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Reverse Geometry in Squaring The Circle

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Abstract:
We use reverse geometry to determine a geometric magnitude what allows us to show how the square that circumscribe the given circle determine the square whose area is equal to that of the given circle.
Key Word: *Pi, area, circle square.*

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I. Introduction

The problem of squaring the circle with a ruler and a compass has kept many geometers awake at night.

In 1882 Ferdinand Lindemann [1] shows that π is transcendental and therefore it is not possible to construct a straight-line segment of length π with a ruler and a compass, and consequently squaring the circle with ruler and compass is impossible.

However, it is possible to construct circles whose area is equal to that of a given square, using the Egyptian Pi $\left(\frac{22}{7}\right)$.

Dr. Sarva Jagannadha Reddy [2, 3] has done extensive work on squaring the circle using the Gayatri Pi what is an exact number $\left(\frac{14-\sqrt{2}}{4}\right)$ and he has achieved the squaring of the circle with simplicity and elegance.

But squaring the circle with the official Pi (π) has resisted for some time.

The purpose of this work is to show how the square that circumscribes the given circle determines the square whose area is equal to the area of the given circle, using reverse geometry, and at the same time to show the geometrical magnitude, very close to that given by Ramanujan, that allow us to square the circle.

II. Procedure

Let R be the radius of given circle, D its diameter and centered at O. Built a square of side D that circumscribe that circle, as shown in the Figure 1

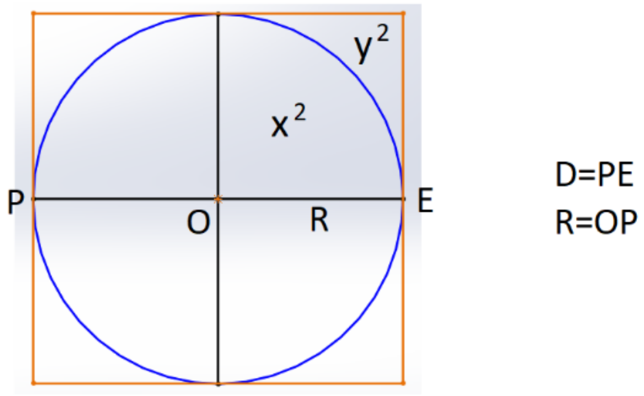

Figure 1

Let $x^2 = \frac{1}{4}\pi R^2$, $y^2 = (1 - \frac{\pi}{4})R^2$

It is clear that

$$x^2 + y^2 = R^2 \qquad \text{……….} \ (2.1)$$

The equation (2.1) strongly suggests the Pythagorean theorem.

From the obvious:

$$4y^2 = (4 - \pi)R^2$$

$$D^2 - 4y^2 = \pi R^2$$

But: $\pi R^2 = L^2$ as required

Whit L the side of the wanted square.

Then: $D^2 - 4y^2 = L^2$

And in this way the four areas between the given circle and the circumscribed square determine the area of the square whose side is L.

III. Reverse geometry

Suppose now that we know the square of side L (Figure 2).

Then: $$x = \frac{L}{2} = \frac{\sqrt{\pi}}{2}R \qquad \text{……….} \ (3.1)$$

$$y = \sqrt{1 - \frac{\pi}{4}}\, R \qquad \text{……….} \ (3.2)$$

$$2y = \sqrt{4 - \pi}\ R \qquad \text{……….} \ (3.3)$$

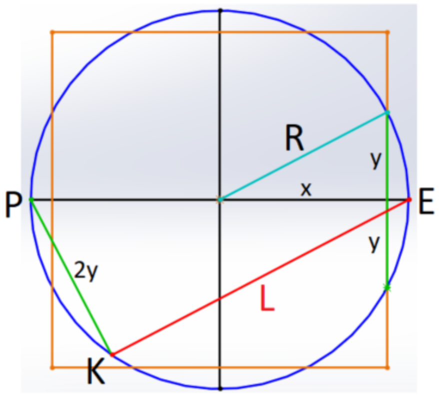

Figure 2.

For the right triangle of sides x, y and R, the Pythagorean theorem states that

$$x^2 + y^2 = R^2 \qquad \text{……………} \ (3.4)$$

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Where: $x^2 = \frac{1}{4}\pi R^2$, $y^2 = (1 - \frac{\pi}{4})R^2$

Now, following the Ramanujan's wonderful geometric construction.

Let: $PK = 2y$ ………. (3.5)

$PK = \sqrt{4 - \pi}\ R$ ………. (3.6)

Then, for the right triangle PEK:

$D^2 - PK^2 = L^2$ ………. (3.7)

And the quadrature is perfect, as is should be.

Now with the help of the compass and supported at the point P draw an arc of circumference of radius PK.

Let OM be the tangent at the point M, α the angle OPM.

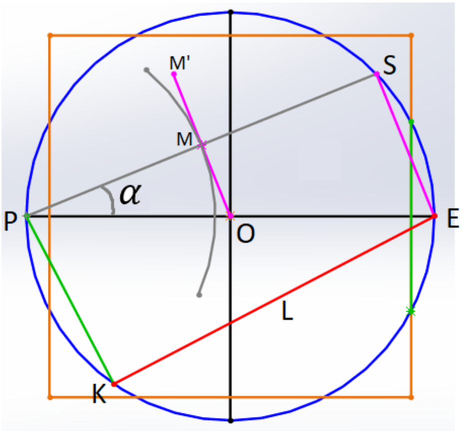

Figure 3

Then: $cos\ \alpha = \frac{PM}{R} = \frac{PK}{R} = \sqrt{4 - \pi}$

$sen\ \alpha = \sqrt{\pi - 3}$

$OM = R\ sen\ \alpha = \sqrt{\pi - 3}\ R$

$OM' = \sqrt{\pi - 3}\ 2R = \sqrt{\pi - 3}\left(\frac{6}{3}R\right)$

$OM' = \frac{\sqrt{36\pi - 108}}{3}\ R$

$OM' = \frac{\sqrt{5.097335529\ldots}}{3}\ R$

This geometric magnitude is fundamental in squaring the circle [4]. Reverse geometry has allowed us to build the length OM', and make it visible.

The magnitude OM' is hidden between the given circle and the square that circumscribe it.

$D^2 - 16y^2 = (\pi - 3)D^2$

$\sqrt{D^2 - 16y^2} = \sqrt{\pi - 3}\, D = \sqrt{\pi - 3}\ (2R)$

$$\sqrt{D^2 - 16y^2} = \frac{\sqrt{36\pi - 108}}{3}\ R$$

$$\sqrt{D^2 - 16y^2} = \frac{\sqrt{5.097335529\ldots}}{3}\ R\ \equiv OM'$$

Now if in Figure 3 we place $ES = OM'$ and follow Ramanujan's [5], magnificent geometric construction, Join P with S, draw OM parallel to ES, place $PM = \frac{PS}{2}$, $PM = PK$ and join E with K, then the we achieve the squaring of the given circle with the official π , $i.e.$:

$$D^2 - PK^2 = L^2$$

And in this way the square that circumscribe the given circle determines the square whose area is equal to that of the given circle.

IV. Conclusion

If we place $PK^2 = 4y^2$ squaring the circle with the official π is achieved.

The geometrical magnitude:

$$ES = \frac{\sqrt{5.097335529\ldots}}{3}\ R \equiv OM'$$

Determine the quadrature of the given circle with the official π.

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