{w_2}\right], & near\ y = 0 \\ \overline{\overline{K_0}}(x)\left[y + \frac{1}{w_2}e^{-w_2 y} - \frac{1}{w_2}\right], & near\ y = b \end{cases} \tag{109}$$

In (105) and (109), exponentially growing terms have been discarded as unmatchable while the functions $\tilde{c_o}(y)$, $\tilde{\tilde{c_o}}(y)$ $\tilde{K}_o(x)$ and $\overline{\overline{K_0}}(x)$ will be determined by matching inner and outer solutions. As such the Van Dyke's matching principle is applied as earlier spelt out. Thus, matching one term outer expansion (101) rewritten in inner variable and expanded for small perturbation parameter ε with one term inner expansion (105) rewritten in outer variable and expanded for small ε (1-1 matching) results in

$$\tilde{C}_o(y) = \tilde{\tilde{C}}_o = \tilde{K}_o = \overline{\overline{K_0}}(x) = 0 \tag{110}$$

$$A_3 = \frac{\tau_4}{\varphi_4(s^2 - c^2\varphi_4^2)}\left(c^2\varphi_4 e^{-s/c} - cse^{-s/c}\sin\varphi_4 - \varphi_4 c^2\right),$$

$$A_4 = -\left[\frac{\tau_4 c^2 e^{-s/c}\cos\varphi_4}{(s^2 - c^2\varphi_4^2)\sin\varphi_4} + \frac{\tau_4 cs(1-\cos\varphi_4)e^{-s/c}}{\varphi_4(s^2 - c^2\varphi_4^2)}\right] \tag{111}$$

$$A_1 = \frac{\tau_3}{\varphi_3}\sin y_0, \qquad A_2 = \frac{\tau_3}{\varphi_3}\frac{[\sin(b-y_0) - \sin y_0\cos\varphi_3 b]}{\sin\varphi_3 b} \tag{112}$$

In view of equations (111) and (112), equation (101) becomes

$$W_o^o(x,y,s) = 2\left[\begin{matrix}\frac{\tau_3}{\varphi_3}\sin y_0\cos\varphi_3 y + \frac{\tau_3}{\varphi_3}\frac{\sin(b-y_0) - \sin\varphi_3 y}{\sin\varphi_3 b} - \frac{\tau_3\sin y_0\cos\varphi_3 b}{\varphi_3\sin\varphi_3 b}\sin\varphi_3 y \\ -\frac{\tau_3}{\varphi_3}\sin(y-y_0)\end{matrix}\right]\sin m\pi x$$

$$+\frac{2}{b}\left[\begin{matrix}\frac{\tau_4 c^2 e^{-\frac{s}{c}}\cos\varphi_4 x}{(s^2-c^2\varphi_4^2)} - \frac{\tau_4 cse^{-\frac{s}{c}}\sin\varphi_4\cos\varphi_4 x}{\varphi_4(s^2-c^2\varphi_4^2)} - \frac{\tau_4 c^2\cos\varphi_4 x}{(s^2-c^2\varphi_4^2)} \\ -\frac{\tau_4 c^2 e^{-\frac{s}{c}}\cos\varphi_4 x\sin\varphi_4 x}{(s^2-c^2\varphi_4^2)\sin\varphi_4} - \frac{\tau_4 cs(1-\cos\varphi_4)e^{-\frac{s}{c}}\sin\varphi_4 x}{\varphi_4(s^2-c^2\varphi_4^2)} + \frac{\tau_4 cs(1-\cos\varphi_4)\sin\varphi_4 x e^{-\frac{s}{c}}}{\varphi_4(s^2+c^2\varphi_4^2)} - \\ \frac{\tau_4 c^2\varphi_4 e^{-\frac{s}{c}}\sin\varphi_4\sin\varphi_4 x}{\varphi_4(s^2+c^2\varphi_4^2)} + \frac{cse^{-\frac{s}{c}}\sin\varphi_4\cos\varphi_4 x}{s^2-c^2\varphi_4^2} - \frac{c^2\varphi_4 e^{-\frac{s}{c}}\cos\varphi_4\cos\varphi_4 x}{s^2-c^2\varphi_4^2} + \frac{c^2\varphi_4\cos\varphi_4 x}{s^2-c^2\varphi_4^2}\end{matrix}\right]\sin\frac{n\pi y}{b}$$

(113)

where

$$\tau_3 = M_0 g\left(\frac{m\pi c^2\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(\beta_2^2 + \alpha_{ot}s^2 + G_0)(s^2 + m^2\pi^2 c^2)}\right) \tag{114}$$

$$\varphi_3 = \sqrt{\left(\frac{m^2\pi^2\beta_1^2 - m^2\pi^2 G_0 - K_0 - (1 - m^2\pi^2\alpha_{ot})s^2}{\alpha_{ot}s^2 + \beta_2^2 + G_0}\right)} \tag{115}$$

$$\tau_4 = \frac{M_0 g \sin\frac{n\pi y_0}{b}}{\beta_1^2 + \alpha_{ot}s^2 + G_0} \tag{116}$$

$$\varphi_4 = \pm\frac{n\pi}{b}\sqrt{\frac{\beta_2^2 + \alpha_{ot}s^2 + G_0}{\beta_1^2 + \alpha_{ot}s^2 + G_0}} \tag{117}$$

$$\beta_2^2 + \alpha_{ot}s^2 + G_0 = \alpha_{ot}[s^2 + \chi_1] \tag{118}$$

$$\chi_1 = \frac{\beta_2^2 + G_0}{\alpha_{ot}} \tag{119}$$

Equation (113) is rewritten as

$$W_0(x,y,s) = 2\left\{\frac{m\pi c^2\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{\sqrt{m^2\pi^2\alpha_{ot}-1}}\left[\frac{\left[\sin y_0 \cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2+x_1)}}y\right]}{(s^2+x_3^2)\sqrt{(s^2+x_1)(s^2-x_2)}} + \frac{\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2+x_1)}}y}{(s^2+x_3^2)\sqrt{(s^2+x_1)(s^2-x_2)}(s^2\pm x_5)} - \frac{\sin y_0\cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2+x_1)}}b}{(s^2+x_3^2)\sqrt{(s^2+x_1)(s^2-x_2)}(s^2\pm x_5)}\frac{\sqrt{\alpha_{ot}(s^2+x_1)}}{\sqrt{(s^2+x_2)}}\frac{\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2+x_1)}}y}{\sqrt{(m^2\pi^2\alpha_{ot}-1)}} - \frac{\sin(y-y_0)}{\left[(s^2+x_3^2)\sqrt{(s^2+x_1)(s^2-x_2)}\right]}\right]\sin m\pi x\right\}$$

$$\frac{2}{b}\left\{\frac{M_0 g\sin\frac{n\pi y}{b}}{\alpha_{ot}}\left[\frac{c^2e^{-\frac{s}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}(s^2+\chi_4)}{(s\pm\chi_8)(s\pm\chi_9)(s\pm\chi_4)} - \frac{cse^{-\frac{s}{c}}\sin\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\sqrt{s^2+\chi_4}}{\frac{n\pi}{b}(s\pm x_8)(s\pm\chi_9)\sqrt{(s^2+\chi_1)(s^2+\chi_4)}} - \frac{c^2\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}x}{(s\pm x\chi_8)(s\pm x\chi_9)(s\pm\chi_4)} - \frac{c^2e^{-\frac{s}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\sin\frac{n\pi y}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}x}{(s\pm\chi_8)(s\pm\chi_9)(s\pm\chi_{13})} - \frac{-bcs\left(1-\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\right)e^{-\frac{s}{c}}\sin\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}x\sqrt{s^2+\chi_4}}{n\pi(s\pm\chi_8)(s\pm\chi_9)\sqrt{s^2+\chi_1}} + \frac{bcs\left(1-\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\right)\sin\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}xe^{-\frac{s}{c}}\sqrt{(s^2+\chi_4)^3}}{n\pi\sqrt{s^2+\chi_1}(s\pm\chi_{10})(s\pm\chi_{11})} - \frac{c^2e^{-\frac{s}{c}}\sin\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\sin\frac{n\pi y}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}x}{(s\pm\chi_{10})(s\pm\chi_{11})} + \frac{cse^{-\frac{s}{c}}\sin\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}x}{(s\pm\chi_8)(s\pm\chi_9)} - \frac{n\pi c^2\sqrt{s^2+\chi_1}e^{-\frac{s}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}}{b\sqrt{s^2+\chi_4}(s\pm\chi_8)(s\pm\chi_9)} + \frac{n\pi c^2\sqrt{s^2+\chi_1}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+\chi_1}{s^2+\chi_4}}x}{b\sqrt{s^2+\chi_4}(s\pm\chi_8)(s\pm\chi_9)}\right]\right. \tag{120}$$

where

$$\chi_1 = \frac{\beta_2^2 + G_0}{\alpha_{ot}} \tag{121.1}$$

$$\chi_2 = \frac{m^2\pi^2 G_0 + k_0 - m^2\pi^2\beta_1^2}{m^2\pi^2\alpha_{ot} - 1} \tag{121.2}$$

$$\chi_3 = \pm m\pi c \tag{121.3}$$

$$\chi_4 = \frac{\beta_1^2 + G_0}{\alpha_{ot}} \tag{121.4}$$

$$\chi_5 = \sqrt{\frac{\gamma^2\pi^2\alpha_{ot}\chi_1 + b^2(m^2\pi^2\alpha_{ot} - 1)}{b^2(m^2\pi^2\alpha_{ot} - 1) - \gamma^2\pi^2\alpha_{ot}}} \tag{121.5}$$

$$\chi_6 = \frac{b^2\chi_4 - c^2n^2\pi^2}{b^2} \tag{121.6}$$

$$\chi_7 = \frac{c^2n^2\pi^2\chi_1}{b^2} \tag{121.7}$$

$$\chi_8 = -\chi_6 + \sqrt{\chi_6{}^2 + 4\chi_7} \tag{121.8}$$

$$x_9 = -x_6 - \sqrt{x_6^2 + 4x_7} \tag{121.9}$$

$$\chi_{10} = \frac{b^2x_4 + c^2n^2\pi^2}{b^2} \tag{121.10}$$

$$\chi_{11} = \frac{-\chi_{10} + \sqrt{\chi_{10}{}^2 - 4\chi_7}}{2} \tag{121.11}$$

$$\chi_{12} = \frac{-\left(\chi_{10} + \sqrt{\chi_{10}{}^2 - 4\chi_7}\right)}{2} \tag{121.12}$$

$$x_{13} = \sqrt{\frac{\gamma^2b^2x_4 - n^2x_1}{n^2 - \gamma^2b^2}} \tag{121.13}$$

Using the Cauchy residual theorem, the Laplace inversion of $W_0(x, y, s)$ is obtained as

$$W_0(x,y,t) = 2\{Pa_1[H_1(x,y,t) + H_2(x,y,t) - H_3(x,y,t) - H_4(x,y,t)]\} + \frac{2}{b}\left\{Pa_2\begin{bmatrix} H_5(x,y,t) - H_6(x,y,t) - H_7(x,y,t) - H_8(x,y,t) - H_9(x,y,t) \\ +H_{10}(x,y,t) - H_{11}(x,y,t) \\ +H_{12}(x,y,t) - H_{13}(x,y,t) + H_{14}(x,y,t) \end{bmatrix}\right\} \tag{122}$$

where

$$Pa_1 = \frac{2m\pi c^2 \sin m\pi x}{\sqrt{(m^2\pi^2\alpha_{ot} - 1)\alpha_{ot}}} \tag{123.1}$$

$$Pa_2 = \frac{2M_0 g \sin\frac{n\pi y}{b}}{b\alpha_{ot}} \tag{123.2}$$

$$H_1(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\sin y_0\cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2+x_1)}}y\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(s^2+x_3^2)\sqrt{(s^2+x_1)(s^2-x_2)}}ds \tag{124.1}$$

$$H_2(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2-x_1)}}y\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(s^2+x_3^2)(s^2\pm x_5)\sqrt{(s^2+x_1)(s^2-x_2)}}ds \tag{124.2}$$

$$H_3(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{\sqrt{\alpha_{ot}}e^{st}\sin y_0\cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2+x_1)}}b\left[1-(-1)^m e^{-\frac{s}{c}}\right]\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(s^2-x_2)}{\alpha_{ot}(s^2+x_1)}}y}{(s^2+x_3^2)(s^2\pm x_5)(s^2-x_2)\sqrt{m^2\pi^2\alpha_{ot}-1}}ds \tag{124.3}$$

$$H_4(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\sin(y-y_0)\left[1-(-1)^m e^{-\frac{s}{c}}\right]}{(s^2+x_3^2)\left(\sqrt{(s^2+x_1)(s^2-x_2)}\right)}ds \tag{124.4}$$

$$H_5(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}c^2e^{-\frac{s}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{(s^2-x_1)}{(s^2+x_4)}}y}{(s\pm x_8)(s\pm x_9)}ds \tag{124.5}$$

$$H_6(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}bce^{-\frac{s}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{(s^2-x_1)}{(s^2+x_4)}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{(s^2-x_1)}{(s^2+x_4)}}\right]}{n\pi(s\pm x_8)(s\pm x_9)\sqrt{(s^2+x_1)(s^2-x_2)}}ds \tag{124.6}$$

$$H_7(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}c^2\cos\frac{n\pi}{b}\sqrt{\frac{(s^2-x_1)}{(s^2+x_4)}}x}{(s\pm x_8)(s\pm x_9)(s^2-x_2)}ds \tag{124.7}$$

$$H_8(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}ce^{-\frac{s}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{(s^2-x_1)}{(s^2+x_4)}}\sin\frac{n\pi}{b}\sqrt{\frac{(s^2-x_1)}{(s^2+x_4)}}}{(s\pm x_8)(s\pm x_9)(s\pm x_{13})}ds \tag{124.8}$$

$$H_9(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}bcs\left(1-\cos\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}\right)e^{-\frac{s}{c}}\sin\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}x\sqrt{s^2+x_4}}{n\pi(s\pm x_8)(s\pm x_9)\sqrt{s^2+x_1}}ds \tag{124.9}$$

$$H_{10}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}bcs\left(1-\cos\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}\right)\sin\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}x\sqrt{(s^2+x_4)^3}e^{-\frac{s}{c}}}{n\pi\sqrt{s^2+x_1}(s\pm x_{10})(s\pm x_{11})}ds \tag{124.10}$$

$$H_{11}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}c^2e^{-\frac{s}{c}}\sin\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}\sin\frac{n\pi y}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}x}{(s\pm x_{10})(s\pm x_{11})}ds \tag{124.11}$$

$$H_{12}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}cse^{-\frac{s}{c}}\sin\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}x}{(s\pm x_8)(s\pm x_9)}ds \tag{124.12}$$

$$H_{13}(x,y,t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{n\pi e^{st}c^2\sqrt{s^2+x_1}e^{-\frac{s}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}x}{b\sqrt{s^2+x_4}(s\pm x_8)(s\pm x_9)}ds \tag{124.13}$$

$$H_{14}(x,y,t) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{n\pi e^{st} c^2\sqrt{s^2+x_1}\cos\frac{n\pi}{b}\sqrt{\frac{s^2+x_1}{s^2+x_4}}x}{b\sqrt{s^2+x_4}(s\pm x_8)(s\pm x_9)}ds \tag{124.14}$$

After the application of the Cauchy Residue theorem, the integrals (124) are evaluated to give

$$\begin{aligned} H_1(x,y,t) = {} & \frac{\sin y_0 \cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_3^2+x_2)}{\alpha_{ot}(x_1-x_3^2)}}y\left[e^{ix_3t}1-(-1)^m e^{\frac{ix_3}{c}}\right]}{2ix_3(x_1-x_3^2)(x_3^2+x_2)} - e^{ix_3t}\left[1-(-1)^m e^{-\frac{ix_3}{c}}\right] \\ & + \frac{\sin y_0}{2\sqrt{x_2}(x_2+x_3^2)\sqrt{(x_2+x_1)}}\left(e^{\sqrt{x_2}t}.\left[1-(-1)^m e^{\frac{-\sqrt{x_2}}{c}}\right]\right. \\ & \left. - e^{-\sqrt{x_2}t}\left[1-(-1)^m e^{\frac{\sqrt{x_2}}{c}}\right]\right) \end{aligned} \tag{125.1}$$

$$\begin{aligned} H_2(x,y,t) = {} & \frac{e^{ix_3t}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_3^2+x_2)}{\alpha_{ot}(x_3^2+x_1)}}y\left[1-(-1)^m e^{-\frac{ix_3}{c}}\right]}{2ix_3(ix_3\pm x_5)\sqrt{(x_3^2+x_2)(x_1-x_3^2)}i} \\ & - \frac{e^{-ix_3t}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_3^2+x_2)}{\alpha_{ot}(x_3^2+x_1)}}y\left[1-(-1)^m e^{\frac{ix_3}{c}}\right]}{2ix_3(-ix_3\pm x_5)i\sqrt{(x_3^2+x_2)(x_1-x_3^2)}} \\ & - \frac{e^{-x_5t}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_5^2-x_2)}{\alpha_{ot}(x_3^2+x_1)}}y\left[1-(-1)^m e^{\frac{x_5}{c}}\right]}{2x_5(x_5^2+x_3^2)\sqrt{(x_5^2+x_1)(x_5^2-x_2)}} \\ & + \frac{e^{i\sqrt{x_1}t}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_1+x_2)}{\alpha_{ot}(x_1+x_1)}}y\left[1-(-1)^m e^{-\frac{i\sqrt{x_1}}{c}}\right]}{(x_3^2-x_1)(i\sqrt{x_1}\pm x_5)i\sqrt{(\sqrt{x_1}+\sqrt{x_1})(x_1+x_2)}} \\ & - \frac{e^{-i\sqrt{x_1}t}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_1+x_2)}{\alpha_{ot}(x_1+x_1)}}y\left[1-(-1)^m e^{\frac{i\sqrt{x_1}}{c}}\right]}{(x_3^2-x_1)(i\sqrt{x_1}\pm x_5)i\sqrt{(\sqrt{x_1}+\sqrt{x_1})(x_1+x_2)}} \\ & + \frac{e^{\sqrt{x_2}t}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_2-x_2)}{\alpha_{ot}(x_2-x_1)}}y\left[1-(-1)^m e^{-\frac{\sqrt{x_2}}{c}}\right]}{(x_2+x_3^2)(\sqrt{x_2}\pm x_5)\sqrt{(\sqrt{x_2}+\sqrt{x_1})(-\sqrt{x_2}-\sqrt{x_2})}} \\ & - \frac{e^{-\sqrt{x_2}t}\sin(b-y_0)\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_2-x_2)}{\alpha_{ot}(x_2-x_1)}}y\left[1-(-1)^m e^{\frac{\sqrt{x_2}}{c}}\right]}{(x_2+x_3^2)(-\sqrt{x_2}\pm x_5)\sqrt{(\sqrt{x_2}+\sqrt{x_1})(\sqrt{x_2}+\sqrt{x_2})}} \end{aligned} \tag{125.2}$$

$$\begin{aligned} H_3(x,y,t) = {} & \frac{e^{ix_3t}\sqrt{\alpha_{ot}}\sin y_0\cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_3^2+x_2)}{\alpha_{ot}(x_3^2+x_1)}}b\left[1-(-1)^m e^{-\frac{ix_3}{c}}\right]}{2ix_3(ix_3\pm x_5)(x_3+x_2)\sqrt{(m^2\pi^2\alpha_{ot}-1)}}\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_3+x_2)y}{\alpha_{ot}(x_3+x_1)}} \\ & + \frac{e^{-ix_3t}\sqrt{\alpha_{ot}}\sin y_0\cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_3^2+x_2)}{\alpha_{ot}(x_3^2+x_1)}}b\left[1-(-1)^m e^{\frac{ix_3}{c}}\right]}{2ix_3(-ix_3\pm x_5)(x_3+x_2)\sqrt{(m^2\pi^2\alpha_{ot}-1)}}\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_3+x_2)y}{\alpha_{ot}(x_3+x_1)}} \\ & + \frac{\sqrt{\alpha_{ot}}e^{x_5t}\sin y_0\cos\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_5^2-x_2)}{\alpha_{ot}(x_5^2-x_1)}}b\left[1-(-1)^m e^{-\frac{x_5}{c}}\right]}{(x_5^2+x_3^2)(x_5^2-x_2)\sqrt{(m^2\pi^2\alpha_{ot}-1)}}\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_5^2-x_2)y}{\alpha_{ot}(x_5^2-x_1)}} \\ & + \frac{\sqrt{\alpha_{ot}}e^{\sqrt{x_2}t}\sin y_0\cos\sqrt{(m^2\pi^2\alpha_{ot}-1)(x_2-x_2)}\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_2-x_2)y}{\alpha_{ot}(x_2-x_1)}}}{2(x_2+x_3^2)(\sqrt{x_2}\pm x_5)\sqrt{x_2(m^2\pi^2\alpha_{ot}-1)}} \\ & - \frac{\sqrt{\alpha_{ot}}e^{-\sqrt{x_2}t}\sin y_0\cos\sqrt{(m^2\pi^2\alpha_{ot}-1)(x_2-x_2)}\sin\sqrt{\frac{(m^2\pi^2\alpha_{ot}-1)(x_2-x_2)y}{\alpha_{ot}(x_2-x_1)}}}{2(x_2+x_3^2)(-\sqrt{x_2}\pm x_5)\sqrt{x_2(m^2\pi^2\alpha_{ot}-1)}} \end{aligned} \tag{125.3}$$

$$
\begin{aligned}
H_4(x,y,t) = &-\frac{e^{ix_8 t}\sin(y-y_0)\left[1-(-1)^m e^{-\frac{ix_8}{c}}\right]}{2ix_3\sqrt{(x_1-x_3^2)(x_3^2+x_2)}}+\frac{e^{ix_8 t}\sin(y-y_0)\left[1-(-1)^m e^{\frac{ix_8}{c}}\right]}{2ix_3\sqrt{(x_1-x_3^2)(x_3^2+x_2)}} \\
&--\frac{e^{i\sqrt{x_1}t}\sin(y-y_0)\left[1-(-1)^m e^{-\frac{i\sqrt{x_1}}{c}}\right]}{2i\sqrt{x_1}(x_3^2-x_1)\sqrt{(x_1-x_2)}}+\frac{e^{-i\sqrt{x_1}t}\sin(y-y_0)\left[1-(-1)^m e^{\frac{i\sqrt{x_1}}{c}}\right]}{2i\sqrt{x_1}(x_3^2-x_1)\sqrt{(x_1-x_2)}} \\
&+\frac{e^{\sqrt{x_2}t}\sin(y-y_0)\left[1-(-1)^m e^{-\frac{\sqrt{x_2}}{c}}\right]}{2\sqrt{x_2}(x_2+x_3^2)\sqrt{(x_2+x_1)}}-\frac{e^{-\sqrt{x_2}t}\sin(y-y_0)\left[1-(-1)^m e^{\frac{\sqrt{x_2}}{c}}\right]}{2\sqrt{x_2}(x_2+x_3^2)\sqrt{(x_2+x_1)}} \qquad (125.4)
\end{aligned}
$$

$$
\begin{aligned}
H_5(x,y,t) = &\frac{e^{x_8 t}c^2 e^{-\frac{x_8}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}}{2x_8(x_8\pm x_9)}-\frac{e^{-x_8 t}c^2 e^{\frac{x_8}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}}{2x_8(x_8\pm x_9)}+\frac{e^{x_9 t}c^2 e^{-\frac{x_9}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}}{2x_9(x_9\pm x_8)} \\
&-\frac{e^{-x_9 t}c^2 e^{\frac{x_9}{c}}\cos\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}}{2x_9(x_9\pm x_8)} \qquad (125.5)
\end{aligned}
$$

$$
\begin{aligned}
H_6(x,y,t) = &\frac{e^{x_8 t}bce^{-\frac{x_8}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]}{2x_8 n\pi(x_8+x_9)\sqrt{(x_8^2+x_1)(x_8^2+x_4)}}+\frac{e^{-x_8 t}bce^{\frac{x_8}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]}{2x_8 n\pi(x_8-x_9)(x_8^2+x_1)(x_8^2+x_4)} \\
&+\frac{e^{x_9 t}bce^{-\frac{x_9}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]}{2x_9 n\pi(x_9-x_8)(x_9^2+x_1)(x_9^2+x_4)}+\frac{e^{-x_9 t}bce^{\frac{x_9}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]}{2x_9 n\pi(x_9+x_8)(x_9^2+x_1)(x_9^2+x_4)} \\
&+\frac{e^{ix_1 t}bce^{-\frac{ix_1}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]}{2ix_1(ix_1+x_8)(ix_1+x_9)(x_1-x_1^2)(x_4-x_1^2)}-\frac{e^{-ix_1 t}bce^{\frac{ix_1}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]}{2ix_1(ix_1-x_8)(ix_1-x_9)(x_1-x_1^2)(x_4-x_1^2)} \\
&+\frac{e^{ix_4 t}bce^{-\frac{ix_4}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]}{2ix_4(ix_4+x_8)(ix_4+x_9)(x_1-x_1^2)(x_4-x_1^2)} \\
&-\frac{e^{-ix_4 t}bce^{\frac{ix_4}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]}{2ix_4(ix_4-x_8)(ix_4-x_9)(x_1-x_1^2)(x_4-x_1^2)} \qquad (125.6)
\end{aligned}
$$

$$
\begin{aligned}
H_7(x,y,t) = &\frac{e^{x_8 t}c^2\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8(x_8\pm x_9)(x_8^2+x_4)}-\frac{e^{-x_8 t}c^2\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8(x_8\pm x_9)(x_8^2+x_4)}+\frac{e^{x_9 t}c^2\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9(x_9\pm x_8)(x_9^2+x_4)} \\
&-\frac{e^{-x_9 t}c^2\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9(x_9\pm x_8)(x_9^2+x_4)}+\frac{e^{ix_4 t}c^2\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_4^2}{x_4-x_4^2}}x\right]}{2ix_4(ix_4\pm x_8)(ix_4\pm x_9)}-\frac{e^{-ix_4 t}c^2\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_4^2}{x_4-x_4^2}}x\right]}{2ix_4(-ix_4\pm x_8)(-ix_4\pm x_9)} \qquad (125.7)
\end{aligned}
$$

$$
\begin{aligned}
H_8(x,y,t) = &\frac{e^{x_8 t}ce^{-\frac{x_8}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8(x_8\pm x_9)(x_8\pm x_{13})}-\frac{e^{-x_8 t}ce^{\frac{x_8}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8(-x_8\pm x_9)(-x_8\pm x_{13})} \\
&+\frac{e^{x_9 t}ce^{-\frac{x_9}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9(x_9\pm x_8)(x_9\pm x_{13})}-\frac{e^{-x_9 t}ce^{\frac{x_9}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9(x_9\pm x_8)(x_9\pm x_{13})} \\
&+\frac{e^{x_{13} t}ce^{-\frac{x_{13}}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_{13}^2+x_1}{x_{13}^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{13}^2+x_1}{x_{13}^2+x_4}}x\right]}{2x_{13}(x_{13}\pm x_8)(x_{13}\pm x_9)} \\
&-\frac{e^{-x_{13} t}ce^{\frac{x_{13}}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_{13}^2+x_1}{x_{13}^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{13}^2+x_1}{x_{13}^2+x_4}}x\right]}{2x_{13}(x_{13}\pm x_8)(x_{13}\pm x_9)} \qquad (125.8)
\end{aligned}
$$

$$H_9(x,y,t) = \frac{e^{x_8 t} bcx_8\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\right)e^{-\frac{x_8}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]\sqrt{x_8^2+x_1}}{2n\pi x_8(x_8 \pm x_9)\sqrt{x_8^2+x_1}}$$
$$+\frac{e^{-x_8 t} bcx_8\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\right)e^{\frac{x_8}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]\sqrt{x_8^2+x_1}}{2n\pi x_8(-x_8 \pm x_9)\sqrt{x_8^2+x_1}}$$
$$+\frac{e^{x_9 t} bcx_9\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\right)e^{-\frac{x_9}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]\sqrt{x_9^2+x_1}}{2n\pi x_9(x_9 \pm x_8)\sqrt{x_9^2+x_1}}$$
$$+\frac{e^{-x_9 t} bcx_9\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\right)e^{\frac{x_9}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]\sqrt{x_9^2+x_1}}{2n\pi x_9(-x_9 \pm x_8)\sqrt{x_9^2+x_1}}$$
$$+\frac{ie^{ix_1 t} bcx_1\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\right)e^{-\frac{ix_1}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}x\right]\sqrt{x_4-x_1^2}}{2in\pi x_1(ix_1 \pm x_8)(ix_1 \pm x_9)}$$
$$+\frac{ie^{-ix_1 t} bcx_1\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\right)e^{\frac{ix_1}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}x\right]\sqrt{x_4-x_1^2}}{2in\pi x_1(-ix_1 \pm x_8)(-ix_1 \pm x_9)} \quad (125.9)$$

$$H_{10}(x,y,t) = \frac{ie^{ix_1 t} bcx_1\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\right)\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}x\right]i\sqrt{(x_1^2+x_4)^3}e^{-\frac{ix_1}{c}}}{n\pi\sqrt{x_1-x_1^2}(ix_1 \pm x_{10})(ix_1 \pm x_{11})}$$
$$-\frac{ie^{-ix_1 t} bcx_1\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}\right]\right)\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_1-x_1^2}{x_4-x_1^2}}x\right]i\sqrt{(x_1^2+x_4)^3}e^{-\frac{ix_1}{c}}}{n\pi\sqrt{x_1-x_1^2}(-ix_1 \pm x_{10})(-ix_1 \pm x_{11})}$$
$$+\frac{e^{x_{10} t} bcx_{10}\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}\right]\right)\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}x\right]\sqrt{(x_{10}^2+x_4)^3}e^{-\frac{x_{10}}{c}}}{2n\pi x_{10}\sqrt{x_{10}^2+x_1}(x_{10} \pm x_{11})}$$
$$+\frac{e^{-x_{10} t} bcx_{10}\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}\right]\right)\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}x\right]\sqrt{(x_{10}^2+x_4)^3}e^{\frac{x_{10}}{c}}}{2n\pi x_{10}\sqrt{x_{10}^2+x_1}(-x_{10} \pm x_{11})}$$
$$+\frac{e^{x_{11} t} bcx_{11}\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}\right]\right)\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}x\right]\sqrt{(x_{11}^2+x_4)^3}e^{-\frac{x_{11}}{c}}}{2n\pi x_{11}\sqrt{x_{11}^2+x_1}(x_{11} \pm x_{10})}$$
$$+\frac{e^{-x_{11} t} bcx_{11}\left(1-\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}\right]\right)\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}x\right]\sqrt{(x_{11}^2+x_4)^3}e^{\frac{x_{11}}{c}}}{2n\pi x_{11}\sqrt{x_{11}^2+x_1}(-x_{11} \pm x_{10})} \quad (125.10)$$

$$H_{11}(x,y,t) = \frac{e^{x_{10}t}c^2e^{-\frac{x_{10}}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}x\right]}{2x_{10}(x_{10}\pm x_{11})} - \frac{e^{-x_{10}t}c^2e^{\frac{x_{10}}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{10}^2+x_1}{x_{10}^2+x_4}}x\right]}{2x_{10}(-x_{10}\pm x_{11})}$$
$$+\frac{e^{x_{11}t}c^2e^{-\frac{x_{11}}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}x\right]}{2x_{11}(x_{11}\pm x_{10})}$$
$$-\frac{e^{-x_{11}t}c^2e^{\frac{x_{11}}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}\right]\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_{11}^2+x_1}{x_{11}^2+x_4}}x\right]}{2x_{11}(-x_{11}\pm x_{10})} \quad (125.11)$$

$$H_{12}(x,y,t) = \frac{e^{x_8t}cse^{-\frac{x_8}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8(x_8\pm x_9)} - \frac{e^{-x_8t}cse^{\frac{x_8}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8(-x_8\pm x_9)}$$
$$+\frac{e^{x_9t}cse^{-\frac{x_9}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9(x_9\pm x_8)}$$
$$-\frac{e^{-x_9t}cse^{\frac{x_9}{c}}\sin\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9(-x_9\pm x_8)} \quad (125.12)$$

$$H_{13}(x,y,t) = 0 + \frac{n\pi e^{x_8t}c^2\sqrt{x_8^2+x_1}e^{-\frac{x_8}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8b\sqrt{x_8^2+x_4}(x_8\pm x_9)}$$
$$-\frac{n\pi e^{-x_8t}c^2\sqrt{x_8^2+x_1}e^{\frac{x_8}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8b\sqrt{x_8^2+x_4}(-x_8\pm x_9)}$$
$$+\frac{n\pi e^{x_9t}c^2\sqrt{x_9^2+x_1}e^{-\frac{x_9}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2bx_9\sqrt{x_9^2+x_4}(x_9\pm x_8)}$$
$$-\frac{n\pi e^{-x_9t}c^2\sqrt{x_9^2+x_1}e^{\frac{x_9}{c}}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}\right]\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2bx_9\sqrt{x_9^2+x_4}(-x_9\pm x_8)} \quad (125.13)$$

$$H_{14}(x,y,t) = 0 + \frac{n\pi e^{x_8t}c^2\sqrt{x_8^2+x_1}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8b\sqrt{x_8^2+x_4}(x_8\pm x_9)} - \frac{n\pi e^{-x_8t}c^2\sqrt{x_8^2+x_1}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_8^2+x_1}{x_8^2+x_4}}x\right]}{2x_8b\sqrt{x_8^2+x_4}(-x_8\pm x_9)}$$
$$+\frac{n\pi e^{x_9t}c^2\sqrt{x_9^2+x_1}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9b\sqrt{x_9^2+x_4}(x_9\pm x_8)}$$
$$-\frac{n\pi e^{-x_9t}c^2\sqrt{x_9^2+x_1}\cos\left[\frac{n\pi}{b}\sqrt{\frac{x_9^2+x_1}{x_9^2+x_4}}x\right]}{2x_9b\sqrt{x_9^2+x_4}(-x_9\pm x_8)} \quad (125.14)$$

Substitution of equations (125) into equation (122) produces the inversion of equation (120) and the general solution of equation (102).

FIRST ORDER CORRECTION

The governing equation for the first order correction when considering the moving force problem (I.e $r_0 = 0$) is

$$-\gamma_1^2\frac{\partial^2 W_1(x,y,s)}{\partial x^2} - \gamma_2^2\frac{\partial^2 W_1(x,y,s)}{\partial y^2} + s^2W_1(x,y,s) - \alpha_{ot}s^2\left[\frac{\partial^2 W_1(x,y,s)}{\partial x^2} + \frac{\partial^2 W_1(x,y,s)}{\partial y^2}\right] = 0 \quad (126)$$

Now one attempt the solution of $W_1(x,y,s)$. The procedure of obtaining this solution is analogous to that of leading order. Following the procedures for obtaining the leading order solution one obtains the first-order correction to the uniformly valid analytical solution in the entire domain of the plate-structure as

$$
\begin{aligned}
W_1(x,y,t) = {} & P_{c_1}\{Q_1(x,y,t)+Q_2(x,y,t)+Q_3(x,y,t)\}+P_{c_2}\{Q_4(x,y,t)+Q_5(x,y,t)+Q_6(x,y,t)\} \\
& +P_{c_3}\{Q_7(x,y,t)+Q_8(x,y,t)+Q_9(x,y,t)+Q_{10}(x,y,t)\}+P_{c_4}\{Q_{11}(x,y,t)+Q_{12}(x,y,t)+Q_{13}(x,y,t)\} \\
& +P_{c_5}\{Q_{14}(x,y,t)+Q_{15}(x,y,t)+Q_{16}(x,y,t)+Q_{17}(x,y,t)+Q_{18}(x,y,t)\} \\
& +P_{c_6}\{Q_{19}(x,y,t)+Q_{20}(x,y,t)+Q_{21}(x,y,t)+Q_{22}(x,y,t)+Q_{23}(x,y,t)\}+P_{c_2}\{Q_{24}(x,y,t)\} \\
& +P_{c_7}\{Q_{25}(x,y,t)+Q_{26}(x,y,t)\}+P_{c_5}\{Q_{27}(x,y,t)+Q_{28}(x,y,t)\} \\
& +P_{c_6}\{Q_{29}(x,y,t)+Q_{30}(x,y,t)+Q_{31}(x,y,t)+Q_{32}(x,y,t)\}+Q_{33}(x,y,t)+Q_{34}(x,y,t)+Q_{35}(x,y,t) \\
& +Q_{36}(x,y,t)+Q_{37}(x,y,t)+Q_{38}(x,y,t)+P_{c_8}\{Q_{39}(x,y,t)+Q_{40}(x,y,t)+Q_{41}(x,y,t)+Q_{42}(x,y,t)\} \\
& +P_{c_9}\{Q_{43}(x,y,t)+Q_{44}(x,y,t)+Q_{45}(x,y,t)+Q_{46}(x,y,t)\} \\
& +bP_{c_2}\{Q_{47}(x,y,t)+Q_{48}(x,y,t)+\cdots+Q_{59}(x,y,t)\} \qquad (127)
\end{aligned}
$$

where,

$$
\begin{aligned}
Q_1(x,y,t) = {} & \frac{e^{i\rho_2 t}\left[1-(-1)^m e^{-\rho_2/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{\sqrt{2i\,\rho_2}(x_1-\rho_2^2)(x_3^2-\rho_2^2)(i\rho_2+\rho_5)(i\rho_2-\rho_5)}-\frac{e^{-i\rho_2 t}\left[1-(-1)^m e^{-i\rho_2/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{\sqrt{2i\,\rho_2}(x_1-\rho_2^2)(x_3^2-\rho_2^2)(i\rho_2-\rho_5)(\rho_5+i\rho_2)}- \\
& \frac{e^{i\sqrt{\chi_1}t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{2i\sqrt{\chi_1}\sqrt{\rho_2-\chi_1}(x_3^2-\chi_1)(i\sqrt{\chi_1}+\rho_5)^2}+\frac{e^{-i\sqrt{\chi_1}t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{2i\sqrt{\chi_1}\sqrt{\rho_2-\chi_1}(x_3^2-\chi_1)(\rho_5-i\sqrt{\chi_1})(\rho_5+i\sqrt{\chi_1})}+ \\
& \frac{e^{i\chi_3 t}\left[1-(-1)^m e^{-i\chi_3/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{2i\chi_3\sqrt{\rho_2-x_3^2}(\chi_1-x_3^2)(\rho_5+i\chi_3)(i\chi_3-\rho_5)}+\frac{e^{-i\chi_3 t}\left[1-(-1)^m e^{i\chi_3/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{2i\chi_3\sqrt{\rho_2-x_3^2}(\chi_1-x_3^2)(\rho_5-i\chi_3)(i\chi_3+\rho_5)}+ \\
& \frac{e^{\rho_5 t}\left[1-(-1)^m e^{-\rho_5/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{2\rho_5\sqrt{\rho_5^2+\rho_2}(\rho_5^2+\chi_1)(\rho_5^2+x_3^2)}-\frac{e^{-\rho_5 t}\left[1-(-1)^m e^{\rho_5/c}\right]\sin(b-y_o)\cos\left(\sqrt{\frac{z(S^2+\rho_1)}{\alpha_{ot}(S^2+\rho_2)}}y\right)}{2\rho_5\sqrt{\rho_5^2+\rho_2}(\rho_5^2+\chi_1)(\rho_5^2+x_3^2)} \qquad (128.1)
\end{aligned}
$$

$$
\begin{aligned}
Q_2(x,y,t) = {} & \frac{e^{i\rho_2 t}\left[1-(-1)^m e^{-\rho_2/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\chi_2-\rho_2^2)}{(\chi_1-\rho_2^2)}}b\right)\cos\left(\sqrt{\frac{z(\rho_1+\rho_2^2)}{\alpha_{ot}(\rho_2+\rho_2^2)}}y\right)}{\sqrt{2i\,\rho_2}(x_1-\rho_2^2)(x_3^2-\rho_2^2)(\rho_5+i\rho_2)(i\rho_2-\rho_5)}+ \\
& \frac{e^{-\rho_2 t}\left[1-(-1)^m e^{-i\rho_2/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\chi_2-\rho_2^2)}{(\chi_1-\rho_2^2)}}b\right)\cos\left(\sqrt{\frac{z(\rho_1+\rho_2^2)}{\alpha_{ot}(\rho_2+\rho_2^2)}}y\right)}{i\sqrt{2i\,\rho_2}(x_1-\rho_2^2)(x_3^2-\rho_2^2)(\rho_5-i\rho_2)(\rho_5-i\rho_2)}-\frac{e^{i\sqrt{\chi_1}t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}b\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}y\right)}{2i\sqrt{\chi_1}\sqrt{\rho_2-\chi_1}(x_3^2-\chi_1)(i\sqrt{\chi_1}+\rho_5)^2}+ \\
& \frac{e^{-i\sqrt{\chi_1}t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}b\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}y\right)}{2i\sqrt{\chi_1}\sqrt{\rho_2-\chi_1}(x_3^2-\chi_1)(\rho_5-i\sqrt{\chi_1})(\rho_5+i\sqrt{\chi_1})}+\frac{e^{i\chi_3 t}\left[1-(-1)^m e^{-i\chi_3/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\chi_2-\chi_3^2)}{(\chi_1-\chi_3^2)}}b\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_3^2)}{\alpha_{ot}(\rho_2-\chi_3^2)}}y\right)}{2i\chi_3\sqrt{\rho_2-\chi_3^2}(x_1-\chi_3^2)(\rho_5+i\chi_3)(i\chi_3-\rho_5)}+ \\
& \frac{e^{-i\chi_3 t}\left[1-(-1)^m e^{i\chi_3/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\chi_2-\chi_3^2)}{(\chi_1-\chi_3^2)}}b\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_3^2)}{\alpha_{ot}(\rho_2-\chi_3^2)}}y\right)}{2i\chi_3\sqrt{\rho_2-\chi_3^2}(x_1-\chi_3^2)(\rho_5-i\chi_3)(i\chi_3+\rho_5)}+\frac{e^{\rho_5 t}\left[1-(-1)^m e^{-\rho_5/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\rho_5^2+\chi_2)}{(\rho_5^2+\chi_1)}}b\right)\cos\left(\sqrt{\frac{z(\rho_5^2+\rho_1)}{\alpha_{ot}(\rho_5^2+\rho_2)}}y\right)}{2\rho_5\sqrt{\rho_5^2+\rho_2}(\rho_5^2+x_1)(\rho_5^2+x_3^2)}+ \\
& \frac{e^{-\rho_5 t}\left[1-(-1)^m e^{\rho_5/c}\right]\sin y_o\cos\left(\sqrt{\frac{(\rho_5^2+\chi_2)}{(\rho_5^2+\chi_1)}}b\right)\cos\left(\sqrt{\frac{z(\rho_5^2+\rho_1)}{\alpha_{ot}(\rho_5^2+\rho_2)}}y\right)}{2\rho_5\sqrt{\rho_5^2+\rho_2}(\rho_5^2+x_1)(\rho_5^2+x_3^2)} \qquad (128.2)
\end{aligned}
$$

$$
\begin{aligned}
Q_3(x,y,t) = -\Bigg[& \frac{e^{i\rho_2 t}\left[1-(-1)^m e^{-i\rho_2/c}\right]\cos y_o\cos\left(\sqrt{\frac{z(\rho_1+\rho_2^2)}{\alpha_{ot}(\rho_2+\rho_2^2)}}y\right)}{\sqrt{2i\,\rho_2(x_1-\rho_2^2)(\chi-\rho_2^2)}(x_3^2-\rho_2^2)}+\frac{e^{-i\rho_2 t}\left[1-(-1)^m e^{i\rho_2/c}\right]\cos y_o\cos\left(\sqrt{\frac{z(\rho_1+\rho_2^2)}{\alpha_{ot}(\rho_2+\rho_2^2)}}y\right)}{\sqrt{2i\,\rho_2(x_1-\rho_2^2)(\chi-\rho_2^2)}(x_3^2-\rho_2^2)}+ \\
& \frac{e^{i\chi_1 t}\left[1-(-1)^m e^{-i\chi_1/c}\right]\sin y_o\cos\left(\sqrt{\frac{z(\rho_1-\chi_1^2)}{\alpha_{ot}(\rho_2-\chi_1^2)}}y\right)}{\sqrt{2i\chi_1\,(\rho_2-\chi_1^2)(x_2-\chi_1^2)}(\chi_3^2-\chi_1^2)}+\frac{e^{-i\chi_1 t}\left[1-(-1)^m e^{i\chi_1/c}\right]\sin y_o\cos\left(\sqrt{\frac{z(\rho_1-\chi_1^2)}{\alpha_{ot}(\rho_2-\chi_1^2)}}y\right)}{i\sqrt{2i\chi\,(\rho_2-\chi_1^2)(x_2-\chi_1^2)}(\chi_3^2-\chi_1^2)}+ \\
& \frac{e^{i\chi_2 t}\left[1-(-1)^m e^{-i\chi_2/c}\right]\sin y_o\cos\left(\sqrt{\frac{z(\rho_1-\chi_2^2)}{\alpha_{ot}(\rho_2-\chi_2^2)}}y\right)}{\sqrt{2i\chi_2\,(\rho_2-\chi_2^2)(x_1-\chi_2^2)}(\chi_3^2-\chi_2^2)}+\frac{e^{-i\chi_2 t}\left[1-(-1)^m e^{i\chi_2/c}\right]\sin y_o\cos\left(\sqrt{\frac{z(\rho_1-\chi_2^2)}{\alpha_{ot}(\rho_2-\chi_2^2)}}y\right)}{i\sqrt{2i\chi_2\,(\rho_2-\chi_2^2)(x_1-\chi_2^2)}(\chi_3^2-\chi_2^2)}+ \\
& \frac{e^{i\chi_3 t}\left[1-(-1)^m e^{-i\chi_3/c}\right]\sin y_o\cos\left(\sqrt{\frac{z(\rho_1-\chi_3^2)}{\alpha_{ot}(\rho_2-\chi_3^2)}}y\right)}{2i\chi_3\sqrt{(\rho_2-\chi_3^2)(x_1-\chi_3^2)(x_2-\chi_3^2)}}+\frac{e^{-i\chi_3 t}\left[1-(-1)^m e^{i\chi_3/c}\right]\sin y_o\cos\left(\sqrt{\frac{z(\rho_1-\chi_3^2)}{\alpha_{ot}(\rho_2-\chi_3^2)}}y\right)}{2i\chi_3\sqrt{(\rho_2-\chi_3^2)(x_1-\chi_3^2)(x_2-\chi_3^2)}}\Bigg] \qquad (128.3)
\end{aligned}
$$

$$
\begin{aligned}
Q_4(x,y,t) = {} & \frac{c^2 n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_1)\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}y\right)}{2ib\sqrt{\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)}-\frac{c^2 n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_1)\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}y\right)}{2ib\sqrt{\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)}+ \\
& \frac{c^2 n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}x\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)}{2ib\sqrt{\chi_8}(\chi_1-\chi_8)(\chi_9-\chi_8)}-\frac{c^2 n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}x\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)}{2ib\sqrt{\chi_8}(\chi_1-\chi_8)(\chi_9-\chi_8)}+ \\
& \frac{c^2 n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}x\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{2ib\sqrt{\chi_9}(\chi_1-\chi_9)(\chi_8-\chi_9)}+\frac{c^2 n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}x\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{2ib\sqrt{\chi_9}(\chi_1-\chi_9)(\chi_8-\chi_9)} \qquad (128.4)
\end{aligned}
$$

$$Q_5(x,y,t) = -\left[\frac{e^{i\rho_2 t}\left[1-(-1)^m e^{-i\rho_2/c}\right]\cos y_o \cos\left(\sqrt{\frac{z(\rho_1+\rho_2^2)}{\alpha_{ot}(\rho_2+\rho_2^2)}}y\right)}{\sqrt{2i\,\rho_2(x_1-\rho_2^2)(\chi-\rho_2^2)(x_3^2-\rho_2^2)}} + \frac{e^{-i\rho_2 t}\left[1-(-1)^m e^{i\rho_2/c}\right]\cos y_o \cos\left(\sqrt{\frac{z(\rho_1+\rho_2^2)}{\alpha_{ot}(\rho_2+\rho_2^2)}}y\right)}{\sqrt{2i\,\rho_2(x_1-\rho_2^2)(\chi-\rho_2^2)(x_3^2-\rho_2^2)}} + \right.$$

$$\frac{e^{i\chi_1 t}\left[1-(-1)^m e^{-i\chi_1/c}\right]\sin y_o \cos\left(\sqrt{\frac{z(\rho_1-\chi_1^2)}{\alpha_{ot}(\rho_2-\chi_1^2)}}y\right)}{\sqrt{2i\chi_1\,(\rho_2-\chi_1^2)(x_2-\chi_1^2)}(\chi_3^2-\chi_1^2)} + \frac{e^{-i\chi_1 t}\left[1-(-1)^m e^{i\chi_1/c}\right]\sin y_o \cos\left(\sqrt{\frac{z(\rho_1-\chi_1^2)}{\alpha_{ot}(\rho_2-\chi_1^2)}}y\right)}{i\sqrt{2i\chi\,(\rho_2-\chi_1^2)(x_2-\chi_1^2)}(\chi_3^2-\chi_1^2)} +$$

$$\frac{e^{i\chi_2 t}\left[1-(-1)^m e^{-i\chi_2/c}\right]\sin y_o \cos\left(\sqrt{\frac{z(\rho_1-\chi_2^2)}{\alpha_{ot}(\rho_2-\chi_2^2)}}y\right)}{\sqrt{2i\chi_2\,(\rho_2-\chi_2^2)(x_1-\chi_2^2)}(\chi_3^2-\chi_2^2)} + \frac{e^{-i\chi_2 t}\left[1-(-1)^m e^{i\chi_2/c}\right]\sin y_o \cos\left(\sqrt{\frac{z(\rho_1-\chi_2^2)}{\alpha_{ot}(\rho_2-\chi_2^2)}}y\right)}{i\sqrt{2i\chi_2\,(\rho_2-\chi_2^2)(x_1-\chi_2^2)}(\chi_3^2-\chi_2^2)} +$$

$$\left.\frac{e^{i\chi_3 t}\left[1-(-1)^m e^{-i\chi_3/c}\right]\sin y_o \cos\left(\sqrt{\frac{z(\rho_1-\chi_3^2)}{\alpha_{ot}(\rho_2-\chi_3^2)}}y\right)}{2i\chi_3\sqrt{(\rho_2-\chi_3^2)(x_1-\chi_3^2)(x_2-\chi_3^2)}} + \frac{e^{-i\chi_3 t}\left[1-(-1)^m e^{i\chi_3/c}\right]\sin y_o \cos\left(\sqrt{\frac{z(\rho_1-\chi_3^2)}{\alpha_{ot}(\rho_2-\chi_3^2)}}y\right)}{2i\chi_3\sqrt{(\rho_2-\chi_3^2)(x_1-\chi_3^2)(x_2-\chi_3^2)}}\right] \tag{128.5}$$

$$Q_6(x,y,t) = \frac{c^2 n\pi\, e^{i\sqrt{X_1}t}(X_4-X_1)\cos\left(\frac{n\pi}{b}\sqrt{\frac{X_1-X_1}{X_4-X_1}}\,x\right)\cos\left(\sqrt{\frac{z(\rho_1-X_1)}{\alpha_{ot}(\rho_2-X_1)}}\,y\right)}{2ib\sqrt{X_1}\,(X_8-X_1)(X_9-X_1)} + -\frac{c^2 n\pi\, e^{-i\sqrt{X_1}t}(X_4-X_1)\cos\left(\frac{n\pi}{b}\sqrt{\frac{X_1-X_1}{X_4-X_1}}\,x\right)\cos\left(\sqrt{\frac{z(\rho_1-X_1)}{\alpha_{ot}(\rho_2-X_1)}}\,y\right)}{2ib\sqrt{X_1}\,(X_8-X_1)(X_9-X_1)} +$$

$$\frac{c^2 n\pi\, e^{i\sqrt{X_8}t}(X_4-X_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{X_1-X_8}{X_4-X_8}}\,x\right)\cos\left(\sqrt{\frac{z(\rho_1-X_8)}{\alpha_{ot}(\rho_2-X_8)}}\,y\right)}{2ib\sqrt{X_8}\,(X_1-X_8)(X_9-X_8)} + -\frac{c^2 n\pi\, e^{-i\sqrt{X_1}t}(X_4-X_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{X_1-X_8}{X_4-X_8}}\,x\right)\cos\left(\sqrt{\frac{z(\rho_1-X_8)}{\alpha_{ot}(\rho_2-X_8)}}\,y\right)}{2ib\sqrt{X_8}\,(X_1-X_8)(X_9-X_8)} +$$

$$\frac{c^2 n\pi\, e^{i\sqrt{X_9}t}(X_4-X_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{X_1-X_9}{X_4-X_9}}\,x\right)\cos\left(\sqrt{\frac{z(\rho_1-X_9)}{\alpha_{ot}(\rho_2-X_9)}}\,y\right)}{2ib\sqrt{X_9}\,(X_1-X_9)(X_8-X_9)} + -\frac{c^2 n\pi\, e^{-i\sqrt{x_9}t}(x_4-x_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{X_1-X_9}{X_4-X_9}}\,x\right)\cos\left(\sqrt{\frac{z(\rho_1-X_9)}{\alpha_{ot}(\rho_2-X_9)}}\,y\right)}{2ib\sqrt{X_9}\,(X_1-X_9)(X_8-X_9)} \tag{128.6}$$

$$Q_7(x,y,t) = \frac{c^2 n\pi e^{i\left(t-\frac{1}{c}\right)\sqrt{\chi_1}}\sqrt{\chi_4-\chi_1}\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}y\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)}{ib\sqrt{2\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)(\rho_5+i\sqrt{\chi_1})(i\sqrt{\chi_1}-\rho_5)} +$$

$$\frac{c^2 n\pi e^{-i\left(t-\frac{1}{c}\right)\sqrt{\chi_1}}\sqrt{\chi_4-\chi_1}\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}y\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)}{ib\sqrt{2\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)(\rho_5-i\sqrt{\chi_1})(\rho_5+i\sqrt{\chi_1})} +$$

$$\frac{c^2 n\pi e^{i\left(t-\frac{1}{c}\right)\sqrt{\chi_8}}\sqrt{\chi_4-\chi_8}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)}{ib\sqrt{2\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)(\rho_5+i\sqrt{\chi_8})(i\sqrt{\chi_8}-\rho_5)} +$$

$$\frac{c^2 n\pi e^{-i\left(t-\frac{1}{c}\right)\sqrt{\chi_8}}\sqrt{\chi_4-\chi_8}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)}{ib\sqrt{2\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)(\rho_5+i\sqrt{\chi_8})(\rho_5-i\sqrt{\chi_8})} -$$

$$\frac{c^2 n\pi e^{i\left(t-\frac{1}{c}\right)\sqrt{\chi_9}}\sqrt{\chi_4-\chi_9}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)}{ib\sqrt{2\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)(\rho_5+i\sqrt{\chi_9})(\rho_5-i\sqrt{\chi_9})} +$$

$$\frac{c^2 n\pi e^{-i\left(t-\frac{1}{c}\right)\sqrt{\chi_9}}\sqrt{\chi_4-\chi_9}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)}{ib\sqrt{2\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)(\rho_5+i\sqrt{\chi_9})(\rho_5-i\sqrt{\chi_9})} +$$

$$\frac{c^2 n\pi e^{-i\left(t-\frac{1}{c}\right)\sqrt{\chi_8}t}\sqrt{(x_1-x_8)(x_4-x_8)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)}{2i\sqrt{\chi_8}\;(\chi_9-\chi_8)} +$$

$$\frac{c^2 n\pi e^{i\left(t-\frac{1}{c}\right)\sqrt{\chi_9}t}\sqrt{(x_1-x_9)(x_4-x_9)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{2i\sqrt{\chi_9}\;(\chi_8-\chi_9)} -$$

$$\frac{c^2 n\pi e^{-i\left(t-\frac{1}{c}\right)\sqrt{\chi_9}t}\sqrt{(x_1-x_9)(x_4-x_9)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{2i\sqrt{\chi_9}\;(\chi_8-\chi_9)} \tag{128.13}$$

$$Q_{14}(x,y,t) = \frac{e^{i\sqrt{\chi_1}t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\chi_1)}{\alpha_{ot}(\chi_4-\chi_1)}}b\right)\sin y_o}{2i\sqrt{\chi_1}(\chi_3^2-\chi_1)\sqrt{\rho_2-\chi_1}(\rho_6+i\sqrt{\chi_1})(i\sqrt{\chi_1}-\rho_6)} - \frac{e^{-i\sqrt{\chi_1}t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\chi_1)}{\alpha_{ot}(\chi_4-\chi_1)}}b\right)\sin y_o}{2i\sqrt{\chi_1}(\chi_3^2-\chi_1)\sqrt{\rho_2-\chi_1}(\rho_6-i\sqrt{\chi_1})(\rho_6+i\sqrt{\chi_1})} +$$

$$\frac{e^{i\chi_3 t}\left[1-(-1)^m e^{-i\chi_3/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\chi_3^2)}{\alpha_{ot}(\chi_4-\chi_3^2)}}b\right)\sin y_o}{2i\chi_3(\chi_1-\chi_3)\sqrt{\rho_2-\chi_3}(\rho_6+i\chi_3)(i\chi_3-\rho_6)} - \frac{e^{-i\chi_3 t}\left[1-(-1)^m e^{i\chi_3/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\chi_3^2)}{\alpha_{ot}(\chi_4-\chi_3^2)}}b\right)\sin y_o}{2i\chi_3(\chi_1-\chi_3)\sqrt{\rho_2-\chi_3}(\rho_6-i\chi_3)(\rho_6+i\chi_3)} +$$

$$\frac{e^{i\rho_6 t}\left[1-(-1)^m e^{-i\rho_6/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\rho_6^2)}{\alpha_{ot}(\chi_4-\rho_6^2)}}b\right)\sin y_o}{2i\chi_6(\chi_1-\rho_6^2)\sqrt{\rho_2-\rho_6^2}(\chi_3^2-\rho_6^2)} - \frac{e^{-i\rho_6 t}\left[1-(-1)^m e^{i\rho_6/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\rho_6^2)}{\alpha_{ot}(\chi_4-\rho_6^2)}}b\right)\sin y_o}{2i\chi_6(\chi_1-\rho_6^2)\sqrt{\rho_2-\rho_6^2}(\chi_3^2-\rho_6^2)} +$$

$$\frac{e^{i\sqrt{\rho_2}t}\left[1-(-1)^m e^{-i\sqrt{\rho_2}/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\rho_2)}{\alpha_{ot}(\chi_4-\rho_2)}}b\right)\sin y_o}{2i\sqrt{\rho_2}(\chi_1-\rho_2)(\chi_3^2-\rho_2)(i\sqrt{\rho_2}+\rho_6)(i\sqrt{\rho_2}-\rho_6)} + \frac{e^{-i\sqrt{\rho_2}t}\left[1-(-1)^m e^{i\sqrt{\rho_2}/c}\right]\sin\left(\sqrt{\frac{z(\chi_1-\rho_2)}{\alpha_{ot}(\chi_4-\rho_2)}}b\right)\sin y_o}{2i\sqrt{\rho_2}(\chi_1-\rho_2)(\chi_3^2-\rho_2)(\rho_6-i\sqrt{\rho_2})(\rho_6+i\sqrt{\rho_2})} \tag{128.14}$$

$$Q_{15}(x,y,t) = \frac{e^{iX_3 t}\left[1-(-1)^{m}e^{-iX_3/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{z(\rho_1-x_3^2)}{\alpha_{\circ t}(\rho_2-x_3^2)}}\right)\sqrt{X_4-X_3^2}}{2n\pi i X_3\sqrt{(X_1-X_3^2)^3}\,(iX_3+\rho_5)(iX_3-\rho_5)(iX_3+\rho_6)(iX_3-\rho_6)} +$$

$$\frac{e^{-iX_3 t}\left[1-(-1)^{m}e^{iX_3/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{z(\rho_1-x_3^2)}{\alpha_{\circ t}(\rho_2-x_3^2)}}\right)\sqrt{X_4-X_3^2}}{-2n\pi i X_3\sqrt{(X_1-X_3^2)^3}\,(\rho_5-iX_3)(\rho_5+iX_3)(\rho_6-iX_3)(\rho_6+iX_3)} + \frac{e^{-\rho_5 t}\left[1-(-1)^{m}e^{\rho_5/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{z(\rho_5^2+\rho_1)}{\alpha_{\circ t}(\rho_5^2+\rho_2)}}\right)\sqrt{\rho_5^2+X_4}}{-2n\pi\rho_5\,(\rho_5^2+X_3^2)\sqrt{(\rho_5^2-X_1)^3}\,(\rho_5-\rho_6)(\rho_5+\rho_6)} +$$

$$\frac{e^{\rho_5 t}\left[1-(-1)^{m}e^{-\rho_5/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{z(\rho_5^2+\rho_1)}{\alpha_{\circ t}(\rho_5^2+\rho_2)}}\right)\sqrt{\rho_5^2+X_4}}{2n\pi\rho_5\,(\rho_5^2+X_3^2)\sqrt{(\rho_5^2-X_1)^3}\,(\rho_6-\rho_5)(\rho_6+\rho_5)} + \frac{e^{\rho_6 t}\left[1-(-1)^{m}e^{\rho_6/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}\right)\sqrt{\rho_6^2+X_4}}{2n\pi\rho_6\,(\rho_6^2+X_3^2)\sqrt{(\rho_6^2-X_1)^3}\,(\rho_6+\rho_5)(\rho_6-\rho_5)} +$$

$$\frac{e^{-\rho_6 t}\left[1-(-1)^{m}e^{-\rho_6/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}\right)\sqrt{\rho_6^2+X_4}}{2n\pi\rho_6\,(\rho_6^2+X_3^2)\sqrt{(\rho_6^2+X_1)^3}\,(\rho_6-\rho_5)(\rho_6+\rho_5)} \tag{128.15}$$

$$Q_{16}(x,y,t) = \frac{e^{iX_3 t}\left[1-(-1)^{m}e^{-iX_3/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{(X_2-X_3^2)}{(X_1-X_3^2)}}\,b\right)\sqrt{X_4-X_3}}{2iX_3\sqrt{(X_1-X_3^2)^3}\,(iX_3+\rho_6)(iX_3-\rho_6)(iX_3+\rho_5)(iX_3-\rho_5)} + \frac{e^{iX_3 t}\left[1-(-1)^{m}e^{iX_3/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{(X_2-X_3^2)}{(X_1-X_3^2)}}\,b\right)\sqrt{X_4-X_3}}{-2iX_3\sqrt{(X_1-X_3^2)^3}\,(\rho_6-iX_3)(\rho_6+iX_3)(\rho_5-iX_3)(\rho_5+iX_3)} +$$

$$\frac{e^{\rho_6 t}\left[1-(-1)^{m}e^{-\rho_6/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{(\rho_6^2+X_2)}{(\rho_6^2+X_1)}}\,b\right)\sqrt{\rho_6^2-X_4}}{2\rho_6\,(\rho_6^2+X_3^2)\sqrt{(\rho_6^2-X_1)^3}\,(\rho_6-\rho_5)(\rho_6+\rho_5)} + \frac{e^{-\rho_6 t}\left[1-(-1)^{m}e^{\rho_6/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{(\rho_6^2+X_2)}{(\rho_6^2+X_1)}}\,b\right)\sqrt{\rho_6^2-X_4}}{2\rho_6\,(\rho_6^2+X_3^2)\sqrt{(\rho_6^2-X_1)^3}\,(\rho_6+\rho_5)(\rho_5-\rho_6)} +$$

$$\frac{e^{\rho_5 t}\left[1-(-1)^{m}e^{-\rho_5/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{(\rho_5^2+X_2)}{(\rho_5^2+X_1)}}\,b\right)\sqrt{\rho_5^2-X_4}}{2\rho_5\,(\rho_5^2+X_3^2)\sqrt{(\rho_5^2-X_1)^3}\,(\rho_5-\rho_6)(\rho_5+\rho_6)} + \frac{e^{-\rho_5 t}\left[1-(-1)^{m}e^{\rho_5/c}\right]\sin(b-y_0)\cos\left(\sqrt{\frac{(\rho_5^2+X_2)}{(\rho_5^2+X_1)}}\,b\right)\sqrt{\rho_5^2-X_4}}{2\rho_5\,(\rho_5^2+X_3^2)\sqrt{(\rho_5^2-X_1)^3}\,(\rho_6-\rho_5)(\rho_6+\rho_5)} \tag{128.16}$$

$$Q_{17}(x,y,t) = \frac{e^{iX_3 t}\left[1-(-1)^{m}e^{-iX_3/c}\right]\sin y_0\cos^2\left(\sqrt{\frac{(X_2-X_3^2)}{(X_1-X_3^2)}}\,b\right)\sqrt{X_4-X_3}}{2iX_3\sqrt{(X_1-X_3)^3}\,(iX_3-\rho_5)(iX_3+\rho_5)(iX_3+\rho_6)(iX_3-\rho_6)} + \frac{e^{-iX_3 t}\left[1-(-1)^{m}e^{iX_3/c}\right]\sin y_0\cos^2\left(\sqrt{\frac{(X_2-X_3^2)}{(X_1-X_3^2)}}\,b\right)\sqrt{X_4-X_3}}{-2iX_3\sqrt{(X_1-X_3)^2}\,(\rho_5-iX_3)(\rho_5+iX_3)(\rho_6-iX_3)(\rho_6+iX_3)} +$$

$$\frac{e^{\rho_5 t}\left[1-(-1)^{m}e^{-\rho_5/c}\right]\sin y_0\cos^2\left(\sqrt{\frac{(\rho_5^2+X_2)}{(\rho_5^2+X_1)}}\,b\right)\sqrt{\rho_5^2+X_4}}{2\rho_5\,(\rho_5^2+X_3^2)\sqrt{(\rho_5^2+X_1)^3}\,(\rho_5+\rho_6)(\rho_5-\rho_6)} + \frac{e^{-\rho_5 t}\left[1-(-1)^{m}e^{\rho_5/c}\right]\sin y_0\cos^2\left(\sqrt{\frac{(\rho_5^2+X_2)}{(\rho_5^2+X_1)}}\,b\right)\sqrt{\rho_5^2+X_4}}{2\rho_5\,(\rho_5^2+X_3^2)\sqrt{(\rho_5^2+X_1)^3}\,(\rho_6-\rho_5)(\rho_6+\rho_5)} +$$

$$\frac{e^{\rho_6 t}\left[1-(-1)^{m}e^{-\rho_6/c}\right]\sin y_0\cos^2\left(\sqrt{\frac{(\rho_6^2+X_2)}{(\rho_6^2+X_1)}}\,b\right)\sqrt{\rho_6^2+X_4}}{2\rho_6\,(\rho_6^2+X_3^2)\sqrt{(\rho_6^2+X_1)^3}\,(\rho_6+\rho_5)(\rho_6-\rho_5)} + \frac{e^{-\rho_6 t}\left[1-(-1)^{m}e^{\rho_6/c}\right]\sin y_0\cos^2\left(\sqrt{\frac{(\rho_6^2+X_2)}{(\rho_6^2+X_1)}}\,b\right)\sqrt{\rho_6^2+X_4}}{2\rho_6\,(\rho_6^2+X_3^2)\sqrt{(\rho_6^2+X_1)^3}\,(\rho_5-\rho_6)(\rho_5+\rho_6)} \tag{128.17}$$

$$Q_{18}(x,y,t) = \frac{e^{iX_3}\left[1-(-1)^{m}e^{-iX_3/c}\right]\sin y_0\cos\left(\sqrt{\frac{(X_2-X_3^2)}{(X_1-X_3^2)}}\,b\right)\cos\left(\sqrt{\frac{z(\rho_1-X_3^2)}{\alpha_{\circ t}(\rho_2-X_1)}}\,b\right)\sqrt{X_4-X_3}}{2in\pi X_3\sqrt{(X_1-X_3)^3}\,(iX_3+\rho_5)(iX_3-\rho_5)(iX_3+\rho_6)(iX_3-\rho_6)} +$$

$$\frac{e^{-iX_3}\left[1-(-1)^{m}e^{iX_3/c}\right]\sin y_0\cos\left(\sqrt{\frac{(X_2-X_3^2)}{(X_1-X_3^2)}}\,b\right)\cos\left(\sqrt{\frac{z(\rho_1-X_3^2)}{\alpha_{\circ t}(\rho_2-X_3^2)}}\,b\right)\sqrt{X_4-X_3}}{-2in\pi X_3\sqrt{(X_1-X_3)^3}\,(\rho_5-iX_3)(\rho_5+iX_3)(\rho_6-iX_3)(\rho_6+iX_3)} +$$

$$\frac{e^{\rho_5 t}\left[1-(-1)^{m}e^{-\rho_5/c}\right]\sin y_0\cos\left(\sqrt{\frac{(\rho_5^2+X_2)}{(\rho_5^2+X_1)}}\,b\right)\cos\left(\sqrt{\frac{z(\rho_5^2+\rho_1)}{\alpha_{\circ t}(\rho_5^2+\rho_2)}}\,b\right)\sqrt{\rho_5^2+X_4}}{2n\pi\rho_5\sqrt{(\rho_5^2+X_1)^3}\,(\rho_5+\rho_6)(\rho_5-\rho_6)} +$$

$$\frac{e^{-\rho_5 t}\left[1-(-1)^{m}e^{\rho_5/c}\right]\sin y_0\cos\left(\sqrt{\frac{(\rho_5^2+X_2)}{(\rho_5^2+X_1)}}\,b\right)\cos\left(\sqrt{\frac{z(\rho_5^2+\rho_1)}{\alpha_{\circ t}(\rho_5^2+\rho_2)}}\,b\right)\sqrt{\rho_5^2+X_4}}{2n\pi\rho_5\sqrt{(\rho_5^2+X_1)^3}\,(\rho_5+\rho_6)(\rho_5-\rho_6)} +$$

$$\frac{e^{\rho_6 t}\left[1-(-1)^{m}e^{-\rho_6/c}\right]\sin y_0\cos\left(\sqrt{\frac{(\rho_6^2+X_2)}{(\rho_6^2+X_1)}}\,b\right)\cos\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}\,b\right)\sqrt{\rho_6^2+X_4}}{2n\pi\rho_6\sqrt{(\rho_6^2+X_1)^3}\,(\rho_6-\rho_5)(\rho_6+\rho_5)} +$$

$$\frac{e^{-\rho_6 t}\left[1-(-1)^{m}e^{\rho_6/c}\right]\sin y_0\cos\left(\sqrt{\frac{(\rho_6^2+X_2)}{(\rho_6^2+X_1)}}\,b\right)\cos\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}\,b\right)\sqrt{\rho_6^2+X_4}}{2n\pi\rho_6\sqrt{(\rho_6^2+X_1)^3}\,(\rho_5-\rho_6)(\rho_5+\rho_6)} \tag{128.18}$$

$$Q_{19}(x,y,t) = \frac{c^2 n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}\,b\right)(x_4-x_1)}{2ib\sqrt{\chi_1}\,\sqrt{\rho_2-\chi_1}(\chi_8-\chi_1)\,(\chi_9-\chi_1)\,(i\sqrt{\chi_1}+\rho_7)(i\sqrt{\chi_1}-\rho_7)}$$

$$-\frac{c^2 n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}\,b\right)(x_4-x_1)}{2ib\sqrt{\chi_1}\,\sqrt{\rho_2-\chi_1}(\chi_8-\chi_1)\,(\chi_9-\chi_1)\,(\rho_7-i\sqrt{\chi_1})(i\sqrt{\chi_1}+\rho_7)}$$

$$+\frac{c^2 n\pi e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)(x_4-\rho_2)}{b\sqrt{2i\sqrt{\rho_2}}\,\sqrt{\chi_1-\rho_2}(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,(\rho_7+i\sqrt{\rho_2})(i\sqrt{\rho_2}-\rho_7)}$$

$$-\frac{c^2 n\pi e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)(x_4-\rho_2)}{b\sqrt{2i\sqrt{\rho_2}}\,\sqrt{\chi_1-\rho_2}(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,(\rho_7-i\sqrt{\rho_2})(\rho_7+i\sqrt{\rho_2})}$$

$$+\frac{c^2 n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)(x_4-\chi_8)}{2ib\sqrt{\chi_8}\,\sqrt{\rho_2-\chi_8}(\chi_9-\chi_8)\,(\rho_7+i\sqrt{\chi_8})(i\sqrt{\chi_8}-\rho_7)}$$

$$2ib\sqrt{\chi_8}\,\sqrt{\rho_2-\chi_8}(\chi_9-\chi_8)\,(\rho_7+i\sqrt{\chi_8})(i\sqrt{\chi_8}-\rho_7)$$

$$-\frac{c^2 n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)(x_4-\chi_8)}{2ib\sqrt{\chi_8}\,\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,(\rho_7+i\sqrt{\chi_8})(\rho_7-i\sqrt{\chi_8})}$$

$$+\frac{c^2 n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,b\right)(x_4-\chi_9)}{2ib\sqrt{\chi_9}\,\sqrt{\chi_1-\chi_9}(\chi_8-\chi_9)\,(\rho_7+i\sqrt{\chi_9})(i\sqrt{\chi_9}-\rho_7)}$$

$$-\frac{c^2 n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,b\right)(x_4-\chi_9)}{2ib\sqrt{\chi_9}\,\sqrt{\chi_1-\chi_9}(\chi_8-\chi_9)\,(\rho_7+i\sqrt{\chi_9})(\rho_7-i\sqrt{\chi_9})}$$

$$+\frac{c^2 n\pi e^{\rho_7\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1+\rho_7^2)}{(\chi_4+\rho_7^2)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1+\rho_7^2)}{\alpha_{ot}(\rho_2+\rho_7^2)}}\,b\right)(x_4+\rho_7^2)}{2b\rho_7\sqrt{\chi_1+\rho_7^2}(\chi_8+\rho_7^2)\,(\chi_9+\rho_7^2)} - \frac{c^2 n\pi e^{-\rho_7\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1+\rho_7^2)}{(\chi_4+\rho_7^2)}}\,\chi\right)\cos\left(\sqrt{\frac{z(\rho_1+\rho_7^2)}{\alpha_{ot}(\rho_2+\rho_7^2)}}\,b\right)(x_4+\rho_7^2)}{2b\rho_7\sqrt{\chi_1+\rho_7^2}(\chi_8+\rho_7^2)\,(\chi_9+\rho_7^2)} \qquad (128.19)$$

$$Q_{20}(x,y,t) = -\left(\frac{ic\sqrt{\rho_2}\,e^{i\sqrt{\rho_2}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\,x\right)\sqrt{(\mathcal{X}_4-\rho_2)^3}}{\sqrt{2i}\sqrt{\rho_2}\sqrt{(\mathcal{X}_1-\rho_2)^3}\,(\mathcal{X}_8+\rho_2)\,(\mathcal{X}_9-\rho_2)\,(i\sqrt{\rho_2}+\rho_7)\,(i\sqrt{\rho_2}-\rho_7)} +\right.$$

$$\frac{c\sqrt{\rho_2}\,e^{-i\sqrt{\rho_2}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\,x\right)\sqrt{(\mathcal{X}_4-\rho_2)^3}}{\sqrt{2i}\sqrt{\rho_2}\sqrt{(\mathcal{X}_1-\rho_2)^3}\,(\mathcal{X}_8+\rho_2)\,(\mathcal{X}_9-\rho_2)\,(\rho_7-i\sqrt{\rho_2})\,(\rho_7+i\sqrt{\rho_2})} +$$

$$\frac{c\sqrt{\mathcal{X}_8}\,e^{i\sqrt{\mathcal{X}_8}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_8)}{\alpha_{ot}(\rho_2-\mathcal{X}_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\,x\right)\sqrt{(\mathcal{X}_4-\mathcal{X}_8)^3}}{2\sqrt{\mathcal{X}_8}\sqrt{(\rho_2-\mathcal{X}_8)}\sqrt{(\mathcal{X}_1-\mathcal{X}_8)^3}\,(\mathcal{X}_9-\mathcal{X}_8)\,(\mathcal{X}_9-\rho_2)\,(i\sqrt{\mathcal{X}_8}+\rho_7)\,(i\sqrt{\mathcal{X}_8}-\rho_7)} +$$

$$\frac{-c\sqrt{\mathcal{X}_8}\,e^{-i\sqrt{\mathcal{X}_8}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_8)}{\alpha_{ot}(\rho_2-\mathcal{X}_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\,x\right)\sqrt{(\mathcal{X}_4-\mathcal{X}_8)^3}}{2\sqrt{\mathcal{X}_8}\sqrt{(\rho_2-\mathcal{X}_8)}\sqrt{(\mathcal{X}_1-\mathcal{X}_8)^3}\,(\mathcal{X}_9-\mathcal{X}_8)\,(\rho_7-i\sqrt{\mathcal{X}_8})\,(\rho_7+i\sqrt{\mathcal{X}_8})} +$$

$$\frac{c\sqrt{\mathcal{X}_9}\,e^{i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_9)}{\alpha_{ot}(\rho_2-\mathcal{X}_9)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\,x\right)\sqrt{(\mathcal{X}_4-\mathcal{X}_9)^3}}{2\sqrt{\mathcal{X}_9}\sqrt{(\rho_2-\mathcal{X}_9)}\sqrt{(\mathcal{X}_1-\mathcal{X}_9)^3}\,(\mathcal{X}_8-\mathcal{X}_9)\,(i\sqrt{\mathcal{X}_9}+\rho_7)\,(i\sqrt{\mathcal{X}_9}+\rho_7)} +$$

$$\frac{-c\sqrt{\mathcal{X}_9}\,e^{i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_9)}{\alpha_{ot}(\rho_2-\mathcal{X}_9)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\,x\right)\sqrt{(\mathcal{X}_4-\mathcal{X}_9)^3}}{2\sqrt{\mathcal{X}_9}\sqrt{(\rho_2-\mathcal{X}_9)}\sqrt{(\mathcal{X}_1-\mathcal{X}_9)^3}\,(\mathcal{X}_8-\mathcal{X}_9)\,(\rho_7-i\sqrt{\mathcal{X}_9})\,(\rho_7+i\sqrt{\mathcal{X}_9})} +$$

$$\frac{c\rho_7\,e^{\rho_7(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_7^2+\mathcal{X}_1}{\rho_7^2+\mathcal{X}_4}}\right)\cos\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_7^2+\mathcal{X}_1}{\rho_7^2+\mathcal{X}_4}}\,x\right)\sqrt{(\rho_7^2+\mathcal{X}_4)^3}}{2\rho_7\sqrt{(\rho_7^2+\rho_2)}\sqrt{(\rho_7^2+\mathcal{X}_1)^3}\,(\rho_7^2+\mathcal{X}_8)\,(\rho_7^2-\mathcal{X}_9)} +$$

$$\left.\frac{c\rho_7\,e^{-\rho_7(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_7^2+\mathcal{X}_1}{\rho_7^2+\mathcal{X}_4}}\right)\cos\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_7^2+\mathcal{X}_1}{\rho_7^2+\mathcal{X}_4}}\,x\right)\sqrt{(\rho_7^2+\mathcal{X}_4)^3}}{2\rho_7\sqrt{(\rho_7^2+\rho_2)}\sqrt{(\rho_7^2+\mathcal{X}_1)^3}\,(\rho_7^2+\mathcal{X}_8)\,(\rho_7^2-\mathcal{X}_9)}\right) \qquad (128.20)$$

$$Q_{21}(x,y,t) = \frac{c^2 n\pi\, e^{i\sqrt{\rho_2}t}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{\circ t}(\rho_2-\rho_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\,x\right)(\chi_4-\rho_2)}{\sqrt{2i}\sqrt{\rho_2}\,(\chi_1-\rho_2)(\chi_8+\rho_2)(\chi_9-\rho_2)(i\sqrt{\rho_2}+\rho_7)(i\sqrt{\rho_2}-\rho_7)} + \frac{c^2 n\pi\, e^{-i\sqrt{\rho_2}t}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{\circ t}(\rho_2-\rho_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\,x\right)(\chi_4-\rho_2)}{i\sqrt{2i}\sqrt{\rho_2}\,(\chi_1-\rho_2)(\chi_8+\rho_2)(\chi_9-\rho_2)(\rho_7-i\sqrt{\rho_2})(\rho_7+i\sqrt{\rho_2})} +$$

$$\frac{c^2 n\pi\, e^{i\sqrt{\chi_1}t}\cos\left(\sqrt{\frac{z(\rho_1-\chi_2)}{\alpha_{\circ t}(\rho_2-\chi_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_1}{\chi_4-\chi_1}}\,x\right)(\chi_4-\chi_1)}{2i\sqrt{\chi_1}(\rho_2-\chi_1)(\chi_8+\chi_1)(\chi_9-\chi_1)(i\sqrt{\chi_1}+\rho_7)(i\sqrt{\chi_1}-\rho_7)} + \frac{c^2 n\pi\, e^{-i\sqrt{\chi_1}t}\cos\left(\sqrt{\frac{z(\rho_1-\chi_2)}{\alpha_{\circ t}(\rho_2-\chi_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_1}{\chi_4-\chi_1}}\,x\right)(\chi_4-\chi_1)}{2i\sqrt{\chi_1}(\rho_2-\chi_1)(\chi_8+\chi_1)(\chi_9-\chi_1)(\rho_7-i\sqrt{\chi_1})(\rho_7+i\sqrt{\chi_1})} +$$

$$\frac{c^2 n\pi\, e^{i\sqrt{\chi_8}t}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{\circ t}(\rho_2-\chi_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\,x\right)(\chi_4-\chi_8)}{2i\sqrt{\chi_8}\sqrt{(\rho_2-\chi_8)}(\chi_9+\chi_8)(\chi_1-\chi_8)(i\sqrt{\chi_8}+\rho_7)(i\sqrt{\chi_8}-\rho_7)} + \frac{c^2 n\pi\, e^{-i\sqrt{\chi_8}t}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{\circ t}(\rho_2-\chi_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\,x\right)(\chi_4-\chi_8)}{2i\sqrt{\chi_8}\sqrt{(\rho_2-\chi_8)}(\chi_9+\chi_8)(\chi_1-\chi_8)(\rho_7-i\sqrt{\chi_1})(\rho_7+i\sqrt{\chi_1})} +$$

$$\frac{c^2 n\pi\, e^{i\sqrt{\chi_9}t}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{\circ t}(\rho_2-\chi_9)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\,x\right)(\chi_4-\chi_9)}{2i\sqrt{\chi_9}\sqrt{(\rho_2-\chi_9)}(\chi_8+\chi_9)(\chi_1-\chi_9)(i\sqrt{\chi_9}+\rho_7)(i\sqrt{\chi_9}-\rho_7)} + \frac{c^2 n\pi\, e^{-i\sqrt{\chi_9}t}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{\circ t}(\rho_2-\chi_9)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\,x\right)(\chi_4-\chi_9)}{2i\sqrt{\chi_9}\sqrt{(\rho_2-\chi_9)}(\chi_8+\chi_9)(\chi_1-\chi_9)(\rho_7-i\sqrt{\chi_9})(\rho_7+i\sqrt{\chi_9})} +$$

$$\frac{c^2 n\pi\, e^{\rho_7 t}\cos\left(\sqrt{\frac{z(\rho_7^2+\chi_1)}{\alpha_{\circ t}(\rho_7^2+\chi_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_7^2+\chi_1}{\rho_7^2+\chi_4}}\,x\right)(\rho_7^2+\chi_4)}{2\rho_7\sqrt{(\rho_7+\rho_2)}(\rho_7^2+\chi_1)(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} + \frac{c^2 n\pi\, e^{-\rho_7 t}\cos\left(\sqrt{\frac{z(\rho_7^2+\chi_1)}{\alpha_{\circ t}(\rho_7^2+\chi_2)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_7^2+\chi_1}{\rho_7^2+\chi_4}}\,x\right)(\rho_7^2+\chi_4)}{2\rho_7\sqrt{(\rho_7+\rho_2)}(\rho_7^2+\chi_1)(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)}. \qquad (128.21)$$

$$Q_{22}(x,y,t) = \frac{c^2 n\pi e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)}{b\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)(\chi_9-\rho_2)(\chi_1-\rho_2)(i\sqrt{\rho_2}-\rho_6)(i\sqrt{\rho_2}+\rho_6)} -$$

$$\frac{c^2 n\pi e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)}{b\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)(\chi_9-\rho_2)(\chi_1-\rho_2)(i\sqrt{\rho_2}+\rho_6)(\rho_6-i\sqrt{\rho_2})} +$$

$$\frac{c^2 n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)}{b\sqrt{2i\sqrt{\chi_8}}\,(\rho_2-\chi_8)(\chi_9-\chi_8)(\chi_1-\chi_8)(i\sqrt{\chi_8}-\rho_6)(i\sqrt{\chi_8}+\rho_6)} -$$

$$\frac{c^2 n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)}{b\sqrt{2i\sqrt{\chi_8}}\,(\rho_2-\chi_8)(\chi_9-\chi_8)(\chi_1-\chi_8)(i\sqrt{\chi_8}+\rho_6)(\rho_6-i\sqrt{\chi_8})} +$$

$$\frac{c^2 n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)}{b\sqrt{2i\sqrt{\chi_9}}\,(\rho_2-\chi_9)(\chi_8-\chi_9)(\chi_1-\chi_9)(i\sqrt{\chi_9}-\rho_6)(i\sqrt{\chi_9}+\rho_6)} -$$

$$\frac{c^2 n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)}{b\sqrt{2i\sqrt{\chi_9}}\,(\rho_2-\chi_9)(\chi_8-\chi_9)(\chi_1-\chi_9)(\rho_6-i\sqrt{\chi_9})(\rho_6+i\sqrt{\chi_9})} +$$

$$\frac{c^2 n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)}{b\sqrt{2i\sqrt{\chi_1}}\,(\rho_2-\chi_1)(\chi_8-\chi_1)(\chi_9-\chi_1)(\rho_6+i\sqrt{\chi_1})(i\sqrt{\chi_1}-\rho_6)} -$$

$$\frac{c^2 n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)}{ib\sqrt{2i\sqrt{\chi_1}}\,(\rho_2-\chi_1)(\chi_8-\chi_1)(\chi_9-\chi_1)(\rho_6+i\sqrt{\chi_1})(\rho_6-i\sqrt{\chi_1})} +$$

$$\frac{c^2 n\pi e^{\rho_6\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1+\rho_6^2)}{\alpha_{ot}(\rho_2+\rho_6^2)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1+\rho_6^2)}{(\chi_4+\rho_6^2)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1+\rho_6^2)}{(\chi_4+\rho_6^2)}}\right)}{2b\rho_6\sqrt{(\rho_6^2+\rho_2)}\,(\rho_6^2+\chi_8)(\rho_6^2+\chi_9)(\rho_6^2+\chi_1)} -$$

$$\frac{c^2 n\pi e^{-\rho_6\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z(\rho_1+\rho_6^2)}{\alpha_{ot}(\rho_2+\rho_6^2)}}b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1+\rho_6^2)}{(\chi_4+\rho_6^2)}}\chi\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1+\rho_6^2)}{(\chi_4+\rho_6^2)}}\right)}{2b\rho_6\sqrt{(\rho_6^2+\rho_2)}\,(\rho_6^2+\chi_8)(\rho_6^2+\chi_9)(\rho_6^2+\chi_1)} \qquad (128.22)$$

$$Q_{23}(x,y,t) = \frac{c\sqrt{\chi_8}\, e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)\right]\sqrt{(\chi_4-\chi_8)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{\circ t}(\rho_2-\chi_8)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\,x\right)}{2\sqrt{\chi_8}(\chi_9-\chi_8)\sqrt{\rho_2-\chi_8}\sqrt{(\chi_1-\chi_8)^3}\,(\rho_6+i\sqrt{\chi_8})(i\sqrt{\chi_8}-\rho_6)} -$$

$$\frac{c\sqrt{\chi_8}\, e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)\right]\sqrt{(\chi_4-\chi_8)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{\circ t}(\rho_2-\chi_8)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\,x\right)}{2\sqrt{\chi_8}(\chi_9-\chi_8)\sqrt{\rho_2-\chi_8}\sqrt{(\chi_1-\chi_8)^3}\,(\rho_6-i\sqrt{\chi_8})(\rho_6+i\sqrt{\chi_8})} +$$

$$\frac{c\sqrt{\chi_9}\, e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)\right]\sqrt{(\chi_4-\chi_9)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{\circ t}(\rho_2-\chi_9)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\,x\right)}{2\sqrt{\chi_9}(\chi_8-\chi_9)\sqrt{\rho_2-\chi_9}\sqrt{(\chi_1-\chi_9)^3}\,(i\sqrt{\chi_9}+\rho_6)(i\sqrt{\chi_9}-\rho_6)} -$$

$$\frac{c\sqrt{\chi_9}\, e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)\right]\sqrt{(\chi_4-\chi_9)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{\circ t}(\rho_2-\chi_9)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\,x\right)}{2\sqrt{\chi_9}(\chi_8-\chi_9)\sqrt{\rho_2-\chi_9}\sqrt{(\chi_1-\chi_9)^3}(\rho_6-i\sqrt{\chi_9})(\rho_6-i\sqrt{\chi_9})} +$$

$$\frac{c\sqrt{\rho_2}\, e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)\right]\sqrt{(\chi_4-\rho_2)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{\circ t}(\rho_2-\rho_2)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\,x\right)}{2\sqrt{\rho_2}(\chi_8-\rho_2)\sqrt{(\chi_1-\rho_2)^3}(i\sqrt{\rho_2}+\rho_6)(i\sqrt{\rho_2}-\rho_6)} +$$

$$-\frac{c\sqrt{\rho_2}\, e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)\right]\sqrt{(\chi_4-\rho_2)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{\circ t}(\rho_2-\rho_2)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\,x\right)}{2\sqrt{\rho_2}(\chi_8-\rho_2)\sqrt{(\chi_1-\rho_2)^3}(\rho_6-i\sqrt{\rho_2})(\rho_6-i\sqrt{\rho_2})} +$$

$$\frac{c\rho_6\, e^{\rho_6\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\right)\right]\sqrt{(\rho_6^2+\chi_4)}\cos\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\,x\right)}{2\rho_6(\rho_6^2+\chi_8)(\rho_6^2+\chi_9)\sqrt{(\rho_6^2+\rho_2)}\sqrt{(\rho_6^2+\chi_1)^3}} \pm$$

$$\frac{c\rho_6\, e^{-\rho_6\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\right)\right]\sqrt{(\rho_6^2+\chi_4)}\cos\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}\,b\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\,x\right)}{2\rho_6(\rho_6^2+\chi_8)(\rho_6^2+\chi_9)\sqrt{(\rho_6^2+\rho_2)}\sqrt{(\rho_6^2+\chi_1)^3}} \qquad (128.23)$$

$$Q_{24}(x,y,t) = \frac{e^{i\sqrt{\chi_8}t}\sqrt{(\chi_4-\chi_8)}}{2i\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)^3}(\chi_9-\chi_8)\sqrt{(\rho_2-\chi_8)}} - \frac{e^{-i\sqrt{\chi_8}t}\sqrt{(\chi_4-\chi_8)}}{2i\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)^3}(\chi_9-\chi_8)\sqrt{(\rho_2-\chi_8)}} + \frac{e^{i\sqrt{\chi_9}t}\sqrt{(\chi_4-\chi_9)}}{2i\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)^3}(\chi_8-\chi_9)\sqrt{(\rho_2-\chi_9)}} - \frac{e^{-i\sqrt{\chi_9}t}\sqrt{(\chi_4-\chi_9)}}{2i\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)^3}(\chi_8-\chi_9)\sqrt{(\rho_2-\chi_9)}} + \frac{e^{i\sqrt{\rho_2}t}\sqrt{(\chi_4-\rho_2)}}{2i\sqrt{\rho_2}\sqrt{(\chi_1-\rho_2)^3}(\chi_8-\rho_2)\sqrt{(\chi_9-\rho_2)}} - \frac{e^{-i\sqrt{\rho_2}t}\sqrt{(\chi_4-\rho_2)}}{2i\sqrt{\rho_2}\sqrt{(\chi_1-\rho_2)^3}(\chi_8-\rho_2)\sqrt{(\chi_9-\rho_2)}} \quad (128.24)$$

$$Q_{25}(x,y,t) = \frac{n\pi c^2 e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)}{2ib\sqrt{\chi_8}(\chi_9-\chi_8)(\chi_1-\chi_8)\sqrt{(\chi_4-\chi_8)}(\rho_2-\chi_8)} - \frac{n\pi c^2 e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)}{2ib\sqrt{\chi_8}(\chi_9-\chi_8)(\chi_1-\chi_8)\sqrt{(\chi_4-\chi_8)}(\rho_2-\chi_8)} + \frac{n\pi c^2 e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)}{2ib\sqrt{\chi_9}(\chi_8-\chi_9)(\chi_1-\chi_9)\sqrt{(\chi_4-\chi_9)}(\rho_2-\chi_9)} + - \frac{n\pi c^2 e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)}{2ib\sqrt{\chi_9}(\chi_8-\chi_9)(\chi_1-\chi_9)\sqrt{(\chi_4-\chi_9)}(\rho_2-\chi_9)} + \frac{n\pi c^2 e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_1}{\chi_4-\chi_1}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_1}{\chi_4-\chi_1}}\right)}{2ib\sqrt{\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)\sqrt{(\chi_4-\chi_1)}(\rho_2-\chi_1)} - \frac{n\pi c^2 e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_1}{\chi_4-\chi_1}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_1}{\chi_4-\chi_1}}\right)}{2ib\sqrt{\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)\sqrt{(\chi_4-\chi_1)}(\rho_2-\chi_1)} +$$

$$\frac{n\pi c^2 e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)}{b\sqrt{2i\sqrt{\rho_2}}(\chi_8-\rho_2)(\chi_9-\rho_2)(\chi_1-\rho_2)\sqrt{(\chi_4-\rho_2)}} + \frac{n\pi c^2 e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)}{ib\sqrt{2i\sqrt{\rho_2}}(\chi_8-\rho_2)(\chi_9-\rho_2)(\chi_1-\rho_2)\sqrt{(\chi_4-\rho_2)}} + \frac{n\pi c^2 e^{i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_4}{\chi_4-\chi_4}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_4}{\chi_4-\chi_4}}\right)}{b\sqrt{2i\sqrt{\chi_4}}(\chi_8-\chi_4)(\chi_9-\chi_4)(\chi_1-\chi_4)\sqrt{(\rho_2-\chi_4)}} + \frac{n\pi c^2 e^{-i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_4}{\chi_4-\chi_4}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_4}{\chi_4-\chi_4}}\right)}{ib\sqrt{2i\sqrt{\chi_4}}(\chi_8-\chi_4)(\chi_9-\chi_4)(\chi_1-\chi_4)\sqrt{(\rho_2-\chi_4)}} \quad (128.25)$$

$$Q_{26}(x,y,t) = -\left(\frac{c\sqrt{\chi_8}\, e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)}{2\sqrt{\chi_8}(\chi_9-\chi_8)\sqrt{\rho_2-\chi_8}\sqrt{(\chi_1-\chi_8)^3}\,(i\sqrt{\chi_8}+\rho_6)(i\sqrt{\chi_8}-\rho_6)} - \frac{c\sqrt{\chi_8}\, e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_8}{\chi_4-\chi_8}}\right)}{2\sqrt{\chi_8}(\chi_9-\chi_8)\sqrt{\rho_2-\chi_8}\sqrt{(\chi_1-\chi_8)^3}\,(\rho_6-i\sqrt{\chi_8})(\rho_6+i\sqrt{\chi_8})} + \frac{c\sqrt{\chi_9}\, e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)}{2\sqrt{\chi_9}(\chi_8-\chi_9)\sqrt{\rho_2-\chi_9}\sqrt{(\chi_1-\chi_9)^3}\,(i\sqrt{\chi_9}+\rho_6)(i\sqrt{\chi_9}-\rho_6)} + - \frac{c\sqrt{\chi_9}\, e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\chi_9}{\chi_4-\chi_9}}\right)}{2\sqrt{\chi_9}(\chi_8-\chi_9)\sqrt{\rho_2-\chi_9}\sqrt{(\chi_1-\chi_9)^3}\,(\rho_6-i\sqrt{\chi_9})(\rho_6+i\sqrt{\chi_9})} + \frac{ic\sqrt{\rho_2}\, e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)}{\sqrt{2i\sqrt{\rho_2}}(\chi_8-\rho_2)(\chi_9-\rho_2)\sqrt{(\chi_1-\rho_2)^3}\,(i\sqrt{\rho_2}+\rho_6)(i\sqrt{\rho_2}-\rho_6)} + \frac{c\sqrt{\rho_2}\, e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\chi_1-\rho_2}{\chi_4-\rho_2}}\right)}{\sqrt{2i\sqrt{\rho_2}}(\chi_8-\rho_2)(\chi_9-\rho_2)\sqrt{(\chi_1-\rho_2)^3}\,(\rho_6+i\sqrt{\rho_2})(\rho_6-i\sqrt{\rho_2})} + \frac{c\rho_6\, e^{\rho_6\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\right)}{2\rho_6(\rho_6^2+\chi_8)(\rho_6^2+\chi_9)\sqrt{(\rho_6^2+\rho_2)(\rho_6^2+\chi_1)^3}} + \frac{-c\rho_6\, e^{-\rho_6\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\right)\right]\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\chi_1}{\rho_6^2+\chi_4}}\right)}{-2\rho_6(\rho_6^2+\chi_8)(\rho_6^2+\chi_9)\sqrt{(\rho_6^2+\rho_2)(\rho_6^2+\chi_1)^3}} \right) \quad (128.26)$$

$$Q_{27}(x,y,t) = \frac{e^{i\sqrt{\chi_1}t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\cos(b-y_o)}{\sqrt{2i\sqrt{\chi_1}}(\chi_3^2-\chi_1)\sqrt{(\chi_2-\chi_1)(\rho_2-\chi_1)}(\rho_6+i\sqrt{\chi_1})(i\sqrt{\chi_1}-\rho_6)} - \frac{e^{-i\sqrt{\chi_1}t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\cos(b-y_o)}{i\sqrt{2i\sqrt{\chi_1}}(\chi_3^2-\chi_1)\sqrt{(\chi_2-\chi_1)(\rho_2-\chi_1)}(\rho_6-i\sqrt{\chi_1})(\rho_6+i\sqrt{\chi_1})} + \frac{e^{i\chi_3 t}\left[1-(-1)^m e^{-i\chi_3/c}\right]\cos(b-y_o)}{2i\chi_3\sqrt{(\chi_1-\chi_3^2)(\chi_2-\chi_3^2)(\rho_2-\chi_3^2)}(\rho_6+i\chi_3)(i\chi_3-\rho_6)} - \frac{e^{-i\chi_3 t}\left[1-(-1)^m e^{i\chi_3/c}\right]\cos(b-y_o)}{2i\chi_3\sqrt{(\chi_1-\chi_3^2)(\chi_2-\chi_3^2)(\rho_2-\chi_3^2)}(\rho_6-i\chi_3)(\rho_6+i\chi_3)} + \frac{e^{i\sqrt{\chi_2}t}\left[1-(-1)^m e^{-i\sqrt{\chi_2}/c}\right]\cos(b-y_o)}{\sqrt{2i\sqrt{\chi_2}}\sqrt{(\chi_1-\chi_2)(\rho_2-\chi_2)}(\chi_3^2-\chi_2)(\rho_6+i\sqrt{\chi_2})(i\sqrt{\chi_2}-\rho_6)} - \frac{e^{-i\sqrt{\chi_2}t}\left[1-(-1)^m e^{i\sqrt{\chi_2}/c}\right]\cos(b-y_o)}{i\sqrt{2i\sqrt{\chi_2}}\sqrt{(\chi_1-\chi_2)(\rho_2-\chi_2)}(\chi_3^2-\chi_2)(\rho_6-i\sqrt{\chi_2})(\rho_6+i\sqrt{\chi_2})} + \frac{e^{i\sqrt{\rho_2}t}\left[1-(-1)^m e^{-i\sqrt{\rho_2}/c}\right]\cos(b-y_o)}{\sqrt{2i\sqrt{\rho_2}}\sqrt{(\chi_1-\rho_2)(\chi_2-\rho_2)}(\chi_3^2-\rho_2)(\rho_6+i\sqrt{\rho_2})(i\sqrt{\rho_2}-\rho_6)} - \frac{e^{-i\sqrt{\rho_2}t}\left[1-(-1)^m e^{i\sqrt{\rho_2}/c}\right]\cos(b-y_o)}{i\sqrt{2i\sqrt{\rho_2}}\sqrt{(\chi_1-\rho_2)(\chi_2-\rho_2)}(\chi_3^2-\rho_2)(\rho_6-i\sqrt{\rho_2})(\rho_6+i\sqrt{\rho_2})} + \frac{e^{\rho_6 t}\left[1-(-1)^m e^{\rho_6/c}\right]\cos(b-y_o)}{2\rho_6\sqrt{(\rho_6^2+\chi_1)(\rho_6^2+\chi_2)(\rho_6^2+\rho_2)}(\rho_6^2+\chi_3^2)} + \frac{e^{-\rho_6 t}\left[1-(-1)^m e^{\rho_6/c}\right]\cos(b-y_o)}{-2\rho_6\sqrt{(\rho_6^2+\chi_1)(\rho_6^2+\chi_2)(\rho_6^2+\rho_2)}(\rho_6^2+\chi_3^2)} \quad (128.27)$$

$$Q_{28}(x,y,t) = \frac{e^{i\sqrt{\chi_1}t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\cos(b-y_o)}{\sqrt{2i\sqrt{\chi_1}}(\chi_3^2-\chi_1)\sqrt{(\chi_2-\chi_1)(\rho_2-\chi_1)}(\rho_6+i\sqrt{\chi_1})(i\sqrt{\chi_1}-\rho_6)} - \frac{e^{-i\sqrt{\chi_1}t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\cos(b-y_o)}{i\sqrt{2i\sqrt{\chi_1}}(\chi_3^2-\chi_1)\sqrt{(\chi_2-\chi_1)(\rho_2-\chi_1)}(\rho_6-i\sqrt{\chi_1})(\rho_6+i\sqrt{\chi_1})} + \frac{e^{i\chi_3 t}\left[1-(-1)^m e^{-i\chi_3/c}\right]\cos(b-y_o)}{2i\chi_3\sqrt{(\chi_1-\chi_3^2)(\chi_2-\chi_3^2)(\rho_2-\chi_3^2)}(\rho_6+i\chi_3)(i\chi_3-\rho_6)} - \frac{e^{-i\chi_3 t}\left[1-(-1)^m e^{i\chi_3/c}\right]\cos(b-y_o)}{2i\chi_3\sqrt{(\chi_1-\chi_3^2)(\chi_2-\chi_3^2)(\rho_2-\chi_3^2)}(\rho_6-i\chi_3)(\rho_6+i\chi_3)} +$$

$$\frac{e^{i\sqrt{\chi_2}t}\left[1-(-1)^m e^{-i\sqrt{\chi_2}/c}\right]\cos(b-y_o)}{\sqrt{2i\sqrt{\chi_2}}\sqrt{(\chi_1-\chi_2)(\rho_2-\chi_2)}(\chi_3^2-\chi_2)(\rho_6+i\sqrt{\chi_2})(i\sqrt{\chi_2}-\rho_6)}-\frac{e^{-i\sqrt{\chi_2}t}\left[1-(-1)^m e^{i\sqrt{\chi_2}/c}\right]\cos(b-y_o)}{i\sqrt{2i\sqrt{\chi_2}}\sqrt{(\chi_1-\chi_2)(\rho_2-\chi_2)}(\chi_3^2-\chi_2)(\rho_6-i\sqrt{\chi_2})(\rho_6+i\sqrt{\chi_2})}+$$
$$\frac{e^{i\sqrt{\rho_2}t}\left[1-(-1)^m e^{-i\sqrt{\rho_2}/c}\right]\cos(b-y_o)}{\sqrt{2i\sqrt{\rho_2}}\sqrt{(\chi_1-\rho_2)(\chi_2-\rho_2)}(\chi_3^2-\rho_2)(\rho_6+i\sqrt{\rho_2})(i\sqrt{\rho_2}-\rho_6)}-\frac{e^{-i\sqrt{\rho_2}t}\left[1-(-1)^m e^{i\sqrt{\rho_2}/c}\right]\cos(b-y_o)}{i\sqrt{2i\sqrt{\rho_2}}\sqrt{(\chi_1-\rho_2)(\chi_2-\rho_2)}(\chi_3^2-\rho_2)(\rho_6-i\sqrt{\rho_2})(\rho_6+i\sqrt{\rho_2})}+$$
$$\frac{e^{\rho_6 t}\left[1-(-1)^m e^{\rho_6/c}\right]\cos(b-y_o)}{2\rho_6\sqrt{(\rho_6^2+\chi_1)(\rho_6^2+\chi_2)(\rho_6^2+\rho_2)}\,(\rho_6^2+\chi_3^2)}+\frac{e^{-\rho_6 t}\left[1-(-1)^m e^{\rho_6/c}\right]\cos(b-y_o)}{-2\rho_6\sqrt{(\rho_6^2+\chi_1)(\rho_6^2+\chi_2)(\rho_6^2+\rho_2)}\,(\rho_6^2+\chi_3^2)} \qquad (128.28)$$

$$Q_{29}(x,y,t) = \frac{c\sqrt{\mathcal{X}_8}\,e^{i\sqrt{\mathcal{X}_8}(t-\frac{1}{c})}\sqrt{(\mathcal{X}_4-\mathcal{X}_8)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_8)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_8)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right)\right]}{2\sqrt{\mathcal{X}_8}(\mathcal{X}_9-\mathcal{X}_8)\sqrt{(\mathcal{X}_1-\mathcal{X}_8)^3}(i\sqrt{\mathcal{X}_8}+\rho_6)(i\sqrt{\mathcal{X}_8}-\rho_6)}+$$
$$-\frac{c\sqrt{\mathcal{X}_8}\,e^{-i\sqrt{\mathcal{X}_8}(t-\frac{1}{c})}\sqrt{(\mathcal{X}_4-\mathcal{X}_8)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_8)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_8)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right)\right]}{2\sqrt{\mathcal{X}_8}(\mathcal{X}_9-\mathcal{X}_8)\sqrt{(\mathcal{X}_1-\mathcal{X}_8)^3}(\rho_6+i\sqrt{\mathcal{X}_8})(\rho_6-i\sqrt{\mathcal{X}_8})}+$$
$$\frac{c\sqrt{\mathcal{X}_9}\,e^{i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sqrt{(\mathcal{X}_4-\mathcal{X}_9)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_9)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_9)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right)\right]}{2\sqrt{\mathcal{X}_9}(\mathcal{X}_8-\mathcal{X}_9)\sqrt{(\mathcal{X}_1-\mathcal{X}_9)^3}(i\sqrt{\mathcal{X}_9}+\rho_6)(i\sqrt{\mathcal{X}_9}-\rho_6)}+$$
$$\frac{c\rho_6\,e^{\rho_6(t-\frac{1}{c})}\sqrt{(\rho_6^2+\mathcal{X}_4)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}x\right)\sin\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\right)\right]}{2\rho_6(\rho_6^2+\mathcal{X}_8)(\rho_6^2+\mathcal{X}_9)\sqrt{(\rho_6^2+\mathcal{X}_1)^3}}+$$
$$\frac{c\rho_6\,e^{-\rho_6(t-\frac{1}{c})}\sqrt{(\rho_6^2+\mathcal{X}_4)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}x\right)\sin\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\right)\right]}{2\rho_6(\rho_6^2+\mathcal{X}_8)(\rho_6^2+\mathcal{X}_9)\sqrt{(\rho_6^2+\mathcal{X}_1)^3}}+$$
$$-\frac{c\sqrt{\mathcal{X}_9}\,e^{-i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sqrt{(\mathcal{X}_4-\mathcal{X}_9)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_9)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_9)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right)\right]}{2\sqrt{\mathcal{X}_9}(\mathcal{X}_8-\mathcal{X}_9)\sqrt{(\mathcal{X}_1-\mathcal{X}_9)^3}(\rho_6-i\sqrt{\mathcal{X}_9})(\rho_6+i\sqrt{\mathcal{X}_9})} \qquad (128.29)$$

$$Q_{30}(x,y,t) = -\left(\frac{n\pi c^2 e^{i\sqrt{\mathcal{X}_1}(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_1}{\mathcal{X}_4-\mathcal{X}_1}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_1)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_1)}}b\right)}{2ib\sqrt{\mathcal{X}_1}(\mathcal{X}_8-\mathcal{X}_1)(\mathcal{X}_9-\mathcal{X}_1)(i\sqrt{\mathcal{X}_1}+\rho_6)(i\sqrt{\mathcal{X}_1}-\rho_6)}+\frac{n\pi c^2 e^{-i\sqrt{\mathcal{X}_1}(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_1}{\mathcal{X}_4-\mathcal{X}_1}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_1)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_1)}}b\right)}{2ib\sqrt{\mathcal{X}_1}(\mathcal{X}_8-\mathcal{X}_1)(\mathcal{X}_9-\mathcal{X}_1)(\rho_6-i\sqrt{\mathcal{X}_1})(\rho_6+i\sqrt{\mathcal{X}_1})}+\right.$$
$$\frac{n\pi c^2 e^{i\sqrt{\mathcal{X}_8}(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_8)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_8)}}b\right)}{2ib\sqrt{\mathcal{X}_8}(\mathcal{X}_1-\mathcal{X}_8)(\mathcal{X}_9-\mathcal{X}_8)(i\sqrt{\mathcal{X}_8}+\rho_6)(i\sqrt{\mathcal{X}_8}-\rho_6)}+\frac{n\pi c^2 e^{-i\sqrt{\mathcal{X}_8}(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_8)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_8)}}b\right)}{2ib\sqrt{\mathcal{X}_8}(\mathcal{X}_1-\mathcal{X}_8)(\mathcal{X}_9-\mathcal{X}_8)(\rho_6-i\sqrt{\mathcal{X}_8})(\rho_6+i\sqrt{\mathcal{X}_8})}+$$
$$\frac{n\pi c^2 e^{i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_9)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_9)}}b\right)}{2ib\sqrt{\mathcal{X}_9}(\mathcal{X}_1-\mathcal{X}_9)(\mathcal{X}_8-\mathcal{X}_9)(i\sqrt{\mathcal{X}_9}-\rho_6)(i\sqrt{\mathcal{X}_9}+\rho_6)}+\frac{n\pi c^2 e^{-i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}x\right)\sin\left(\sqrt{\frac{z(\rho_1-\mathcal{X}_9)}{\alpha_{\circ t}(\rho_2-\mathcal{X}_9)}}b\right)}{2ib\sqrt{\mathcal{X}_9}(\mathcal{X}_1-\mathcal{X}_9)(\mathcal{X}_8-\mathcal{X}_9)(\rho_6-i\sqrt{\mathcal{X}_9})(\rho_6+i\sqrt{\mathcal{X}_9})}+$$
$$\left.\frac{n\pi c^2 e^{\rho_6(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}x\right)\sin\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}b\right)}{2b\,\rho_6(\rho_6^2+\mathcal{X}_1)(\rho_6^2+\mathcal{X}_8)(\rho_6^2+\mathcal{X}_9)}-\frac{n\pi c^2 e^{-\rho_6(t-\frac{1}{c})}\sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}x\right)\sin\left(\sqrt{\frac{z(\rho_6^2+\rho_1)}{\alpha_{\circ t}(\rho_6^2+\rho_2)}}b\right)}{2b\,\rho_6(\rho_6^2+\mathcal{X}_1)(\rho_6^2+\mathcal{X}_8)(\rho_6^2+\mathcal{X}_9)}\right) \qquad (128.30)$$

$$Q_{31}(x,y,t) = \frac{(-1)^n cb^2 e^{i\sqrt{\psi_8}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\psi_1-\psi_8}{\psi_4-\psi}}x\right)\sqrt{(\psi_4-\psi_8)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right)\right]}{2(\mathcal{X}_9-\mathcal{X}_8)(\rho_2-\mathcal{X}_8)\sqrt{(\mathcal{X}_1-\mathcal{X}_8)^3}(i\sqrt{\mathcal{X}_8}-\rho_6)(i\sqrt{\mathcal{X}_8}+\rho_6)}+$$
$$\frac{(-1)^n cb^2 e^{-i\sqrt{\mathcal{X}_8}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}x\right)\sqrt{(\mathcal{X}_4-\mathcal{X}_8)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right)\right]}{2(\mathcal{X}_9-\mathcal{X}_8)(\rho_2-\mathcal{X}_8)\sqrt{(\mathcal{X}_1-\mathcal{X}_8)^3}(\rho_6-i\sqrt{\mathcal{X}_8})(\rho_6+i\sqrt{\mathcal{X}_8})}+$$
$$\frac{(-1)^n cb^2 e^{i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}x\right)\sqrt{(\mathcal{X}_4-\mathcal{X}_9)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right)\right]}{2(\mathcal{X}_8-\mathcal{X}_9)(\rho_2-\mathcal{X}_9)\sqrt{(\mathcal{X}_1-\mathcal{X}_9)^3}(i\sqrt{\mathcal{X}_9}-\rho_6)(i\sqrt{\mathcal{X}_9}+\rho_6)}+$$
$$\frac{(-1)^n cb^2 e^{-i\sqrt{\mathcal{X}_9}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}x\right)\sqrt{(\mathcal{X}_4-\mathcal{X}_9)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right)\right]}{2(\mathcal{X}_8-\mathcal{X}_9)(\rho_2-\mathcal{X}_9)\sqrt{(\mathcal{X}_1-\mathcal{X}_9)^3}(\rho_6-i\sqrt{\mathcal{X}_9})(\rho_6+i\sqrt{\mathcal{X}_9})}+$$
$$\frac{(-1)^n cb^2 e^{i\sqrt{\rho_2}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}x\right)\sqrt{(\mathcal{X}_4-\rho_2)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\right)\right]}{2(\mathcal{X}_8-\rho_2)(\mathcal{X}_9-\rho_2)\sqrt{(\mathcal{X}_1-\rho_2)^3}(i\sqrt{\rho_2}-\rho_6)(i\sqrt{\rho_2}+\rho_6)}+$$

$$\frac{(-1)^n cb^2 e^{-i\sqrt{\rho_2}(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}x\right)\sqrt{(\mathcal{X}_4-\rho_2)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\right)\right]}{2(\mathcal{X}_8-\rho_2)(\mathcal{X}_9-\rho_2)\sqrt{(\mathcal{X}_1-\rho_2)^3}(\rho_6-i\sqrt{\rho_2})(\rho_6+i\sqrt{\rho_2})}+$$
$$\frac{(-1)^n cb^2 e^{\rho_6(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}x\right)\sqrt{\rho_6^2+\mathcal{X}_4}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\right)\right]}{2(\rho_6^2+\mathcal{X}_8)(\rho_6^2+\mathcal{X}_9)\sqrt{(\rho_6^2+\mathcal{X}_1)^3}(\rho_6^2+\rho_2)}+$$
$$\frac{(-1)^n cb^2 e^{-\rho_6(t-\frac{1}{c})}\sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}x\right)\sqrt{\rho_6^2+\mathcal{X}_4}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\right)\right]}{2(\rho_6^2+\mathcal{X}_8)(\rho_6^2+\mathcal{X}_9)\sqrt{(\rho_6^2+\mathcal{X}_1)^3}(\rho_6^2+\rho_2)} \qquad (128.31)$$

$$Q_{32}(x,y,t) = \frac{(-1)^n n\pi b\, c^2\, e^{i\sqrt{\mathcal{X}_8}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right) sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\,x\right)}{2i\sqrt{\mathcal{X}_8}\,(\mathcal{X}_9-\mathcal{X}_8)\,(\mathcal{X}_1-\mathcal{X}_8)\sqrt{(\rho_2-\mathcal{X}_8)}\,(i\sqrt{\mathcal{X}_8}+\rho_6)\,(i\sqrt{\mathcal{X}_8}-\rho_6)} +$$

$$\frac{(-1)^n n\pi b\, c^2\, e^{-i\sqrt{\mathcal{X}_8}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\right) sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_8}{\mathcal{X}_4-\mathcal{X}_8}}\,x\right)}{2i\sqrt{\mathcal{X}_8}\,(\mathcal{X}_9-\mathcal{X}_8)\,(\mathcal{X}_1-\mathcal{X}_8)\sqrt{(\rho_2-\mathcal{X}_8)}\,(\rho_6-i\sqrt{\mathcal{X}_8})\,(\rho_6+i\sqrt{\mathcal{X}_8})} + \frac{(-1)^n n\pi b\, c^2\, e^{i\sqrt{\mathcal{X}_9}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right) sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\,x\right)}{2i\sqrt{\mathcal{X}_9}\,(\mathcal{X}_8-\mathcal{X}_9)\,(\mathcal{X}_1-\mathcal{X}_9)\sqrt{(\rho_2-\mathcal{X}_9)}\,(i\sqrt{\mathcal{X}_9}+\rho_6)\,(i\sqrt{\mathcal{X}_9}-\rho_6)} +$$

$$\frac{(-1)^n n\pi b\, c^2\, e^{-i\sqrt{\mathcal{X}_9}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\right) sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_9}{\mathcal{X}_4-\mathcal{X}_9}}\,x\right)}{2i\sqrt{\mathcal{X}_9}\,(\mathcal{X}_8-\mathcal{X}_9)\,(\mathcal{X}_1-\mathcal{X}_9)\sqrt{(\rho_2-\mathcal{X}_9)}\,(\rho_6-i\sqrt{\mathcal{X}_9})\,(\rho_6+i\sqrt{\mathcal{X}_9})} + \frac{(-1)^n n\pi b\, c^2\, e^{i\sqrt{\mathcal{X}_1}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_1}{\mathcal{X}_4-\mathcal{X}_1}}\right) sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_1}{\mathcal{X}_4-\mathcal{X}_1}}\,x\right)}{2i\sqrt{\mathcal{X}_1}\,(\mathcal{X}_8-\mathcal{X}_1)\,(\mathcal{X}_9-\mathcal{X}_1)\sqrt{(\rho_2-\mathcal{X}_1)}\,(i\sqrt{\mathcal{X}_1}+\rho_6)\,(i\sqrt{\mathcal{X}_1}-\rho_6)} +$$

$$\frac{(-1)^n n\pi b\, c^2\, e^{-i\sqrt{\mathcal{X}_1}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_1}{\mathcal{X}_4-\mathcal{X}_1}}\right) sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\mathcal{X}_1}{\mathcal{X}_4-\mathcal{X}_1}}\,x\right)}{2i\sqrt{\mathcal{X}_1}\,(\mathcal{X}_8-\mathcal{X}_1)\,(\mathcal{X}_9-\mathcal{X}_1)\sqrt{(\rho_2-\mathcal{X}_1)}\,(\rho_6-i\sqrt{\mathcal{X}_1})\,(\rho_6+i\sqrt{\mathcal{X}_1})} + \frac{(-1)^n n\pi b c^2\, e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\right)\, \sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\,x\right)}{2i\sqrt{\rho_2}(\mathcal{X}_8-\rho_2)(\mathcal{X}_9-\rho_2)\,(\mathcal{X}_1-\rho_2)\,(i\sqrt{\rho_2}+\rho_6)\,(i\sqrt{\rho_2}-\rho_6)} +$$

$$\frac{(-1)^n n\pi b c^2\, e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\right)\, \sin\left(\frac{n\pi}{b}\sqrt{\frac{\mathcal{X}_1-\rho_2}{\mathcal{X}_4-\rho_2}}\,x\right)}{2i\sqrt{\rho_2}(\mathcal{X}_8-\rho_2)(\mathcal{X}_9-\rho_2)\,(\mathcal{X}_1-\rho_2)\,(\rho_6-i\sqrt{\rho_2})\,(\rho_6+i\sqrt{\rho_2})} + \frac{(-1)^n n\pi b c^2\, e^{\rho_6\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\right)\, \sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\,x\right)}{2\rho_6\,(\rho_6^2+\mathcal{X}_8)\,(\rho_6^2+\mathcal{X}_9)\,(\rho_6^2+\mathcal{X}_1)\sqrt{\rho_6^2+\rho_2}} +$$

$$-\frac{(-1)^n n\pi b c^2\, e^{-\rho_6\left(t-\frac{1}{c}\right)}\, sin^2\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\right)\, \sin\left(\frac{n\pi}{b}\sqrt{\frac{\rho_6^2+\mathcal{X}_1}{\rho_6^2+\mathcal{X}_4}}\,x\right)}{2\rho_6\,(\rho_6^2+\mathcal{X}_8)\,(\rho_6^2+\mathcal{X}_9)\,(\rho_6^2+\mathcal{X}_1)\sqrt{\rho_6^2+\rho_2}} \tag{128.32}$$

$$Q_{33}(x,y,t) = \frac{ic^2 n\pi\sqrt{\rho_2}e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)(\chi_4-\rho_2)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,y\right)}{2b\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,(i\sqrt{\rho_2}+\rho_7)(i\sqrt{\rho_2}-\rho_7)}$$

$$+\frac{ic^2 n\pi\sqrt{\rho_2}e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)(\chi_4-\rho_2)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,y\right)}{2b\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,(\rho_7-i\sqrt{\rho_2})(\rho_7+i\sqrt{\rho_2})}$$

$$+\frac{ic^2 n\pi\sqrt{\chi_8}e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,\chi\right)(\chi_4-\chi_8)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,y\right)}{4ib\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,(i\sqrt{\chi_8}+\rho_7)(i\sqrt{\chi_8}-\rho_7)}$$

$$+\frac{ic^2 n\pi\sqrt{\chi_8}e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,\chi\right)(\chi_4-\chi_8)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,y\right)}{-4ib\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,(\rho_7-i\sqrt{\chi_8})(i\sqrt{\chi_8}+\rho_7)} + \frac{ic^2 n\pi\sqrt{\chi_9}e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,\chi\right)(\chi_4-\chi_9)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,y\right)}{4ib\sqrt{\chi_9}\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,(i\sqrt{\chi_9}+\rho_7)(i\sqrt{\chi_9}-\rho_7)} -$$

$$\frac{ic^2 n\pi\sqrt{\chi_9}e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,\chi\right)(\chi_4-\chi_9)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,y\right)}{4ib\sqrt{\chi_9}\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,(i\sqrt{\chi_9}+\rho_7)(\rho_7-i\sqrt{\chi_9})} +$$

$$\frac{ic^2 n\pi\rho_7 e^{\rho_7\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\,\chi\right)(\rho_7^2+\chi_4)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\cos\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,y\right)}{2b\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} +$$

$$\frac{ic^2 n\pi\rho_7 e^{-\rho_7\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\,\chi\right)(\rho_7^2+\chi_4)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\cos\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,y\right)}{2b\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} \tag{128.33}$$

$$Q_{34}(x,y,t)=\frac{(-1)^n icn\pi\sqrt{\rho_2}e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}y\right)}{2b\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,\left(i\sqrt{\rho_2}+\rho_7\right)\left(i\sqrt{\rho_2}-\rho_7\right)}$$

$$-\frac{(-1)^n icn\pi\sqrt{\rho_2}e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}y\right)}{2b\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,\left(\rho_7-i\sqrt{\rho_2}\right)\left(\rho_7+i\sqrt{\rho_2}\right)}$$

$$+\frac{(-1)^n icn\pi\sqrt{\chi_8}e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)}{4ib\sqrt{\chi_8}\,\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,\left(i\sqrt{\chi_8}+\rho_7\right)\left(i\sqrt{\chi_8}-\rho_7\right)}$$

$$-\frac{(-1)^n icn\pi\sqrt{\chi_8}e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)}{4ib\sqrt{\chi_8}\,\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,\left(i\sqrt{\chi_8}+\rho_7\right)\left(i\sqrt{\chi_8}-\rho_7\right)}$$

$$+\frac{(-1)^n icn\pi\sqrt{\chi_9}e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{4ib\sqrt{\chi_9}\,\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,\left(i\sqrt{\chi_9}+\rho_7\right)\left(i\sqrt{\chi_9}-\rho_7\right)}$$

$$-\frac{(-1)^n icn\pi\sqrt{\chi_9}e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{4ib\sqrt{\chi_9}\,\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,\left(i\sqrt{\chi_9}+\rho_7\right)\left(\rho_7-i\sqrt{\chi_9}\right)}$$

$$+\frac{(-1)^n icn\pi\rho_7e^{\rho_7\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}y\right)}{2b\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)}+$$

$$\frac{(-1)^n icn\pi\rho_7e^{-\rho_7\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\chi\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}y\right)}{2b\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} \qquad (128.34)$$

$$Q_{35}(x,y,t)=\frac{c^2n^2\pi^2e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)\sqrt{(\chi_4-\rho_2)(\chi_1-\rho_2)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}b\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}y\right)}{4ib^2\sqrt{\rho_2}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,\left(i\sqrt{\rho_2}+\rho_7\right)\left(i\sqrt{\rho_2}-\rho_7\right)}$$

$$+\frac{c^2n^2\pi^2e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,\chi\right)\sqrt{(\chi_4-\rho_2)(\chi_1-\rho_2)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}b\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}y\right)}{4ib^2\sqrt{\rho_2}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,\left(\rho_7-i\sqrt{\rho_2}\right)\left(\rho_7+i\sqrt{\rho_2}\right)}$$

$$c^2n^2\pi^2e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,\chi\right)\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)$$

$$+\frac{\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)}{4ib^2\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,(i\sqrt{\chi_8}+\rho_7)(i\sqrt{\chi_8}-\rho_7)}$$

$$+\frac{c^2n^2\pi^2e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\chi\right)\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}y\right)}{4ib^2\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,(\rho_7-i\sqrt{\chi_8})(i\sqrt{\chi_8}+\rho_7)}$$

$$+\frac{c^2n^2\pi^2e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{4ib^2\sqrt{\chi_9}\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,(i\sqrt{\chi_9}+\rho_7)(i\sqrt{\chi_9}-\rho_7)}$$

$$+\frac{c^2n^2\pi^2e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}y\right)}{4ib^2\sqrt{\chi_9}\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,(i\sqrt{\chi_9}+\rho_7)(\rho_7-i\sqrt{\chi_9})}+$$

$$\frac{c^2n^2\pi^2e^{\rho_7\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\chi\right)\sqrt{(\rho_7^2+\chi_4)(\rho_7^2+\chi_1)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\cos\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}b\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}y\right)}{4b^2\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)}+$$

$$\frac{c^2n^2\pi^2e^{-\rho_7\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\chi\right)\sqrt{(\rho_7^2+\chi_4)(\rho_7^2+\chi_1)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\cos\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}b\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}y\right)}{-4b^2\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} \qquad (128.35)$$

$$Q_{36}(x,y,t)=\frac{(-1)^nc^2n^2\pi^2e^{i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\chi\right)\sqrt{(\chi_4-\rho_2)(\chi_1-\rho_2)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\right)}{2b\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,(i\sqrt{\rho_2}+\rho_7)(i\sqrt{\rho_2}-\rho_7)}$$

$$-\frac{(-1)^nc^2n^2\pi^2e^{-i\sqrt{\rho_2}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\chi\right)\sqrt{(\chi_4-\rho_2)(\chi_1-\rho_2)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\right)}{2ib\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)\,(\chi_9-\rho_2)\,(\rho_7-i\sqrt{\rho_2})(\rho_7+i\sqrt{\rho_2})}+$$

$$\frac{(-1)^nc^2n^2\pi^2e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\chi\right)\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\right)}{4ib\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,(i\sqrt{\chi_8}+\rho_7)(i\sqrt{\chi_8}-\rho_7)}+$$

$$\frac{(-1)^nc^2n^2\pi^2e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\chi\right)\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\right)}{4ib\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)\,(\rho_7-i\sqrt{\chi_8})(i\sqrt{\chi_8}+\rho_7)}+$$

$$\frac{(-1)^nc^2n^2\pi^2e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\right)}{4ib\sqrt{\chi_9}\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,(i\sqrt{\chi_9}+\rho_7)(i\sqrt{\chi_9}-\rho_7)}+$$

$$\frac{(-1)^nc^2n^2\pi^2e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\chi\right)\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\right)}{4ib\sqrt{\chi_9}\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)\,(i\sqrt{\chi_9}+\rho_7)(\rho_7-i\sqrt{\chi_9})}+$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{\rho_7\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\,x\right)\sqrt{(\rho_7^2+\chi_4)(\rho_7^2+\chi_1)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\right)}{4b\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} - \frac{(-1)^n c^2 n^2 \pi^2 e^{-\rho_7\left(t-\frac{1}{c}\right)} \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\,x\right)\sqrt{(\rho_7^2+\chi_4)(\rho_7^2+\chi_1)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\right)}{4b\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} \qquad (128.36)$$

$$Q_{37}(x,y,t) = \frac{c^2 n^2 \pi^2 e^{i\sqrt{\rho_2}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,x\right)\sqrt{(\chi_4-\rho_2)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,y\right)}{2b^2\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)(\chi_9-\rho_2)(i\sqrt{\rho_2}+\rho_7)(i\sqrt{\rho_2}-\rho_7)}$$

$$-\frac{c^2 n^2 \pi^2 e^{-i\sqrt{\rho_2}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,x\right)\sqrt{(\chi_4-\rho_2)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,y\right)}{2ib^2\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)(\chi_9-\rho_2)(\rho_7-i\sqrt{\rho_2})(\rho_7+i\sqrt{\rho_2})}$$

$$+\frac{c^2 n^2 \pi^2 e^{i\sqrt{\chi_8}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,x\right)\sqrt{(\chi_4-\chi_8)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,y\right)}{4ib^2\sqrt{\chi_8}\,\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)(i\sqrt{\chi_8}+\rho_7)(i\sqrt{\chi_8}-\rho_7)}$$

$$+\frac{c^2 n^2 \pi^2 e^{-i\sqrt{\chi_8}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,x\right)\sqrt{(\chi_4-\chi_8)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,y\right)}{4ib^2\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)(\rho_7-i\sqrt{\chi_8})(i\sqrt{\chi_8}+\rho_7)}$$

$$+\frac{c^2 n^2 \pi^2 e^{i\sqrt{\chi_9}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,x\right)\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,y\right)}{4ib^2\sqrt{\chi_9}\,\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)(i\sqrt{\chi_9}+\rho_7)(i\sqrt{\chi_9}-\rho_7)} + \frac{c^2 n^2 \pi^2 e^{-i\sqrt{\chi_9}t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,x\right)\sqrt{(\chi_4-\chi_9)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,y\right)}{4ib^2\sqrt{\chi_9}\,\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)(i\sqrt{\chi_9}+\rho_7)(\rho_7-i\sqrt{\chi_9})} +$$

$$\frac{c^2 n^2 \pi^2 e^{\rho_7 t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\,x\right)\sqrt{(\rho_7^2+\chi_4)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,y\right)}{4b^2\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} - \frac{c^2 n^2 \pi^2 e^{-\rho_7 t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\,x\right)\sqrt{(\rho_7^2+\chi_4)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,b\right)\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,y\right)}{4b^2\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} \qquad (128.37)$$

$$Q_{38}(x,y,t) = \frac{(-1)^n c^2 n^2 \pi^2 \cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,x\right)\sqrt{(\chi_4-\rho_2)(\chi_1-\rho_2)}\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,y\right)e^{i\sqrt{\rho_2}\,t}}{2b^2\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)(\chi_9-\rho_2)(i\sqrt{\rho_2}+\rho_7)(i\sqrt{\rho_2}-\rho_7)} -$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{i\sqrt{\rho_2}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_2)}{(\chi_4-\rho_2)}}\,x\right)\sqrt{(\chi_4-\rho_2)(\chi_1-\rho_2)}\sin\left(\sqrt{\frac{z(\rho_1-\rho_2)}{\alpha_{ot}(\rho_2-\rho_2)}}\,y\right)}{2ib^2\sqrt{2i\sqrt{\rho_2}}\,(\chi_8-\rho_2)(\chi_9-\rho_2)(\rho_7-i\sqrt{\rho_2})(\rho_7+i\sqrt{\rho_2})} +$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{i\sqrt{\chi_8}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,x\right)\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,y\right)}{4ib^2\sqrt{\chi_8}\,\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)(i\sqrt{\chi_8}+\rho_7)(i\sqrt{\chi_8}-\rho_7)} +$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{-i\sqrt{\chi_8}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,x\right)\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}\sin\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\,y\right)}{4ib^2\sqrt{\chi_8}\sqrt{\rho_2-\chi_8}\,(\chi_9-\chi_8)(\rho_7-i\sqrt{\chi_8})(i\sqrt{\chi_8}+\rho_7)} +$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{i\sqrt{\chi_9}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,x\right)\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,y\right)}{4ib^2\sqrt{\chi_9}\,\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)(i\sqrt{\chi_9}+\rho_7)(i\sqrt{\chi_9}-\rho_7)} +$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{-i\sqrt{\chi_9}\,t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,x\right)\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}\sin\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\,y\right)}{4ib^2\sqrt{\chi_9}\,\sqrt{\rho_2-\chi_9}(\chi_8-\chi_9)(i\sqrt{\chi_9}+\rho_7)(\rho_7-i\sqrt{\chi_9})} +$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{\rho_7 t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\,x\right)\sqrt{(\rho_7^2+\chi_4)(\rho_7^2+\chi_1)}\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}\,y\right)}{4b^2\rho_7\sqrt{(\rho_7^2+\rho_2)}\,(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} -$$

$$\frac{(-1)^n c^2 n^2 \pi^2 e^{-\rho_7 t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_7^2+\chi_1)}{(\rho_7^2+\chi_4)}}\chi\right)\sqrt{(\rho_7^2+\chi_4)(\rho_7^2+\chi_1)}\sin\left(\sqrt{\frac{z(\rho_7^2+\rho_1)}{\alpha_{ot}(\rho_7^2+\rho_2)}}y\right)}{4b^2\rho_7\sqrt{(\rho_7^2+\rho_2)}(\rho_7^2+\chi_8)(\rho_7^2+\chi_9)} \tag{128.38}$$

$$Q_{39}(x,y,t)=\frac{c^2n^2\pi^2 e^{i\sqrt{\rho_4}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_4)}{(\chi_4-\rho_4)}}\right)\sqrt{(\chi_4-\rho_4)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_4)}{\alpha_{ot}(\rho_2-\rho_4)}}\chi\right)}{b\sqrt{2i\sqrt{\rho_4}}(\chi_8-\rho_4)(\chi_9-\rho_4)\sqrt{(\chi_1-\rho_4)}}+$$
$$\frac{c^2n^2\pi^2 e^{-i\sqrt{\rho_4}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_4)}{(\chi_4-\rho_4)}}\right)\sqrt{(\chi_4-\rho_4)}\cos\left(\sqrt{\frac{z(\rho_1-\rho_4)}{\alpha_{ot}(\rho_2-\rho_4)}}\chi\right)}{ib\sqrt{2i\sqrt{\rho_4}}(\chi_8-\rho_4)(\chi_9-\rho_4)\sqrt{(\chi_1-\rho_4)}}+\frac{c^2n^2\pi^2 e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\sqrt{(\chi_4-\chi_1)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}\chi\right)}{ib\sqrt{\chi_1}\sqrt{\rho_4-\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c^2n^2\pi^2 e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\sqrt{(\chi_4-\chi_1)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_1)}{\alpha_{ot}(\rho_2-\chi_1)}}\chi\right)}{ib\sqrt{\chi_1}\sqrt{\rho_4-\chi_1}(\chi_8-\chi_1)(\chi_9-\chi_1)}+\frac{c^2n^2\pi^2 e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sqrt{(\chi_4-\chi_8)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\chi\right)}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)}-$$
$$\frac{c^2n^2\pi^2 e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sqrt{(\chi_4-\chi_8)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_8)}{\alpha_{ot}(\rho_2-\chi_8)}}\chi\right)}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)}+\frac{c^2n^2\pi^2 e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sqrt{(\chi_4-\chi_9)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\chi\right)}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)}-$$
$$\frac{c^2n^2\pi^2 e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sqrt{(\chi_4-\chi_9)}\cos\left(\sqrt{\frac{z(\rho_1-\chi_9)}{\alpha_{ot}(\rho_2-\chi_9)}}\chi\right)}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)} \tag{128.39}$$

$$Q_{40}(x,y,t)=\frac{in\pi c\sqrt{\chi_4}e^{i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_4)}{\alpha_{ot}(\rho_4-\chi_4)}}x\right)}{b\sqrt{2i\sqrt{\chi_4}(\rho_4-\chi_4)(\chi_1-\chi_4)}}-\frac{nc\sqrt{\chi_4}e^{-i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_4)}{\alpha_{ot}(\rho_4-\chi_4)}}x\right)}{b\sqrt{2i\sqrt{\chi_4}(\rho_4-\chi_4)(\chi_1-\chi_4)}}+$$
$$\frac{in\pi c\sqrt{\rho_4}e^{i\sqrt{\rho_4}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_4)}{(\chi_4-\rho_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\rho_4)}{\alpha_{ot}(\rho_4-\rho_4)}}x\right)}{b\sqrt{2i\sqrt{\rho_4}(\chi_4-\rho_4)(\chi_1-\rho_4)}}-\frac{n\pi c\sqrt{\rho_4}e^{-i\sqrt{\rho_4}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_4)}{(\chi_4-\rho_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\rho_4)}{\alpha_{ot}(\rho_4-\rho_4)}}x\right)}{b\sqrt{2i\sqrt{\rho_4}(\chi_4-\rho_4)(\chi_1-\rho_4)}}+$$
$$\frac{in\pi c\sqrt{\chi_1}e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_1)}{\alpha_{ot}(\rho_4-\chi_1)}}x\right)}{b\sqrt{2i\sqrt{\chi_1}(\chi_4-\chi_1)(\rho_4-\chi_1)}}+\frac{n\pi c\sqrt{\chi_1}e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_1)}{\alpha_{ot}(\rho_4-\chi_1)}}x\right)}{b\sqrt{2i\sqrt{\chi_1}(\chi_4-\chi_1)(\rho_4-\chi_1)}} \tag{128.40}$$

$$Q_{41}(x,y,t)=\frac{ic\sqrt{\rho_4}e^{i\sqrt{\rho_4}\left(t-\frac{1}{c}\right)}(\chi_4-\rho_4)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_4)}{(\chi_4-\rho_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\rho_4)}{\alpha_{ot}(\rho_4-\rho_4)}}x\right)}{\sqrt{2i\rho_4}(\chi_4-\rho_4)(\chi_8-\rho_4)(\chi_9-\rho_4)}-$$
$$\frac{ic\sqrt{\rho_4}e^{-i\sqrt{\rho_4}\left(t-\frac{1}{c}\right)}(\chi_4-\rho_4)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_4)}{(\chi_4-\rho_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\rho_4)}{\alpha_{ot}(\rho_4-\rho_4)}}x\right)}{\sqrt{2i\rho_4}(\chi_4-\rho_4)(\chi_8-\rho_4)(\chi_9-\rho_4)}+\frac{c\sqrt{\chi_4}e^{i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_4)}{\alpha_{ot}(\rho_4-\chi_4)}}x\right)}{2i\sqrt{\chi_4}\sqrt{(\rho_4-\chi_4)}(\chi_8-\chi_4)(\chi_9-\chi_4)}+$$
$$\frac{ic\sqrt{\chi_4}e^{-i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_4)}{\alpha_{ot}(\rho_4-\chi_4)}}x\right)}{2i\sqrt{\chi_4}\sqrt{(\rho_4-\chi_4)}(\chi_8-\chi_4)(\chi_9-\chi_4)}+\frac{ic\sqrt{\chi_8}e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_8)}{\alpha_{ot}(\rho_4-\chi_8)}}x\right)}{2i\sqrt{\chi_8}\sqrt{(\rho_4-\chi_8)}(\chi_4-\chi_8)(\chi_9-\chi_8)}+$$
$$\frac{ic\sqrt{\chi_8}e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_8)}{\alpha_{ot}(\rho_4-\chi_8)}}x\right)}{2i\sqrt{\chi_8}\sqrt{(\rho_4-\chi_8)}(\chi_4-\chi_8)(\chi_9-\chi_8)}+\frac{ic\sqrt{\chi_9}e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_9)}{\alpha_{ot}(\rho_4-\chi_9)}}x\right)}{2i\sqrt{\chi_9}\sqrt{(\rho_4-\chi_9)}(\chi_4-\chi_9)(\chi_8-\chi_9)}+$$
$$\frac{ic\sqrt{\chi_9}e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\right]\cos\left(\sqrt{\frac{z(\rho_8-\chi_9)}{\alpha_{ot}(\rho_4-\chi_9)}}x\right)}{2i\sqrt{\chi_9}\sqrt{(\rho_4-\chi_9)}(\chi_4-\chi_9)(\chi_8-\chi_9)} \tag{128.41}$$

$$Q_{42}(x,y,t)=0 \tag{128.42}$$

$$Q_{43}(x,y,t)=\frac{e^{i\sqrt{\rho_4}t}\left[1-(-1)^m e^{-i\sqrt{\rho_4}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}y\right)}{\sqrt{2i\sqrt{\rho_4}}(\chi_8-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}(i\sqrt{\rho_4}+\rho_8)(i\sqrt{\rho_4}-\rho_8)}-\frac{e^{-i\sqrt{\rho_4}t}\left[1-(-1)^m e^{i\sqrt{\rho_4}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}y\right)}{i\sqrt{2i\sqrt{\rho_4}}(\chi_8-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}(\rho_8-i\sqrt{\rho_4})(\rho_8+i\sqrt{\rho_4})}+$$
$$\frac{e^{i\sqrt{\chi_8}t}\left[1-(-1)^m e^{-i\sqrt{\chi_8}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_8)}{(\chi_1-\chi_8)}}y\right)}{2i\sqrt{\chi_8}\sqrt{(\rho_4-\chi_8)(\chi_1-\chi_8)(\chi_2-\chi_8)}(i\sqrt{\chi_8}+\rho_8)(i\sqrt{\chi_8}-\rho_8)}+\frac{e^{-i\sqrt{\chi_8}t}\left[1-(-1)^m e^{i\sqrt{\chi_8}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_8)}{(\chi_1-\chi_8)}}y\right)}{2i\sqrt{\chi_8}\sqrt{(\rho_4-\chi_8)(\chi_1-\chi_8)(\chi_2-\chi_8)}(\rho_8-i\sqrt{\chi_8})(\rho_8+i\sqrt{\chi_8})}+$$
$$\frac{e^{i\sqrt{\chi_1}t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}y\right)}{\sqrt{2i\sqrt{\chi_1}}\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_8-\chi_1)(i\sqrt{\chi_1}+\rho_8)(i\sqrt{\chi_1}-\rho_8)}-\frac{e^{-i\sqrt{\chi_1}t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}y\right)}{\sqrt{2i\sqrt{\chi_1}}\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_8-\chi_1)(\rho_8-i\sqrt{\chi_1})(\rho_8+i\sqrt{\chi_1})}+$$
$$\frac{e^{i\sqrt{\chi_2}t}\left[1-(-1)^m e^{-i\sqrt{\chi_2}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}y\right)}{\sqrt{2i\sqrt{\chi_2}}\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_8-\chi_2)(i\sqrt{\chi_2}+\rho_8)(i\sqrt{\chi_2}-\rho_8)}-\frac{e^{-i\sqrt{\chi_2}t}\left[1-(-1)^m e^{i\sqrt{\chi_2}/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}y\right)}{\sqrt{2i\sqrt{\chi_2}}\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_8-\chi_2)(\rho_8-i\sqrt{\chi_2})(\rho_8+i\sqrt{\chi_2})}+$$
$$\frac{e^{\rho_8 t}\left[1-(-1)^m e^{-\rho_8/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}y\right)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_8)}-\frac{e^{-\rho_8 t}\left[1-(-1)^m e^{\rho_8/c}\right]\sin y_o\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}y\right)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_8)} \tag{128.43}$$

$$
\begin{aligned}
Q_{44}(x,y,t) = {} & \frac{e^{i\sqrt{\rho_4}\,t}\left[1-(-1)^m e^{-i\sqrt{\rho_4}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}\,y\right)}{\sqrt{2i\sqrt{\rho_4}}\,(\chi_3-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}\left(i\sqrt{\rho_4}+\rho_8\right)\left(i\sqrt{\rho_4}-\rho_8\right)} \\
& - \frac{e^{-i\sqrt{\rho_4}\,t}\left[1-(-1)^m e^{i\sqrt{\rho_4}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}\,y\right)}{i\sqrt{2i\sqrt{\rho_4}}\,(\chi_3-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}\left(\rho_8-i\sqrt{\rho_4}\right)\left(\rho_8+i\sqrt{\rho_4}\right)} \\
& + \frac{e^{i\sqrt{\chi_3}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_3}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_3)}{(\chi_1-\chi_3)}}\,y\right)}{2i\sqrt{\chi_3}\,\sqrt{(\rho_4-\chi_3)(\chi_1-\chi_3)(\chi_2-\chi_3)}\left(i\sqrt{\chi_3}+\rho_8\right)\left(i\sqrt{\chi_3}-\rho_8\right)} \\
& + \frac{e^{-i\sqrt{\chi_3}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_3}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_3)}{(\chi_1-\chi_3)}}\,y\right)}{2i\sqrt{\chi_3}\,\sqrt{(\rho_4-\chi_3)(\chi_1-\chi_3)(\chi_2-\chi_3)}\left(\rho_8-i\sqrt{\chi_3}\right)\left(\rho_8+i\sqrt{\chi_3}\right)} \\
& + \frac{e^{i\sqrt{\chi_1}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}\,y\right)}{\sqrt{2i\sqrt{\chi_1}}\,\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_3-\chi_1)(i\sqrt{\chi_1}+\rho_8)(i\sqrt{\chi_1}-\rho_8)} \\
& - \frac{e^{-i\sqrt{\chi_1}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}\,y\right)}{\sqrt{2i\sqrt{\chi_1}}\,\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_3-\chi_1)(\rho_8-i\sqrt{\chi_1})(\rho_8+i\sqrt{\chi_1})} \\
& + \frac{e^{i\sqrt{\chi_2}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_2}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}\,y\right)}{\sqrt{2i\sqrt{\chi_2}}\,\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_3-\chi_2)(i\sqrt{\chi_2}+\rho_8)(i\sqrt{\chi_2}-\rho_8)} \\
& - \frac{e^{-i\sqrt{\chi_2}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_2}/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}\,y\right)}{\sqrt{2i\sqrt{\chi_2}}\,\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_3-\chi_2)(\rho_8-i\sqrt{\chi_2})(\rho_8+i\sqrt{\chi_2})} \\
& + \frac{e^{\rho_8 t}\left[1-(-1)^m e^{-\rho_8/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}\,y\right)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_3)} - \frac{e^{-\rho_8 t}\left[1-(-1)^m e^{\rho_8/c}\right]\sin(b-y_o)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}\,y\right)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_3)} \qquad (128.44)
\end{aligned}
$$

$$Q_{45}(x,y,t) = \frac{e^{i\sqrt{\rho_4}\,t}\left[1-(-1)^m e^{-i\sqrt{\rho_4}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}\,b\right)}{\sqrt{2i\sqrt{\rho_4}}\,(\chi_3-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}(i\sqrt{\rho_4}+\rho_8)(i\sqrt{\rho_4}-\rho_8)}$$
$$-\frac{e^{-i\sqrt{\rho_4}\,t}\left[1-(-1)^m e^{i\sqrt{\rho_4}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\rho_4)}{(\chi_1-\rho_4)}}\,b\right)}{i\sqrt{2i\sqrt{\rho_4}}\,(\chi_3-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}(\rho_8-i\sqrt{\rho_4})(\rho_8+i\sqrt{\rho_4})}$$
$$+\frac{e^{i\sqrt{\chi_3}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_3}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_3)}{(\chi_1-\chi_3)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_3)}{(\chi_1-\chi_3)}}\,b\right)}{2i\sqrt{\chi_3}\,\sqrt{(\rho_4-\chi_3)(\chi_1-\chi_3)(\chi_2-\chi_3)}(i\sqrt{\chi_3}+\rho_8)(i\sqrt{\chi_3}-\rho_8)}$$

$$+\frac{e^{-i\sqrt{\chi_3}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_3}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_3)}{(\chi_1-\chi_3)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_3)}{(\chi_1-\chi_3)}}\,b\right)}{2i\sqrt{\chi_3}\,\sqrt{(\rho_4-\chi_3)(\chi_1-\chi_3)(\chi_2-\chi_3)}(\rho_8-i\sqrt{\chi_3})(\rho_8+i\sqrt{\chi_3})}+$$
$$\frac{e^{i\sqrt{\chi_1}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}\,b\right)}{\sqrt{2i\sqrt{\chi_1}}\,\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_3-\chi_1)(i\sqrt{\chi_1}+\rho_8)(i\sqrt{\chi_1}-\rho_8)}-\frac{e^{-i\sqrt{\chi_1}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_1)}{(\chi_1-\chi_1)}}\,b\right)}{\sqrt{2i\sqrt{\chi_1}}\,\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_3-\chi_1)(\rho_8-i\sqrt{\chi_1})(\rho_8+i\sqrt{\chi_1})}+$$
$$\frac{e^{i\sqrt{\chi_2}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_2}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}\,b\right)}{\sqrt{2i\sqrt{\chi_2}}\,\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_3-\chi_2)(i\sqrt{\chi_2}+\rho_8)(i\sqrt{\chi_2}-\rho_8)}-\frac{e^{-i\sqrt{\chi_2}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_2}/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_2-\chi_2)}{(\chi_1-\chi_2)}}\,b\right)}{\sqrt{2i\sqrt{\chi_2}}\,\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_3-\chi_2)(\rho_8-i\sqrt{\chi_2})(\rho_8+i\sqrt{\chi_2})}+$$
$$\frac{e^{\rho_8 t}\left[1-(-1)^m e^{-\rho_8/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}\,b\right)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_3)}-\frac{e^{-\rho_8 t}\left[1-(-1)^m e^{\rho_8/c}\right]\sin y_o \sin\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}\,y\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\rho_8^2+\chi_2)}{(\rho_8^2+\chi_1)}}\,b\right)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_3)} \quad (128.45)$$

$$Q_{46}(x,y,t) = \frac{e^{i\sqrt{\rho_4}\,t}\left[1-(-1)^m e^{-i\sqrt{\rho_4}/c}\right]\sin(b-y_o)}{\sqrt{2i\sqrt{\rho_4}}\,(\chi_3-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}(i\sqrt{\rho_4}+\rho_8)(i\sqrt{\rho_4}-\rho_8)}-\frac{e^{-i\sqrt{\rho_4}\,t}\left[1-(-1)^m e^{i\sqrt{\rho_4}/c}\right]\sin(b-y_o)}{i\sqrt{2i\sqrt{\rho_4}}\,(\chi_3-\rho_4)\sqrt{(\chi_1-\rho_4)(\chi_2-\rho_4)}(\rho_8-i\sqrt{\rho_4})(\rho_8+i\sqrt{\rho_4})}+$$
$$\frac{e^{i\sqrt{\chi_3}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_3}/c}\right]\sin(b-y_o)}{2i\sqrt{\chi_3}\,\sqrt{(\rho_4-\chi_3)(\chi_1-\chi_3)(\chi_2-\chi_3)}(i\sqrt{\chi_3}+\rho_8)(i\sqrt{\chi_3}-\rho_8)}+\frac{e^{-i\sqrt{\chi_3}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_3}/c}\right]\sin(b-y_o)}{2i\sqrt{\chi_3}\,\sqrt{(\rho_4-\chi_3)(\chi_1-\chi_3)(\chi_2-\chi_3)}(\rho_8-i\sqrt{\chi_3})(\rho_8+i\sqrt{\chi_3})}+$$
$$\frac{e^{i\sqrt{\chi_1}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_1}/c}\right]\sin(b-y_o)}{\sqrt{2i\sqrt{\chi_1}}\,\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_3-\chi_1)(i\sqrt{\chi_1}+\rho_8)(i\sqrt{\chi_1}-\rho_8)}-\frac{e^{-i\sqrt{\chi_1}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_1}/c}\right]\sin(b-y_o)}{\sqrt{2i\sqrt{\chi_1}}\,\sqrt{(\rho_4-\chi_1)(\chi_2-\chi_1)}(\chi_3-\chi_1)(\rho_8-i\sqrt{\chi_1})(\rho_8+i\sqrt{\chi_1})}+$$
$$\frac{e^{i\sqrt{\chi_2}\,t}\left[1-(-1)^m e^{-i\sqrt{\chi_2}/c}\right]\sin(b-y_o)}{\sqrt{2i\sqrt{\chi_2}}\,\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_3-\chi_2)(i\sqrt{\chi_2}+\rho_8)(i\sqrt{\chi_2}-\rho_8)}-\frac{e^{-i\sqrt{\chi_2}\,t}\left[1-(-1)^m e^{i\sqrt{\chi_2}/c}\right]\sin(b-y_o)}{\sqrt{2i\sqrt{\chi_2}}\,\sqrt{(\rho_4-\chi_2)(\chi_1-\chi_2)}(\chi_3-\chi_2)(\rho_8-i\sqrt{\chi_2})(\rho_8+i\sqrt{\chi_2})}+$$
$$\frac{e^{\rho_8 t}\left[1-(-1)^m e^{-\rho_8/c}\right]\sin(b-y_o)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_3)}-\frac{e^{-\rho_8 t}\left[1-(-1)^m e^{\rho_8/c}\right]\sin(b-y_o)}{2\rho_8\sqrt{(\rho_8^2+\rho_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_2)}(\rho_8^2+\chi_3)} \quad (128.46)$$

$$Q_{47}(x,y,t) = \frac{c^2 n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)\sqrt{\chi_4-\chi_1}}{b\sqrt{2i\sqrt{\chi_1}}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c^2 n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)\sqrt{\chi_4-\chi_1}}{ib\sqrt{2i\sqrt{\chi_1}}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c^2 n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)\sqrt{\chi_4-\chi_8}}{2ib\sqrt{\chi_8}\,\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}-\frac{c^2 n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)\sqrt{\chi_4-\chi_8}}{2ib\sqrt{\chi_8}\,\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)} \quad (128.47)$$

$$Q_{48}(x,y,t) = \frac{c\sqrt{\chi_1}e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)(\chi_4-\chi_1)}{2\sqrt{\chi_1}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c\sqrt{\chi_1}e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)(\chi_4-\chi_1)}{2\sqrt{\chi_1}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c\sqrt{\chi_8}e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)(\chi_4-\chi_8)}{2\sqrt{\chi_8}\,\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}-\frac{c\sqrt{\chi_8}e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)(\chi_4-\chi_8)}{2\sqrt{\chi_8}\,\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+\frac{c\sqrt{\chi_9}e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)(\chi_4-\chi_9)}{2\sqrt{\chi_9}\,\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}+$$
$$\frac{c\sqrt{\chi_9}e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)(\chi_4-\chi_9)}{2\sqrt{\chi_9}\,\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}+\frac{c^2 n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)\sqrt{\chi_4-\chi_9}}{2ib\sqrt{\chi_9}\,\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}-\frac{c^2 n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)\sqrt{\chi_4-\chi_9}}{2ib\sqrt{\chi_9}\,\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \quad (128.48)$$

$$Q_{49}(x,y,t) = \frac{c^2 n\pi e^{i\sqrt{\chi_1}\,t}\sqrt{\chi_4-\chi_1}}{b\sqrt{2i\sqrt{\chi_1}}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c^2 n\pi e^{-i\sqrt{\chi_1}\,t}\sqrt{\chi_4-\chi_1}}{ib\sqrt{2i\sqrt{\chi_1}}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c^2 n\pi e^{i\sqrt{\chi_8}\,t}\sqrt{\chi_4-\chi_8}}{2ib\sqrt{\chi_8}\,\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}$$
$$-\frac{c^2 n\pi e^{-i\sqrt{\chi_8}\,t}\sqrt{\chi_4-\chi_8}}{2ib\sqrt{\chi_8}\,\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+\frac{c^2 n\pi e^{i\sqrt{\chi_9}\,t}\sqrt{\chi_4-\chi_9}}{2ib\sqrt{\chi_9}\,\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}$$
$$-\frac{c^2 n\pi e^{-i\sqrt{\chi_9}\,t}\sqrt{\chi_4-\chi_9}}{2ib\sqrt{\chi_9}\,\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \quad (128.49)$$

$$Q_{50}(x,y,t)=\frac{c\sqrt{\chi_1}e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_1)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\right]}{2\sqrt{\chi_1}(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c\sqrt{\chi_1}e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_1)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\right]}{2\sqrt{\chi_1}(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c\sqrt{\chi_8}e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_8)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\right]}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+$$
$$\frac{c\sqrt{\chi_8}e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_8)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\right]}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+\frac{c\sqrt{\chi_9}e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_9)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\right]}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}+$$
$$\frac{c\sqrt{\chi_9}e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_9)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\right]}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \qquad (128.50)$$

$$Q_{51}(x,y,t)=\frac{c^2n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_1)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)}{b\sqrt{2i\sqrt{\chi_1}}(\chi_8+\chi_1)(\chi_9-\chi_1)}-$$
$$\frac{c^2n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_1)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)}{b\sqrt{2i\sqrt{\chi_1}}(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c^2n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_8)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_8)}{b^2\alpha_{ot}(\rho_4-\chi_8)}}\right)}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}-$$
$$\frac{c^2n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_8)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_8)}{b^2\alpha_{ot}(\rho_4-\chi_8)}}\right)}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+\frac{c^2n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_9)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_9)}{b^2\alpha_{ot}(\rho_4-\chi_9)}}\right)}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}-$$
$$\frac{c^2n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_9)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_9)}{b^2\alpha_{ot}(\rho_4-\chi_9)}}\right)}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \qquad (128.51)$$

$$Q_{52}(x,y,t)=\frac{c\sqrt{\chi_1}e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_1)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\right]\sin\frac{n\pi y}{b}}{2\sqrt{\chi_1}(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c\sqrt{\chi_1}e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_1)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\right]\sin\frac{n\pi y}{b}}{2\sqrt{\chi_1}(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c\sqrt{\chi_8}e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\right]\sin\frac{n\pi y}{b}}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+$$
$$\frac{c\sqrt{\chi_8}e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\right]\sin\frac{n\pi y}{b}}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+$$

$$\frac{c\sqrt{\chi_9}e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\right]\sin\frac{n\pi y}{b}}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}+$$
$$\frac{c\sqrt{\chi_9}e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}(\chi_4-\chi_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\right]\sin\frac{n\pi y}{b}}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \qquad (128.52)$$

$$Q_{53}(x,y,t)=\frac{c^2n\pi e^{i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}b\right)\sin\frac{n\pi y}{b}}{b\sqrt{2i\sqrt{\chi_4}}\sqrt{(\chi_1-\chi_4)}(\chi_8+\chi_4)(\chi_9-\chi_4)}+\frac{c^2n\pi e^{-i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}b\right)\sin\frac{n\pi y}{b}}{ib\sqrt{2i\sqrt{\chi_4}}\sqrt{(\chi_1-\chi_4)}(\chi_8+\chi_4)(\chi_9-\chi_4)}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}b\right)\sin\frac{n\pi y}{b}}{b\sqrt{2i\sqrt{\chi_1}}\sqrt{(\chi_4-\chi_1)}(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c^2n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}b\right)\sin\frac{n\pi y}{b}}{ib\sqrt{2i\sqrt{\chi_1}}\sqrt{(\chi_4-\chi_1)}(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)}-\frac{c^2n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)}-\frac{c^2n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)} \qquad (128.53)$$

$$Q_{54}(x,y,t)=\frac{c^2n\pi e^{i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_4)}{b^2\alpha_{ot}(\rho_4-\chi_4)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)}{b\sqrt{2i\sqrt{\chi_4}}\sqrt{(\chi_1-\chi_4)}(\chi_8+\chi_4)(\chi_9-\chi_4)(i\sqrt{\chi_4}+\rho_8)(i\sqrt{\chi_4}-\rho_8)}-\frac{c^2n\pi e^{-i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_4)}{b^2\alpha_{ot}(\rho_4-\chi_4)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)}{ib\sqrt{2i\sqrt{\chi_4}}\sqrt{(\chi_1-\chi_4)}(\chi_8+\chi_4)(\chi_9-\chi_4)(\rho_8-i\sqrt{\chi_4})(\rho_8+i\sqrt{\chi_4})}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)}{b\sqrt{2i\sqrt{\chi_1}}\sqrt{(\chi_4-\chi_1)}(\chi_8+\chi_1)(\chi_9-\chi_1)(i\sqrt{\chi_1}+\rho_8)(i\sqrt{\chi_1}-\rho_8)}-\frac{c^2n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)}{ib\sqrt{2i\sqrt{\chi_1}}\sqrt{(\chi_4-\chi_1)}(\chi_8+\chi_1)(\chi_9-\chi_1)(\rho_8-i\sqrt{\chi_1})(\rho_8+i\sqrt{\chi_1})}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_8)}{b^2\alpha_{ot}(\rho_4-\chi_8)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)(i\sqrt{\chi_8}+\rho_8)(i\sqrt{\chi_8}-\rho_8)}-\frac{c^2n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_8)}{b^2\alpha_{ot}(\rho_4-\chi_8)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)(\rho_8-i\sqrt{\chi_8})(\rho_8+i\sqrt{\chi_8})}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_9)}{b^2\alpha_{ot}(\rho_4-\chi_9)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)(i\sqrt{\chi_9}+\rho_8)(i\sqrt{\chi_9}-\rho_8)}-\frac{c^2n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_9)}{b^2\alpha_{ot}(\rho_4-\chi_9)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)(\rho_8-i\sqrt{\chi_9})(\rho_8+i\sqrt{\chi_9})}+$$
$$\frac{c^2n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_9)}{b^2\alpha_{ot}(\rho_4-\chi_9)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)(\rho_8-i\sqrt{\chi_9})(\rho_8+i\sqrt{\chi_9})}-\frac{c^2n\pi e^{-i\rho_8\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\rho_8)}{b^2\alpha_{ot}(\rho_4-\rho_8)}}\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_8)}{(\chi_4-\rho_8)}}\right)}{2b\rho_8\sqrt{(\rho_8^2+\chi_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_8)(\rho_8^2+\chi_9)}} \qquad (128.54)$$

$$Q_{55}(x,y,t)=\frac{c\sqrt{\chi_1}e^{i\sqrt{\chi_1}t}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)e^{-i\sqrt{\chi_1}/c}\right]}{2\sqrt{\chi_1}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{c\sqrt{\chi_1}e^{-i\sqrt{\chi_1}t}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_1)}{b^2\alpha_{ot}(\rho_4-\chi_1)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)e^{i\sqrt{\chi_1}/c}\right]}{2\sqrt{\chi_1}\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+\frac{c\sqrt{\chi_8}e^{i\sqrt{\chi_8}t}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_8)}{b^2\alpha_{ot}(\rho_4-\chi_8)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)e^{-i\sqrt{\chi_8}/c}\right]}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+$$
$$\frac{c\sqrt{\chi_8}e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_8)}{b^2\alpha_{ot}(\rho_4-\chi_8)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)e^{i\sqrt{\chi_8}/c}\right]}{2\sqrt{\chi_8}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+\frac{c\sqrt{\chi_9}e^{i\sqrt{\chi_9}t}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_9)}{b^2\alpha_{ot}(\rho_4-\chi_9)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)e^{-i\sqrt{\chi_8}/c}\right]}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}+$$
$$\frac{c\sqrt{\chi_9}e^{-i\sqrt{\chi_9}t}\cos\left(\sqrt{\frac{z_1(\rho_8-\chi_9)}{b^2\alpha_{ot}(\rho_4-\chi_9)}}\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)e^{i\sqrt{\chi_8}/c}\right]}{2\sqrt{\chi_9}\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \qquad (128.55)$$

$$Q_{56}(x,y,t)=\frac{c^2n\pi e^{i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\sin\frac{n\pi y}{b}}{b\sqrt{2i\sqrt{\chi_4}\sqrt{(\chi_1-\chi_4)}(\chi_8+\chi_4)(\chi_9-\chi_4)\,(i\sqrt{\chi_4}+\rho_8)(i\sqrt{\chi_4}-\rho_8)}}+\frac{c^2n\pi e^{-i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\sin\frac{n\pi y}{b}}{ib\sqrt{2i\sqrt{\chi_4}\sqrt{(\chi_1-\chi_4)}(\chi_8+\chi_4)(\chi_9-\chi_4)\,(\rho_8-i\sqrt{\chi_4})(\rho_8+i\sqrt{\chi_4})}}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\sin\frac{n\pi y}{b}}{b\sqrt{2i\sqrt{\chi_1}\sqrt{(\chi_4-\chi_1)}(\chi_8+\chi_1)(\chi_9-\chi_1)\,(i\sqrt{\chi_1}+\rho_8)(i\sqrt{\chi_1}-\rho_8)}}+\frac{c^2n\pi e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\sin\frac{n\pi y}{b}}{ib\sqrt{2i\sqrt{\chi_1}\sqrt{(\chi_4-\chi_1)}\,(\chi_8+\chi_1)(\chi_9-\chi_1)\,(\rho_8-i\sqrt{\chi_1})(\rho_8+i\sqrt{\chi_1})}}+$$
$$\frac{c^2n\pi e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)\,(i\sqrt{\chi_8}+\rho_8)(i\sqrt{\chi_8}-\rho_8)}+\frac{c^2n\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)(\chi_1-\chi_8)}(\chi_9-\chi_8)\,(\rho_8-i\sqrt{\chi_8})(\rho_8+i\sqrt{\chi_8})}+$$

$$\frac{c^2n\pi e^{i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)\,(i\sqrt{\chi_9}+\rho_8)(i\sqrt{\chi_9}-\rho_8)}+\frac{c^2n\pi e^{-i\sqrt{\chi_9}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\frac{n\pi y}{b}}{2ib\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)(\chi_1-\chi_9)}(\chi_8-\chi_9)\,(\rho_8-i\sqrt{\chi_9})(\rho_8+i\sqrt{\chi_9})}+$$
$$\frac{c^2n\pi e^{i\rho_8\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_8)}{(\chi_4-\rho_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_8)}{(\chi_4-\rho_8)}}\right)\sin\frac{n\pi y}{b}}{2b\rho_8\sqrt{(\rho_8^2+\chi_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_8)(\rho_8^2+\chi_9)}}-\frac{c^2n\pi e^{-i\rho_8\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_8)}{(\chi_4-\rho_8)}}\,b\right)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\rho_8)}{(\chi_4-\rho_8)}}\right)\sin\frac{n\pi y}{b}}{2b\rho_8\sqrt{(\rho_8^2+\chi_4)(\rho_8^2+\chi_1)(\rho_8^2+\chi_8)(\rho_8^2+\chi_9)}} \qquad (128.56)$$

$$Q_{57}(x,y,t)=\frac{ce^{i\sqrt{\chi_1}t}(\chi_4-\chi_1)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)e^{-i\sqrt{\chi_1}/c}\right]\sin\frac{n\pi y}{b}}{2\,(\chi_8+\chi_1)(\chi_9-\chi_1)}$$
$$+\frac{ce^{-i\sqrt{\chi_1}t}(\chi_4-\chi_1)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\,b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)e^{i\sqrt{\chi_1}/c}\right]\sin\frac{n\pi y}{b}}{2\,(\chi_8+\chi_1)(\chi_9-\chi_1)}+$$
$$\frac{ce^{i\sqrt{\chi_8}t}(\chi_4-\chi_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)e^{-i\sqrt{\chi_8}/c}\right]\sin\frac{n\pi y}{b}}{2\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+$$
$$\frac{ce^{-i\sqrt{\chi_8}t}(\chi_4-\chi_8)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\,b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)e^{i\sqrt{\chi_8}/c}\right]\sin\frac{n\pi y}{b}}{2\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)}+$$
$$\frac{ce^{i\sqrt{\chi_9}t}(\chi_4-\chi_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)e^{-i\sqrt{\chi_8}/c}\right]\sin\frac{n\pi y}{b}}{2\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)}+$$
$$\frac{ce^{-i\sqrt{\chi_9}t}(\chi_4-\chi_9)\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\,b\right)\left[1-\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)e^{i\sqrt{\chi_8}/c}\right]\sin\frac{n\pi y}{b}}{2\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \qquad (128.57)$$

$$
\begin{aligned}
Q_{58}(x,y,t) = {} & \frac{icn\pi\sqrt{\chi_1}e^{i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_1)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}b\right)\sin\frac{n\pi y}{b}}{2\sqrt{2i\sqrt{\chi_1}}(\chi_8+\chi_1)(\chi_9-\chi_1)} + \\
& \frac{icn\pi\sqrt{\chi_1}e^{-i\sqrt{\chi_1}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_1)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_1)}{(\chi_4-\chi_1)}}b\right)\sin\frac{n\pi y}{b}}{2\sqrt{2i\sqrt{\chi_1}}(\chi_8+\chi_1)(\chi_9-\chi_1)} + \\
& \frac{icn\pi\cancel{\sqrt{\chi_8}}e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_8)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\sin\frac{n\pi y}{b}}{4\cancel{\sqrt{\chi_8}}\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)} + \frac{icn\pi e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\sqrt{(\chi_4-\chi_8)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\sin\frac{n\pi y}{b}}{4\sqrt{(\chi_1-\chi_8)}(\chi_9-\chi_8)} + \\
& \frac{ce^{i\sqrt{\chi_9}t}\sqrt{(\chi_4-\chi_9)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\sin\frac{n\pi y}{b}}{4\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} + \frac{ce^{-i\sqrt{\chi_9}t}\sqrt{(\chi_4-\chi_9)}\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\sin\frac{n\pi y}{b}}{4\sqrt{(\chi_1-\chi_9)}(\chi_8-\chi_9)} \qquad (128.58)
\end{aligned}
$$

$$
\begin{aligned}
Q_{59}(x,y,t) = {} & \frac{c^2n^2\pi^2e^{i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}b\right)\sin\frac{n\pi y}{b}}{2i\sqrt{\chi_4}(\chi_8+\chi_4)(\chi_9-\chi_4)} - \\
& \frac{c^2n^2\pi^2e^{-i\sqrt{\chi_4}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_4)}{(\chi_4-\chi_4)}}b\right)\sin\frac{n\pi y}{b}}{2i\sqrt{\chi_4}(\chi_8+\chi_4)(\chi_9-\chi_4)} \\
& + \frac{c^2n^2\pi^2e^{i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\sin\frac{n\pi y}{b}}{2i\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)}(\chi_9-\chi_8)} - \frac{c^2n^2\pi^2e^{-i\sqrt{\chi_8}\left(t-\frac{1}{c}\right)}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_8)}{(\chi_4-\chi_8)}}b\right)\sin\frac{n\pi y}{b}}{2i\sqrt{\chi_8}\sqrt{(\chi_4-\chi_8)}(\chi_9-\chi_8)} + \\
& \frac{c^2n^2\pi^2e^{i\sqrt{\chi_9}t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\sin\frac{n\pi y}{b}}{2i\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)}(\chi_8-\chi_9)} - \frac{c^2n^2\pi^2e^{-i\sqrt{\chi_9}t}\cos\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}\right)\sin\left(\frac{n\pi}{b}\sqrt{\frac{(\chi_1-\chi_9)}{(\chi_4-\chi_9)}}b\right)\sin\frac{n\pi y}{b}}{2i\sqrt{\chi_9}\sqrt{(\chi_4-\chi_9)}(\chi_8-\chi_9)} \qquad (128.59)
\end{aligned}
$$

Thus, the composite solution which is uniformly valid in the entire domain of the highly prestressed orthotropic rectangular plate acted upon by moving force is

$$W(x,y,t) = W_0(x,y,t) + \varepsilon W_1(x,y,t) \qquad (129)$$

Remarks on theory

Equations (58) and (84) are the leading order and the first order (transformed) solutions of the moving mass problem while (122) and (127) respectively are for the moving force problem. The leading order and the first order solutions are combined in equations (87) and (129) to form the composite solution which is uniformly valid in the entire domain of the highly prestressed orthotropic rectangular plate.

It is observed from the leading order and first order solutions that fully clamped highly prestressed orthotropic plate traversed by moving concentrated masses and resting on Pasternak foundation reached the resonant state whenever

$$C_{s1}(m,\pi) = \sqrt{\frac{(m\pi)^2(\beta_1^2+G_0)-K_0}{(m\pi)^2\Gamma_0\delta(y-y_0)}} \qquad (130-1)$$

$$C_{s2}(m,\pi) = \frac{1}{m\pi}\sqrt[4]{\frac{\beta_1^2+G_0}{\alpha_{ot}}} \qquad (130-2)$$

$$C_{s3}(m,\pi) = \frac{J_1}{J_2} \pm \frac{1}{2}\sqrt{J_3-4J_4} \qquad (130-3)$$

where

$$J_1 = -(m\pi)^2 \propto_{ot} - 1 - \Gamma_0\delta(y-y_0)$$

$$J_2 = 2m\pi\Gamma_0\delta(y-y_0)$$

$$J_3 = \left(\frac{(m\pi)^2 \propto_{ot} - 1 - \Gamma_0\delta(y-y_0)}{m\pi\Gamma_{0\delta(y-y_0)}}\right)^2$$

$$J_4 = \frac{(m\pi)^2\beta_1^2 + (m\pi)^2G_0 - K_0}{(m\pi)^2\Gamma_{0\delta(y-y_0)}}$$

Similarly, the system under moving force operates at a frequency which equals the natural frequency to display an enhanced oscillation at the following critical speeds

$$C_{r1} = \sqrt{\frac{\beta_2^2 + G_0}{m^2\pi^2}} \qquad (131-1)$$

,

$$C_{r2} = \sqrt{\frac{K_0 - m^2\pi^2(\beta_1^2 - G_0)}{m^2\pi^2(1 - m^2\pi^2 \propto_{ot})}} \qquad (131-2)$$

$$C_{r3} = \frac{b}{n\pi}\sqrt{\frac{\beta_1^2 + G_0}{4\beta_2^2 + 4G_0 + \propto_{ot}}} \qquad (131-3)$$

$$C_{r4} = \frac{\beta_1^2 + G_0}{m\pi \propto_{ot}} \qquad (132-4)$$

$$C_{r5} = \frac{\gamma_2^2}{m\pi \propto_{ot}} \qquad (131-5)$$

$$C_{r6} = n\pi\sqrt{\frac{(m^2\pi^2 \propto_{ot} - 1)(\beta_2^2 + G_0) + m^2\pi^2 \propto_{ot} (\beta_1^2 - G_0)}{\propto_{ot} (m^2\pi^2 \propto_{ot} - 1)}} \qquad (131-6)$$

From (130) and (131), it is observed that the resonance conditions of the plate are dependent on anisotropic prestress, mass ratio, rotatory inertia correction factor, shear and foundation moduli.

At this juncture, the critical speeds for the system of a highly prestressed orthotropic rectangular plate under the action of travelling masses or moving force are sought. The critical speeds that exist in the dynamical system are as given above.

Numerical simulation

In order to illustrate the analytical results, for instance, the orthotropic rectangular plate is taken to be of length 1.0 m and width 0.9 m. Other values used for the analysis are b = 0.65 m , g = 9.81, $\pi = \frac{22}{7}$, $y_0 = 0.2$, $c = 8.128\frac{m}{s}$. The values of the prestress in the x-direction range between 0 and 30 000 N. The critical velocities are plotted against prestress for various values of other parameters. The process is repeated for mass ratio, shear and foundation moduli in turn.

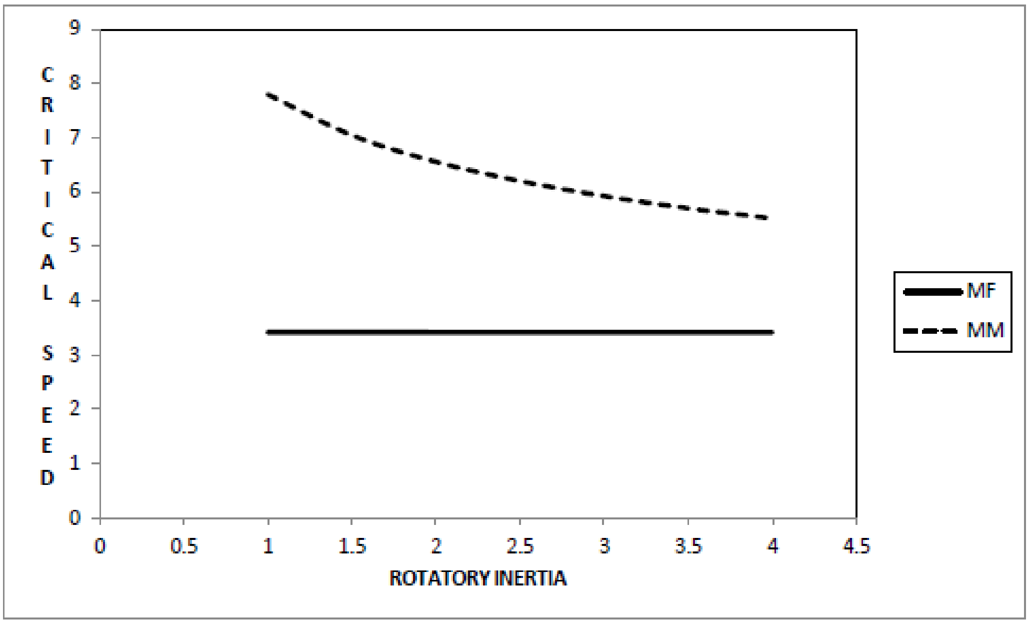


Figure 1: Comparison of critical speed of moving force and moving mass cases for highly prestressed rectangular plate with varying rotatory inertia correction factor.

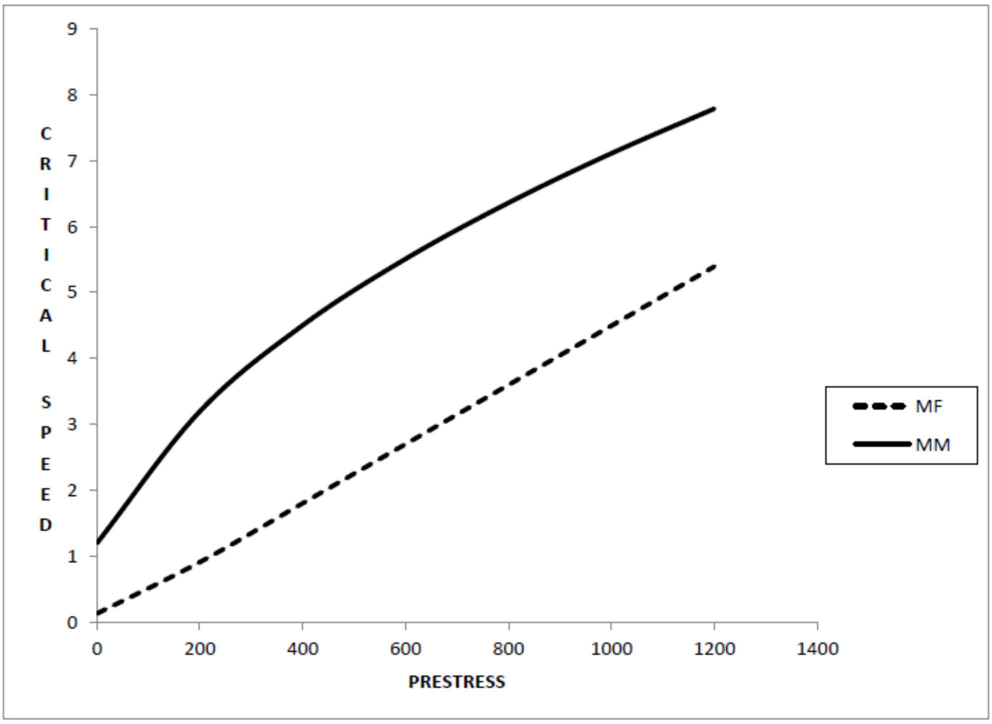


Figure 2: Comparison of critical speed of moving force and moving mass cases for highly prestressed rectangular plate

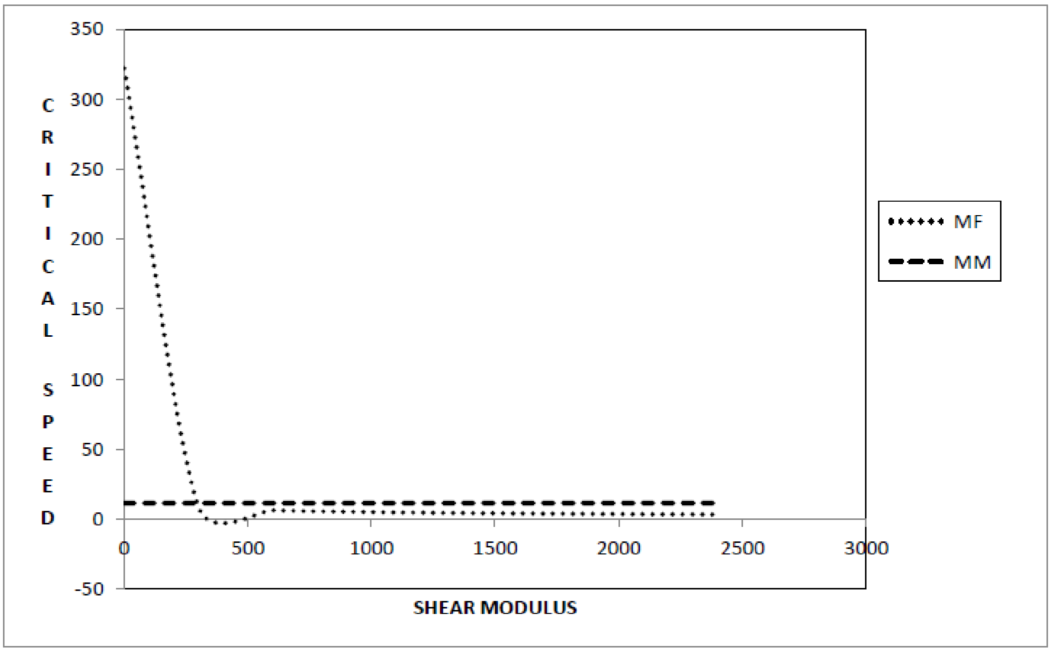
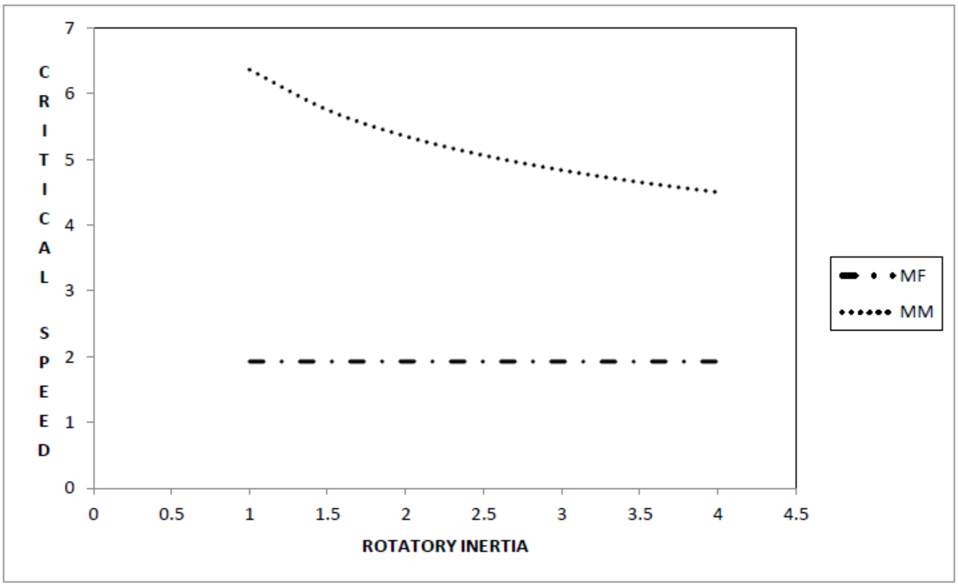


Figure 3: Comparison of critical speed of moving force and moving mass cases for highly prestressed rectangular plate for varying shear modulus.



Figgure 4: Comparison of critical speed of moving force and moving mass cases for highly prestressed rectangular plate with varying rotatory inertia correction factor at fixed value of prestress

Conclusion

This study concerns the problem of the dynamic response of a highly prestressed orthotropic rectangular plate resting on elastic foundation and traversed by either concentrated moving mass or moving force. The problem is governed by a fourth order non-homogeneous partial differential equation. For the purpose of solution, the equation of motion of the plate problem is presented in a non-dimensional form. It is observed that a small parameter multiplied the highest derivatives in the governing differential equation. In accordance with the principle that the behaviour of solutions is governed by the highest order term, the choice of a suitable method of solution is made. Thus, this type of dynamical problem is usually amenable to singular perturbation technique. In particular the Method of Matched Asymptotic Expansions (MMAE) is used. This technique constructs outer (core) and inner (boundary layer) solutions that are valid in partly disjoint domains. These solutions are then matched using the Van Dyke matching process. Consequently, an approximate uniformly valid solution in the entire domain of definition of the rectangular plate is obtained with the rigorous use of Laplace transformation, finite Fourier sine transform and the Cauchy residue theorem. This solution is analysed and some resonance conditions are obtained for the dynamical system.

A numerical simulation is carried out and the study reveals the following results:

(a) the leading order solution and the first order correction are affected by the mass ratio, anisotropic prestress, shear and foundation modulus. However, the effects of rotatory inertia correction factor is not appreciably pronounced.

(b) As the prestress or shear modulus increases , the critical speed of the orthotropic rectangular plate traversed by moving concentrated mass also increases. Also, as the rotatory inertia or mass ratio increases, the critical speed decreases.

.(c) there may be more than one resonance condition in a dynamical system such as this which involves plate flexure under moving concentrated masses or moving forces.

(d) the smaller the mass ratio the better the improvement in critical speed.

(e) The influence of prestress, mass ratio, shear and foundation moduli on the response of the highly prestressed orthotropic rectangular plate has been indicated by this study.

Finally, this work has showcased the use of a valuable method for the solution of this class of dynamical problems.

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