

# Study of Coupled Fixed Point Results in Complex Valued Dislocated Metric Spaces

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**Abstract:** In this paper, a coupled fixed-point theorem for maps satisfying contractive conditions in perspective of complex valued dislocated metric spaces have been established. Our main result generalizes, extends and improves the some known result in the existing literature of [10] and [16]. Also provide an example in support of our main result.

**Key words:** Coupled fixed point, Complex valued dislocated metric spaces, Contractive condition.

**MSC:**47H10, 54 H25

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## I. Introduction:

The concept of couple fixed point was extended by Guo and Lakshmikantham [1] and proved the existence theorem of the coupled fixed point for monotone iterations techniques. He also gave application to the initial value problems or ordinary differential equations with dis continuous right-hand sides. After that, they introduced the concept for partially order set. Bhaskar and Lakshmikantham [2] studied the existence and uniqueness of a coupled fixed point results in partially ordered metric space, using a weak counteractivity type of assumption. Coupled fixed point is also extended in different spaces like metric space, G-metric space, b-metric space, partially ordered metric space, fuzzy metric spaces, cone metric space, complex valued metric space, complex valued b- metric space etc.

The concept of Bhaskar and Lakshmikantham [2] are also extended to tripled fixed point by Berinde and Borcut [3] and to quadrupled fixed point by Karapinar [4]. After that, coupled fixed point theorem have vast application in the recent development of the fixed-point theory. Some of the results are noted in [11],[12],[13],[14],[15], [16].

The concept of dislocated metric space was introduced by Hitzler, P. and Seda, A [5, 6] and generalized Banach contraction principle [7]. Since Banach contraction principle is the most useful way for solution of existence problems in mathematical analysis. Dislocated has a significant role in topology, logical programming electronics engineering.

On the other hand, in 2011, Azam, et al. [8] and Rauzkard, et al. [9] defined the concept of complex valued metric space and gave common fixed point result for mappings. Naturally, this new idea can be utilized to define complex valued normed spaces and complex valued inner product spaces, which in turn, offer a wide scope for further investigations. Recently, in (2018) E. Ozgur and Karaca, Ismet [10] introduced complex valued dislocated metric spaces and prove Banach, Kannan and Chatterjea type fixed point theorems in this space.

In this paper, we prove and generalize some coupled fixed-point theorem for contractive type mappings in the perspective complex valued dislocated metric spaces. Our results improve and generalize the comparable result in the existing literature of [10] and [16]

## II. Preliminaries:

In this section, we introduced the notion of complex valued dislocated metric space.

Let  $\mathbb{C}$  be the set of complex numbers and  $z_1, z_2 \in \mathbb{C}$ . Define a partial order  $\preceq$  on  $\mathbb{C}$  as follows:

$$z_1 \preceq z_2 \text{ if and only if } \operatorname{Re}(z_1) \leq \operatorname{Re}(z_2) \text{ and } \operatorname{Im}(z_1) \leq \operatorname{Im}(z_2).$$

It follows that  $z_1 \leq z_2$  if one of the following conditions is satisfied.

$$(C_1) \operatorname{Re}(z_1) = \operatorname{Re}(z_2) \text{ and } \operatorname{Im}(z_1) = \operatorname{Im}(z_2);$$

$$(C_2) \operatorname{Re}(z_1) < \operatorname{Re}(z_2) \text{ and } \operatorname{Im}(z_1) = \operatorname{Im}(z_2);$$

$$(C_3) \operatorname{Re}(z_1) = \operatorname{Re}(z_2) \text{ and } \operatorname{Im}(z_1) < \operatorname{Im}(z_2);$$

$(C_4) \operatorname{Re}(z_1) < \operatorname{Re}(z_2)$  and  $\operatorname{Im}(z_1) < \operatorname{Im}(z_2)$ .

In Particular, we will write  $z_1 \lesssim z_2$  if  $z_1 \neq z_2$  and one of  $(C_2)$ ,  $(C_3)$  and  $(C_4)$  is satisfied and we will write  $x_1 < z_2$  if only  $(C_4)$  is satisfied.

**Remark 2.1.** we obtained that the following statements hold:

- (1) If  $a, b \in \mathbb{R}$  with  $a \leq b$ , then  $az < bz$  for all  $z \in \mathbb{C}$ .
- (2) If  $0 \lesssim z_1 \lesssim z_2$ , then  $|z_1| < |z_2|$ .
- (3) If  $z_1 \lesssim z_2$  and  $z_2 \lesssim z_3$ , then  $z_1 < z_3$ .

**Definition 2.2[2].** Let  $X$  be a non – empty set. A Mapping  $d : X \times X \rightarrow \mathbb{C}$  satisfies the following conditions:

- (i)  $d(x, y) = d(y, x)$ ;
- (ii)  $d(x, y) = d(y, x) = 0$  implies  $x = y$ ;
- (iii)  $d(x, y) \lesssim d(x, z) + d(z, y)$  for all  $x, y, z \in X$ .

Then  $d$  is said to be complex valued dislocated metric on  $X$  and the pair  $(X, d)$  is called a complex valued dislocated metric space.

**Example 2.2.** Let  $d: X \times X \rightarrow \mathbb{C}$  be defined by  $d(x, y) = \max\{x, y\}$ , where  $X = \mathbb{C}$ . It is clear that  $d$  is a complex valued dislocated metric.

**Definition 2.3[10].** Let  $(X, d)$  be a complex-valued dislocated metric space. Let  $\{x_n\}$  be a sequence in  $X$  and  $x \in X$ . Then

- (a)  $\{x_n\}$  is called complex valued dislocated convergent in  $(X, d)$  and converges to  $x$ , if for every  $\epsilon > 0$  there exist  $n_0 \in \mathbb{N}$  such that  $d(x_n, x) < \epsilon$  for all  $n > n_0$  and is denoted by  $x_n \rightarrow x$  or  $\lim x_n \rightarrow x$  or  $\lim x_n = x$ .
- (i)  $\{x_n\}$  is called Cauchy sequence in  $(X, d)$  if  $\lim_{n \rightarrow \infty} d(x_n, x_{n+p}) = 0$  for all  $p > 0$ .
- (ii) If every complex valued Cauchy sequence in  $X$  is converges to some  $x \in X$ . Then  $(X, d)$  is called a complete complex valued dislocated metric Space.

**Lemma 2.5[10].** Let  $(X, d)$  be a complex valued dislocated metric space and let  $\{x_n\}$  be a sequence in  $X$ . Then  $\{x_n\}$  converges to  $x$  if and only if  $|d(x_n, x)| \rightarrow 0$  as  $n \rightarrow \infty$ .

**Lemma 2.6.** Let  $(X, d)$  be a complex valued dislocated metric space and let  $\{x_n\}$  be a sequence in  $X$ . Then  $\{x_n\}$  is a complex valued Cauchy sequence if and only  $|d(x_n, x_{n+m})| \rightarrow 0$  as  $n \rightarrow \infty$ .

**Definition 2.7.** Let  $(X, d)$  be a complex valued dislocated metric space. Then an element  $(x, y) \in X \times X$  is said to be a coupled fixed point of the mapping  $T: X \times X \rightarrow \mathbb{C}$  if  $T(x, y) = x$  and  $T(y, x) = y$ .

### III. Main Results

Our Main results as follows:

**Theorem 3.1.** Let  $(X, d)$  be a complex valued dislocated metric space. Suppose that the mapping  $T: X \times X \rightarrow X$  satisfies

$$d(T(x, y), T(u, v)) \lesssim \alpha d(x, u) + \beta d(y, v) + \gamma [d(x, T(x, y)) + d(u, T(u, v))]$$

for all  $x, y, u, v \in X$ , where  $\alpha, \beta, \gamma$  are non-negative constants with  $\alpha + \beta + 2\gamma < 1$ .  $T$  has a unique coupled fixed point.

Proof: Let  $x_0, y_0 \in X$ . and set  $x_1 = T(x_0, y_0)$ ,  $y_1 = T(y_0, x_0)$ ,  
 $= \dots = \dots$

$$x_{n+1} = T(x_n, y_n), \quad y_{n+1} = T(y_n, x_n)$$

From (3.1), we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(T(x_{n-1}, y_{n-1}), T(x_n, y_n)) \\ &\lesssim \alpha d(x_{n-1}, x_n) + \beta d(y_{n-1}, y_n) + \gamma [d(x_{n-1}, T(x_{n-1}, y_{n-1})) + d(x_n, T(x_n, y_n))] \\ &\lesssim \alpha d(x_{n-1}, x_n) + \beta d(y_{n-1}, y_n) + \gamma [d(x_{n-1}, x_n) + d(x_n, x_{n+1})] \\ (1 - \gamma)d(x_n, x_{n+1}) &\lesssim (\alpha + \gamma)d(x_{n-1}, x_n) + \beta d(y_{n-1}, y_n) \\ d(x_n, x_{n+1}) &\lesssim \frac{(\alpha + \gamma)}{1 - \gamma} d(x_{n-1}, x_n) + \frac{\beta}{1 - \gamma} d(y_{n-1}, y_n) \quad \dots \end{aligned} \tag{3.1.1}$$

Similarly, we can show that

$$d(y_n, y_{n+1}) \lesssim \frac{(\alpha + \gamma)}{1 - \gamma} d(y_{n-1}, y_n) + \frac{\beta}{1 - \gamma} d(x_{n-1}, x_n) \quad \dots \tag{3.1.22}$$

No form (1) and (2), we get

$$\begin{aligned} d(x_n, x_{n+1}) + d(y_n, y_{n+1}) &\lesssim \frac{(\alpha + \beta + \gamma)}{1 - \gamma} [d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] \\ &\lesssim h [d(x_{n-1}, x_n) + d(y_{n-1}, y_n)], \end{aligned}$$

where  $\frac{(\alpha + \beta + \gamma)}{1 - \gamma} = h < 1$ .

Therefore, by letting  $d(x_n, x_{n+1}) + d(y_n, y_{n+1}) = d_n$ , we have

$$d_n = h d_{n-1} \dots \tag{3.1.3}$$

In general, we have  $n = 0, 1, 2, \dots$

$$d_n \lesssim h d_{n-1} \lesssim h^2 d_{n-2} \lesssim \dots \lesssim h^n d_0.$$

Now, for all  $m > n$

$$d(x_n, x_m) \lesssim d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \dots + d(x_{m-1}, x_m)$$

and

$$d(y_n, y_m) \lesssim d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{m-1}, y_m)$$

Therefore, we have

$$d(x_n, x_m) + d(y_n, y_m) \lesssim h^n d_0 + h^{n+1} d_0 + \dots + h^{m-1} d_0.$$

Thus, we have

$$|d(x_n, x_m) + d(y_n, y_m)| \leq \frac{h^n}{1-h} |d_0|$$

Since  $h < 1$ , taking limit as  $n \rightarrow \infty$ , then  $\frac{h^n}{1-h} |d_0| \rightarrow 0$ , i.e.  $|d(x_n, x_m) + d(y_n, y_m)| \rightarrow 0$ .

From lemma 2.6, we conclude that  $\{x_n\}$  and  $\{y_n\}$  are Cauchy sequence in  $(X, d)$ . So, by completeness of  $X$  there exists  $u, v \in X$  such that  $\lim_{n \rightarrow \infty} x_n = u$  and  $\lim_{n \rightarrow \infty} y_n = v$ . i.e  $x_n \rightarrow u$  and  $y_n \rightarrow v$ . Next, we claim that  $(u, v)$  coupled fixed point of  $T$ . Then we have

$$\begin{aligned} d(T(u, v), u) &\lesssim d(T(u, v), x_{n+1}) + d(x_{n+1}, u) \\ &= d(T(u, v), T(x_n, y_n)) + d(x_{n+1}, u) \\ &\lesssim \alpha d(x_n, u) + \beta d(y_n, v) + \gamma [x_n, T(x_n, y_n) + d(u, T(u, v))] + d(x_{n+1}, u) \\ &\lesssim \alpha d(x_n, u) + \beta d(y_n, v) + \gamma [d(x_n, x_{n+1}) + d(u, T(u, v))] + d(x_{n+1}, u) \\ &\lesssim \frac{\alpha + \gamma}{1 - \gamma} [\alpha d(x_n, u)] + \frac{\beta}{1 - \gamma} d(y_n, v) + \frac{1}{1 - \gamma} d(x_{n+1}, u). \end{aligned}$$

Since  $x_n \rightarrow u$  and  $y_n \rightarrow v$ , then we obtain  $|d(T(u, v), u)| \leq 0$  for  $n \rightarrow \infty$ . i.e.  $d(T(u, v), u) = 0$  and hence  $T(u, v) = u$ . Similarly, we have  $(u, v) = v$ . Thus  $(u, v)$  is a coupled fixed point of  $T$ . Finally, we need to prove the uniqueness of fixed point.

If  $(u^*, v^*)$  is another coupled fixed point of  $T$  Then

$$\begin{aligned} d(u^*, u) &= d(T(u^*, v^*), T(u, v)) \\ &\lesssim \alpha d(u^*, u) + \beta d(v^*, v) + \gamma [d(u^*, T(u^*, v^*)) + d(u, T(u, v))] \end{aligned}$$

and

$$\begin{aligned} d(v^*, v) &= d(T(v^*, u^*), T(v, u)) \\ &\lesssim \alpha d(v^*, v) + \beta d(u^*, u) + \gamma [d(v^*, T(v^*, u^*)) + d(v, T(v, u))]. \end{aligned}$$

Thus, we have

$$d(u^*, u) + d(v^*, v) \lesssim (\alpha + \beta) [d(u^*, u) + d(v^*, v)],$$

Which implies that

$$|1 - d(u^*, u) + d(v^*, v)| \leq 0 \text{ as } n \rightarrow \infty,$$

i.e.  $d(u^*, u) + d(v^*, v) = 0 \Rightarrow d(u^*, u) = d(v^*, v) = 0 \Rightarrow u^* = u, v^* = v$ .

Hence  $(u^*, v^*) = (u, v)$ . Therefore,  $T$  has a unique coupled fixed point. The proof is completed.

From theorem 3.1, with  $\alpha = \beta$  and  $\gamma = 0$ , we get the following corollary:

**Corollary 3.2.** Let  $(X, d)$  be a complex valued dislocated metric space. Suppose that the mapping  $T: X \times X \rightarrow X$  satisfies

$$d(T(x, y), T(u, v)) \lesssim \alpha [d(x, u) + d(y, v)]$$

for all  $x, y, u, v \in X$ , where  $\alpha, \beta, \gamma$  are non-negative constants with  $\alpha + \beta + 2\gamma < 1$ .  $T$  has a unique coupled fixed point.

**Example 3.3.** Let  $X = \mathbb{C}$  and defined  $d: X \times X \rightarrow \mathbb{C}$  by  $d(x, y) = i|x - y|$  is a complete complex valued  $d$ -metric space. Consider the mapping  $T: X \times X \rightarrow X$  with  $T(x, y) = \frac{x+y}{6} i$ . Then  $T$  satisfies the contractive condition (3.2) for  $\frac{1}{3} \lesssim \alpha \lesssim \frac{1}{2}$ , i.e.  $m$

$$d(T(x, y), T(u, v)) \lesssim \alpha [d(x, u) + d(y, v)].$$

Hence  $(0, 0)$  is the unique coupled fixed point of  $T$ .

**Theorem 3.4.** Let  $(X, d_{cq})$  be a complex valued dislocated metric space. Suppose that the mapping  $T: X \times X \rightarrow X$  satisfies

$$d(T(x, y), T(u, v)) \lesssim \sigma \max [d(x, u), d(y, v), d(x, T(x, y)), d(u, T(u, v))]. \dots (3.4.1)$$

for all  $x, y, u, v \in X$ , where  $\sigma \in [0, 1)$ .  $T$  has a unique coupled fixed point.

**Proof:** Let  $x_0, y_0 \in X$  and  $x_1 = T(x_0, y_0), y_1 = T(y_0, x_0)$

$$\begin{aligned} &= \dots = \dots \\ x_{n+1} &= T(x_n, y_n), y_{n+1} = T(y_n, x_n). \text{ Then from (3.4.1), we have} \\ d_{cq}(x_{n+1}, x_{n+2}) &= d_{cq}(T(x_n, y_n), T(x_{n+1}, y_{n+1})) \\ &\lesssim \sigma \max [d_{cq}(x_n, x_{n+1}), d_{cq}(y_n, y_{n+1}), d_{cq}(x_n, T(x_n, y_n)), \\ &\quad d_{cq}(x_{n+1}, T(x_{n+1}, y_{n+1}))] \\ &\lesssim \sigma \max [d_{cq}(x_n, x_{n+1}), d_{cq}(y_n, y_{n+1}), d_{cq}(x_n, x_{n+1}), d_{cq}(x_{n+1}, x_{n+2})] \end{aligned}$$

$$\lesssim \sigma \max [d_{cq}(x_n, x_{n+1}), d_{cq}(y_n, y_{n+1})],$$

Which implies that

$$|d_{cq}(x_{n+1}, x_{n+2})| \leq \sigma \max \{|d_{cq}(x_n, x_{n+1})|, |d_{cq}(y_n, y_{n+1})|\}. \dots \tag{3.4.2}$$

and

$$|d_{cq}(y_{n+1}, y_{n+2})| \leq \sigma \max \{|d_{cq}(y_n, y_{n+1})|, |d_{cq}(x_n, x_{n+1})|\}. \dots\dots \tag{3.4.3}$$

From (3.4.2) and (3.4.3), we get

$$\max\{|d_{cq}(x_{n+1}, x_{n+2})|, |d_{cq}(y_{n+1}, y_{n+2})|\} \leq \sigma \max \{|d_{cq}(x_n, x_{n+1})|, |d_{cq}(y_n, y_{n+1})|\}.$$

Continuing this process, we obtain

$$\begin{aligned} \max \{|d_{cq}(x_n, x_{n+1})|, |d_{cq}(y_n, y_{n+1})|\} &\leq \sigma \max \{|d_{cq}(x_{n-1}, x_n)|, |d_{cq}(y_{n-1}, y_n)|\} \\ &\leq \sigma^2 \max \{|d_{cq}(x_{n-2}, x_{n-1})|, |d_{cq}(y_{n-2}, y_{n-1})|\} \\ &\vdots \\ &\leq \sigma^n \max \{|d_{cq}(x_0, x_1)|, |d_{cq}(y_0, y_1)|\}. \end{aligned}$$

Therefore

$$\begin{aligned} \lim_{n \rightarrow \infty} \max \{|d_{cq}(x_n, x_{n+1})|, |d_{cq}(y_n, y_{n+1})|\} &= 0, \text{ which gives that,} \\ \lim_{n \rightarrow \infty} |d_{cq}(x_n, x_{n+1})| &= 0, \text{ and } \lim_{n \rightarrow \infty} |d_{cq}(y_n, y_{n+1})| = 0. \end{aligned}$$

Now  $n > m$ , we have

$$d_{cq}(x_n, x_m) \lesssim d_{cq}(x_n, x_{n-1}) + d_{cq}(x_{n-1}, x_{n-2}) + \dots + d_{cq}(x_{m+1}, x_m)$$

Which implies that

$$|d_{cq}(x_n, x_m)| \lesssim |d_{cq}(x_n, x_{n-1})| + |d_{cq}(x_{n-1}, x_{n-2})| + \dots + |d_{cq}(x_{m+1}, x_m)|.$$

As  $n, m \rightarrow \infty$ , we obtain that

$$\lim_{n, m \rightarrow \infty} |d_{cq}(x_n, x_m)| = 0.$$

Similarly, we can show that

$$\lim_{n, m \rightarrow \infty} |d_{cq}(y_n, y_m)| = 0,$$

which implies that  $\{x_n\}$  and  $\{y_n\}$  are Cauchy sequence in  $(X, d_{cq})$ . Since complex valued metric space  $(X, d_{cq})$  is complete, there exists  $u, v \in X$  such that

$$\{x_n\} \rightarrow u \text{ and } \{y_n\} \rightarrow v \text{ as } n \rightarrow \infty.$$

Next, we claim that  $(u, v)$  is coupled fixed point of  $T$ . Now let us consider

$$\begin{aligned} d_{cq}(u, T(u, v)) &\lesssim d_{cq}(u, x_{n+1}) + d_{cq}(x_{n+1}, T(u, v)) \\ &= d_{cq}(u, x_{n+1}) + d_{cq}(T(x_n, y_n), T(u, v)) \\ &\lesssim d_{cq}(u, x_{n+1}) + \sigma \max \{d_{cq}(x_n, u), d_{cq}(y_n, v), d_{cq}(x_n, T(x_n, y_n)), \\ &\quad d_{cq}(u, T(u, v))\} \\ &= d_{cq}(u, x_{n+1}) + \sigma \max \{d_{cq}(x_n, u), d_{cq}(y_n, v), d_{cq}(x_n, x_{n+1}), \\ &\quad d_{cq}(u, T(u, v))\}, \end{aligned}$$

which implies that

$$|d_{cq}(u, T(u, v))| \leq |d_{cq}(u, x_{n+1})| + \sigma \max \{|d_{cq}(x_n, u)|, |d_{cq}(y_n, v)|, |d_{cq}(x_n, x_{n+1})|, |d_{cq}(u, T(u, v))|\} \rightarrow 0. \text{ As } n \rightarrow \infty.$$

Implies that,  $|d_{cq}(u, T(u, v))| \leq 0$ . Since  $\sigma \in [0, 1)$ . Therefore,  $|d_{cq}(u, T(u, v))| = 0$ .

Thus,  $u = T(u, v)$ . Similarly, we can prove that  $v = T(v, u)$ . Hence  $(u, v)$  is coupled fixed point of  $T$ . Now to prove uniqueness, if  $(u^*, v^*)$  is another fixed point of  $T$ , then

$$\begin{aligned} d_{cq}(u, u^*) &= d_{cq}(T(u, v), T(u^*, v^*)) \\ &\lesssim \sigma \max \{d_{cq}(u, u^*), d_{cq}(v, v^*), d_{cq}(u, T(u, v)), d_{cq}(u^*, T(u^*, v^*))\} \\ &\lesssim \sigma \max \{d_{cq}(u, u^*), d_{cq}(v, v^*), d_{cq}(u, u), d_{cq}(u^*, u^*)\}. \end{aligned}$$

Therefore,

$$d_{cq}(u, u^*) \lesssim \sigma \max \{d_{cq}(u, u^*), d_{cq}(v, v^*)\}.$$

Implies that,  $|d_{cq}(u, u^*)| \leq \sigma \max \{|d_{cq}(u, u^*)|, |d_{cq}(v, v^*)|\} \dots \dots \dots \tag{3.4.4}$

Similarly, we can prove that

$$|d_{cq}(v, v^*)| \leq \sigma \max \{|d_{cq}(u, u^*)|, |d_{cq}(v, v^*)|\} \dots \dots \dots \tag{3.4.5}$$

From (3.4.4) and (3.4.5), we get

$$\max \{|d_{cq}(u, u^*)|, |d_{cq}(v, v^*)|\} \leq \sigma \max \{|d_{cq}(u, u^*)|, |d_{cq}(v, v^*)|\}.$$

Since  $0 \leq \sigma < 1$ , we have  $\max\{|d_{cq}(u, u^*)|, |d_{cq}(v, v^*)|\} = 0$ . So,  $d_{cq}(u, u^*) = 0$  and  $d_{cq}(v, v^*) = 0$ .

Therefore  $u = u^*$  and  $v = v^*$ . implies that  $(u, v) = (u^*, v^*)$  is unique coupled fixed point of  $T$ .

**Example 3.5.** Let  $X = \mathbb{C}$  and defined  $d: X \times X \rightarrow \mathbb{C}$  by  $d(x, y) = \max\{|x, y|\}(1 + i)$ . Then  $(X, d)$  is a complex valued dislocated metric space. Consider the mapping  $T: X \times X \rightarrow X$  defined by  $T(x, y) = \frac{|x-y|}{2}$ . For  $x, y, u, v \in X$ , we have

$$\begin{aligned} d(T(x, y), T(u, v)) &= \frac{1}{2} \max\{|x - y|, |u - v|\}(1 + i) \\ &= \frac{1}{2} \max\{x - y, y - x, u - v, v - u\}(1 + i) \\ &= \frac{1}{2} \max\{x, y, u, v\}(1 + i) \\ &= \frac{1}{2} \max\{d(x, y), d(u, v)\} \\ &\lesssim \frac{1}{2} \max\{d(x, y), d(u, v), d(x, T(x, y)), d(u, T(u, v))\}. \end{aligned}$$

Thus  $T$  has a unique coupled fixed point. Hence  $(0, 0)$  is the unique fixed point of  $T$ .

#### IV. Conclusion

we prove and generalize some coupled fixed-point theorem for contractive type mappings in the perspective complex valued dislocated metric spaces. Our results improve and generalize the comparable result in the existing literature of [10] and [16].

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