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Analysis of Physiological Non-Newtonian Blood Flow through Abdominal Aortic Aneurysm

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Abstract: *A numerical simulation is performed to look into Newtonian physiological flows pattern on three dimensional idealized double abdominal aortic aneurysms (DAAA). The wall vessel is fixed as rigid during calculation.Physiological and parabolic velocity profiles are set out to fix the conditions of inlet boundaries of artery. On the other way, physiological waveform is an significant part of compilation and it is successfully done by the act of utilizing of Fourier series having sixteen harmonics. The inquiry has a Reynolds number range of 77 to 894. Low Reynolds number* $k-\omega$ *model has been used as governing equation. The investigation has been carried out to characterize two Non Newtonian constitutive equations of blood, namely, (i) Carreau and (ii) Cross models. The Newtonian model has also been investigated to study the physics of fluid. The results of Newtonian model are compared with the Non-Newtonian models. The numerical results are presented in terms of pressure, wall shear stress distributions and the streamlines contours. At early systole pressure differences between Newtonian and Non-Newtonian models are observed at aneurysm regions. In the case of wall shear stress, some differences between Newtonian and Non-Newtonian models are observed when the flows are minimum such as at early systole or diastole.*

Keywords- *Atherosclerosis, Carreau model, Cross model, Physiological flow, Aneurysm, Viscoelastic fluid*.

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I. Introduction

Blood flow over normal physiological situation is a vital field of study, as blood flow under diseased circumstances. The majority of deaths are caused from cardiovascular diseases in developed countries and most of them are connected with some form of irregular blood flow in arteries. Hemodynamic is the study of physical forces involved in blood circulation. It refers to physiological factors governing the flow of blood in circulatory system. Aneurysm is a balloonlike dilation found on the walls of a blood vessel or a sac formed by the localized dilatation of the wall of an artery or in a vein, or in the heart. Unprocessed aneurysm may rupture under insistent internal pressure, causing fatality or severe disability. Even an unruptured aneurysm can lead to damage by inter-rupting the flow of blood or by impinging on the wall of the vessel, in some cases eroding nearby blood vessel, organs, or bone. Abdominal Aortic Aneurysms (AAA) is seen most often in large arteries such as the iliac, femoral, popliteal, carotid or renal arteries. Hemodynamic parameters are considered to be responsible for initiating growth and rupture of aneurysms. In arterial walls, if increased blood flow occurs, it leads to enlargement of vessel diameter and reduction of shear forces. To understand aneurysm behavior, the flow dynamics have been studied in multitude experimental models. Multiple aneurysms can grow from the same location of an artery, and the interaction between these aneurysms raises the risk of rupture.

Patients with various aneurysm geometries have been accomplished by several authors in the fact is that [1, 2]. Blood flow has been numerically analyzed by Wille [3] in moderately dilated rigid blood vessels to trace stream lines of the flow. Similarly, the path lines of the flow particles were analyzed by Perktold [4,5]. Moreover, Khanafer Khalil,et al. [6] has analyzed the turbulent flow effect and the wall stress on the aortic aneurysm. Double aneurysm has also been analyzed for different dilations by Guzma´n, A., Moraga, N., and Amon [7]. Knowledge on flow partition, pressure and shear stress may provide a better understanding of the relationship between the fluid dynamics in pulsatile blood flow and arterial diseases.

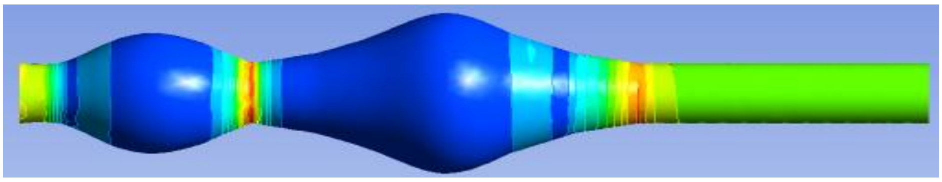
As the flow in aneurysms is complex with presence of vortices, secondary flows and strong amplification of instability so the aim of this study is to investigate the flow dynamics for single and multiple abdominal aneurysms using computational fluid dynamics (CFD). The numerical simulation of Newtonian and non-Newtonian blood flow patterns in AAA would be simulated. It is performed in the 3D model of AAA with aneurysm along with its peripheral branches for systolic and diastolic cardiac phase. Limitation on the amount of the dilation is ignored. Since arterial wall is gently elastic, we neglect the wall dispensability.

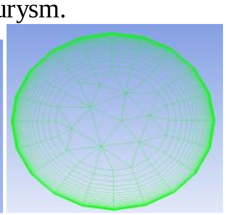
Numerical calculation is patterns of blood flow and hemodynamic stresses are performed in a two-aneurysm in the eatery of Abdominal Aortic Aneurysms (AAAs). This simulation is to analyzed Newtonian and Non-Netonian physiological flows characteristic on three dimensional idealized Abdominal Aortic Aneurysms (AAAs). The artery of wall vessel is consider as rigid during simulation. Parabolic and physiological velocity profiles are set the conditions of inlet boundaries of artery. In other way, physiological waveform is an important part of compilation and it is successfully done by utilization of Fourier series having sixteen harmonics. The investigation has a Reynolds number range of 77 to 896.Low Reynolds number k-ω model has been used as governing equation. The investigation has been carried out to characterize the flow behavior of blood in Abdominal Aortic Aneurysms (AAAs). The Newtonian model has been used to study the physics of fluid. The findings of the model are thoroughly observe there behavioral sequence of flows. The numerical results were presented in terms of velocity, pressure, wall shear stress distributions and cross sectional velocities as well as the streamlines contour. Aneurysms disturb the normal pattern of blood flow through the artery as dilated area. At aneurysm region velocity and peak Reynolds number rapidly decrease and Reynolds number reach transitional and turbulent region. These flow fluctuation and turbulence have bad effect to the blood vessel which makes to accelerate the progress of aneurysms. .

II. Meterials And Method

Geometry

Three dimensional double abdominal aortic aneurysms (DAAA) are used as geometry for this study shown in Fig. 1 (a). The geometry of generated model in this study has ainlet diameter of the blood vessel AAA (1.7cm), diameter of the 1st aneurysm (3.5cm), diameter of the 2nd aneurysm (4.6cm),a length(19.975cm).The wall is considered to be rigid. The flow field mesh consists of 206788 nodes and 415536 elements for the geometry. Fig. 1(a),(b) shows the mesh in cross-sectional plane of a abdominal aortic aneurysm.





a b

Figure.1.(a) Representation of the axisymmetric model of the Double aneurysm of Abdominal aorta,b) mesh in cross sectional plane of a Abdominal aorta.

Blood Properties

Blood is taken as fluid where the blood is considered incompressible. The density of the blood is 1050kg/m3. In a Newtonian model for the blood viscosity, the value of μ is treated as a constant usually set t $\mu = 3.45 \times 10^{-3}\ pa.s$. But when blood is considered non-Newtonian fluid then the viscosity of the blood is calculated from two models such as Carreau model and cross model.

Cross Model for the Non-Newtonian Blood Viscosity

For non-Newtonian fluid Malcolm M. Cross [15] proposed a shear rate dependent viscosity model called Cross model. The Cross model is defined by

$$\mu = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\frac{\dot{\gamma}}{\gamma_c})^m]^{-1}$$

Carreau Model for the Non-Newtonian Blood Viscosity

TheCarreau model was proposedbyPierre Carreau [16] is defined by

$$\mu = \mu_\infty + (\mu_0 - \mu_\infty)[1 + (\gamma_c \dot{\gamma})^2]^{n-1/2}$$

Governing Equation

Due to enlargement blood passes through the aneurysm region with low velocity create high pressure. Flow velocity at post aneurysm region increases but pressure of that region decreases. Neither laminar flow modeling nor standard two-equation models are suitable for this kind of blood flow. So, Wilcox low-Returbulence model is more acceptable for flow analysis found by Varghese and Frankel [14]. Therefore low Re $k-\omega$ turbulent model is taken for calculation.

Now, the Navier-Stokes equation can be given by-

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial^2 u_i}{\partial x_j \partial x_i} \qquad (2)$$

Since each term of this equation is time averaged, the equation is referred to as a Reynolds averaged Navier-Stokes (RANS) equation. During this procedure several additional unknown parameters appear which require additional equations to be introduced as turbulence models. The set of RANS equations are-

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x} = 0 \qquad (3)$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\left(\frac{\partial u_1}{\partial x_1}\right)\right)\right] + \frac{\partial}{\partial t}\left(-\rho \overline{u_i^{/} u_j^{/}}\right) \qquad (4)$$

In this equation $-\rho \overline{u_i^{/} u_j^{/}}$ is an additional term known as the Reynolds's stress tensor, which can be approximated by using Boussinesq's hypothesis-

$$-\rho \overline{u_i^{/} u_j^{/}} = \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\left(\rho k + \mu_t \frac{\partial u_k}{\partial x_k}\right)$$

(5)

Eddy viscosity can be modeled as a function of the turbulence kinetic energy (k) and specific dissipation rate (ω); therefore it is referred to as the two-equation turbulent model. The turbulence kinetic energy k and specific dissipation rate ω of standard $k-\omega$ model are determined by following two equations:

The k equation: $$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left(\Gamma_k \frac{\partial k}{\partial x_i}\right) + G_k - Y_k + S_k$$

(6)

The ω equation: $$\frac{\partial}{\partial t}(\rho\omega) + \frac{\partial}{\partial x_i}(\rho\omega u_i) = \frac{\partial}{\partial x_j}\left(\Gamma_\omega \frac{\partial \omega}{\partial x_i}\right) + G_\omega - Y_\omega + S_\omega$$

(7)

In these equations, G_k represents the generation of turbulence kinetic energy due to mean velocity gradients. G_ω represents the generation of ω.Γ_k and Γ_ω represent the effective diffusivity of k and ω, respectively. Y_k and Y_ω represent the dissipation of k and ω due to turbulence. S_k and S_ω are user-defined source terms.

A low Reynolds number correction factor controls the influence on the overall structure of the flow field depending upon local conditions, and it is given as-

$$\alpha^* = \alpha_\infty^*\left(\frac{\alpha_0^* + \mathrm{Re}_t / R_k}{1 + \mathrm{Re}_t / R_k}\right)$$

(8)

Where, $\mathrm{Re}_t = \frac{\rho k}{\mu\omega}, R_k = 8, \alpha_0^* = \frac{\beta_i}{4}, \beta_i = 0.072, \alpha_\infty^* = 1.$

Closure Coefficient for the Transitional $k-\omega$ Model are-

$\alpha_\infty^* = 1, \alpha_\infty = 0.52, \alpha_0 0.1111, \beta_\infty^* = 0.09, \beta_i = 0.072, R_k = 8 \; and \; R_\beta = 8$

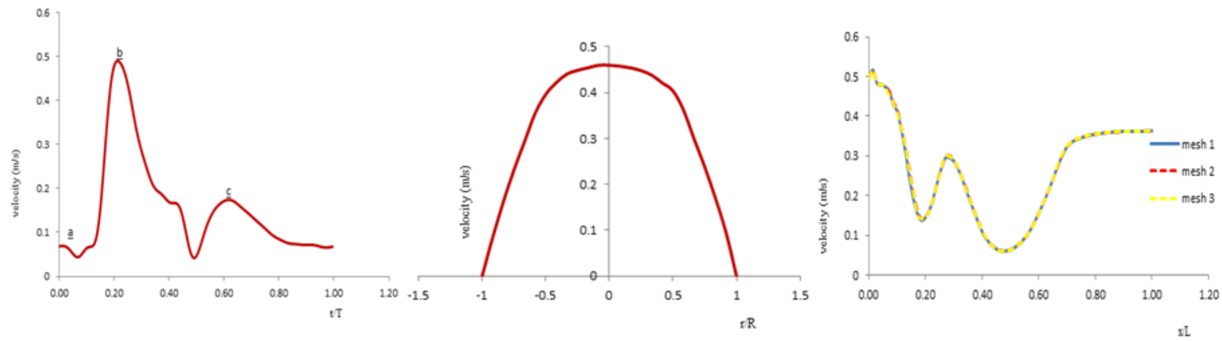


Figure. 2(a) Oscillatory physiological waveform, (b) parabolic inlet velocity profile and (c) velocity distribution in AAA from different mesh sizes.

Boundary condition

Usually, the blood flow through aorta is an unsteady phenomenon. Since, the blood flow to be fully developed at inlet region, so inlet velocity profile is taken as parabolic. Again, sinusoidal or oscillatory physiological velocity profile has been imposed at inlet for unsteady condition. For this purpose an user defined function has been written in C++ compiler that demonstrate the unsteady parabolic nature of velocity using the relation given in following equations, $u_x = u\left(1 - \frac{y^2 + z^2}{radius^2}\right)$. Where $u = \sum_{n=1}^{n=16} (A_n \cos(\omega t) + B_n \sin(\omega t))$; when velocity profile is physiological. Here, A_n and B_n are the coefficients. Where, Reynolds number varies from 77 to 894

TABLE 1. Harmonic coefficients for pulsatile waveform shown in Figure 2(a).

n	A_n	B_n	n	A_n	B_n	n	A_n	B_n
0	0.166667	0	6	-0.01735	0.01915	12	-0.00341	0.005463
1	-0.03773	0.0985	7	-0.00648	0.002095	13	-0.00194	0.000341
2	-0.10305	0.012057	8	-0.01023	-0.0078	14	-0.00312	-0.00017
3	0.007745	-0.06763	9	0.008628	-0.00663	15	0.000157	-0.00299
4	0.025917	-0.02732	10	0.002267	0.001817	16	0.001531	0.000226
5	0.037317	0.024517	11	0.005723	0.003352			

Since cardiac pulse cycle is 0.82sec, ω is found from the calculation $\omega = \frac{2\pi}{0.82} = 7.66\ \mathrm{rad/sec}$. Figure. 3.5(a), 3.5(b) and 3.5(c) show the sinusoidal, physiological waveform and parabolic inlet velocity profile respectively. In Figure 3.5(a, b), a, b, and c represent the positions of early systole, peak systole, and diastole respectively.

Numerical scheme

The numerical simulations are performed by well-known software ANSYS Fluent 14.5. A pressure based algorithm is chosen as the solver type. This solver is generally selected for an incompressible fluid. As there is no heat transfer in the blood flow process energy equation is not solved. In solution methods the SIMPLE algorithm is selected for pressure-velocity coupling. First order upwind scheme is employed as a numerical scheme for discretization of the momentum equation. The time step is set to 0.00041 with 2000 number of total time steps. Maximum 10 iterations are performed per each time step.

Grid independence check

An extensive test is carried out with different sizes of mesh such as mesh 1 (200259 element), mesh2 (415536 element) and mesh3 (990556 element) respectively. Fig. 2(c) shows the center line velocity distributions for Abdominal Aortic Aneurysm with mentioned mesh sizes at peak systole (0.1804 sec). In all cases, the velocity distributions are same. It implies that the solution is grid independence.

III. Result And Discussion

In this study simulations have been conducted with a three dimensional idealized artery with DAAA. The investigation has been carried out to characterize two non Newtonian constitutive equations of blood, namely, (a) Carreau and (b) Cross models. The Newtonian model has also been investigated to study the physics of fluid. The results of Newtonian model are compared with the non-Newtonian models. The numerical results are presented in terms of pressure and wall shear stress distributions. The computational results are conducted to study the influence of aneurysm on the flow behavior. The flow parameters like pressure and wss are observed from longitudinal contours at specific instants of pulse cycle for comparing the flow variation. The discussion is categorized with the observations of flow variation starting from early systole, peak systole and diastole, respectively.

It is known that blood is Bingham plastic fluid. So the viscosity of blood will decrease with increase in shear rate and when shear rate will be greater than 100 then viscosity will be constant. The viscosity of Newtonian model is less than that of non-Newtonian model when shear rate is less than 100, but viscosity of all models is equal when shear rate is equal to 100. When Reynolds number is very low then pressure and WSS of Newtonian model will be less than that of non-Newtonian model but opposite scenario will be seen for velocity distribution. Since early systole and diastole there are comparatively low Reynolds numbers, the results of Newtonian and non-Newtonian condition may be different at early systole and diastole. On the other hand maximum Reynolds number is seen at peak systole. So the results of Newtonian and non Newtonian condition follow each other at peak systole.

Wall Shear Stress (WSS) is another important parameter to apprehend the condition of severity of arterial stenosis. It is also responsible for the growth of arterial diseases. WSS is defined as $\tau = \mu \frac{du}{dr}$, where μ is the viscosity and $\frac{du}{dr}$is the velocity gradient. So, WSS of Non-Newtonian fluid depends on the viscosity of the fluid and velocity gradient but WSS of Newtonian fluid does not depend on the viscosity because viscosity is a constant property of Newtonian fluid. Thus the results of WSS for Newtonian and Non-Newtonian model may be different.

Figure 3, 4, and 5 represent the distribution of wall shear stress for Non-Newtonian (Carreau & Cross) and Newtonian model at early systole, peak systole and diastole respectively. The wall shear stress distributions for all models are fairly similar. The difference in WSS magnitude is dependent on Reynolds number; therefore the largest difference occurs during the minimum flow such as early systole or diastole.

Figure 6, 7, and 8 reveal the centreline pressure distribution for Non-Newtonian (Carreau and Cross) and Newtonian models at early systole, peak systole and diastole respectively. At early systole significant pressure difference between Newtonian and Non-Newtonian models are observed at aneurysmregion. Pressure of the Non-Newtonian (Carreau and cross) models are higher than that of Newtonian model which is natural and meets our expectation. Due to high Reynolds number or high shear rate at peak systole,viscosity of Carreau, Cross and Newtonian model are same. Thus, pressure of the Non-Newtonian (Carreau and Cross) models follow the pressure of the Newtonian model. High pressure is noticed at the aneurysm region,and it continues after the throat region. At peak systole pressure of the Cross model mostly follow the pressure of the Newtonian model but very little difference is observed in Carreau model. No significant pressure fall is occurred due to very low Reynolds number.

The results of pressure and WSS distribution are discussed below with respective figure.

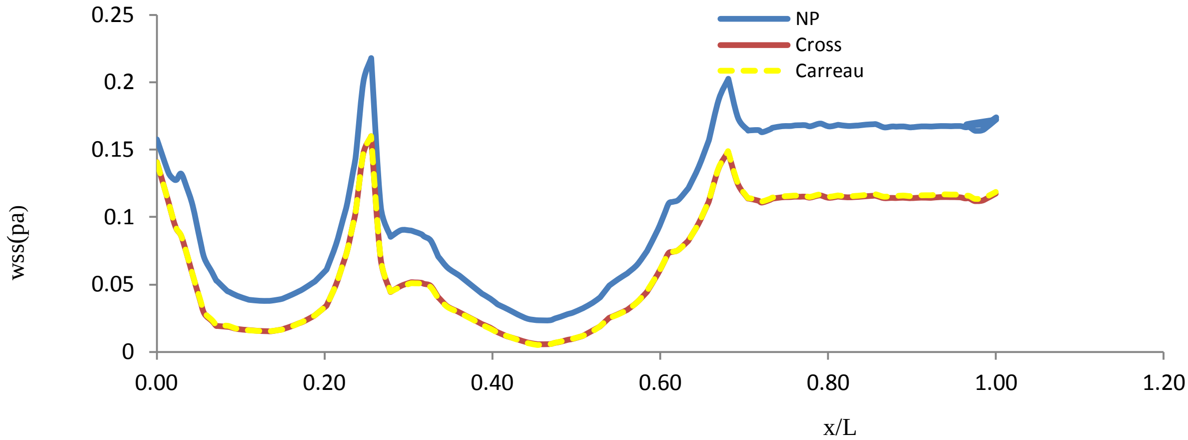


Figure 3 : Distribution of wall shear stress for Newtonian , Cross, and Carreau model at early systole.

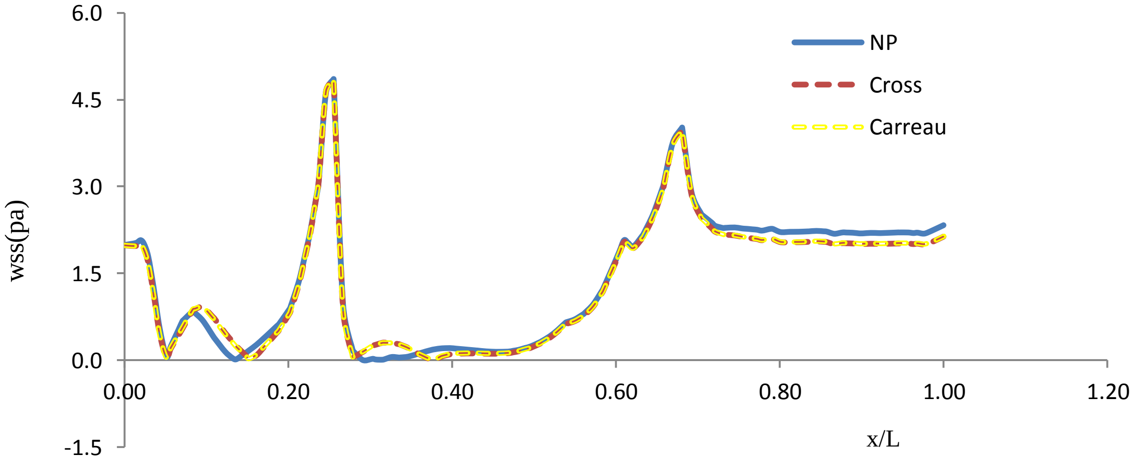


Figure 4: Distribution of wall shear stress for Newtonian , Cross, and Carreau model at peak systole.

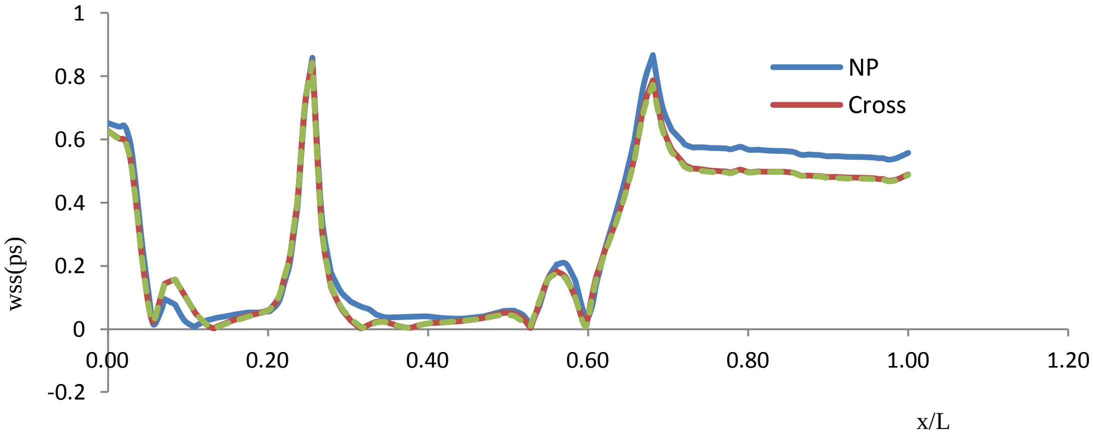


Figure 5 :Distribution of wall shear stress for Newtonian , Cross, and Carreau model at Diastole.

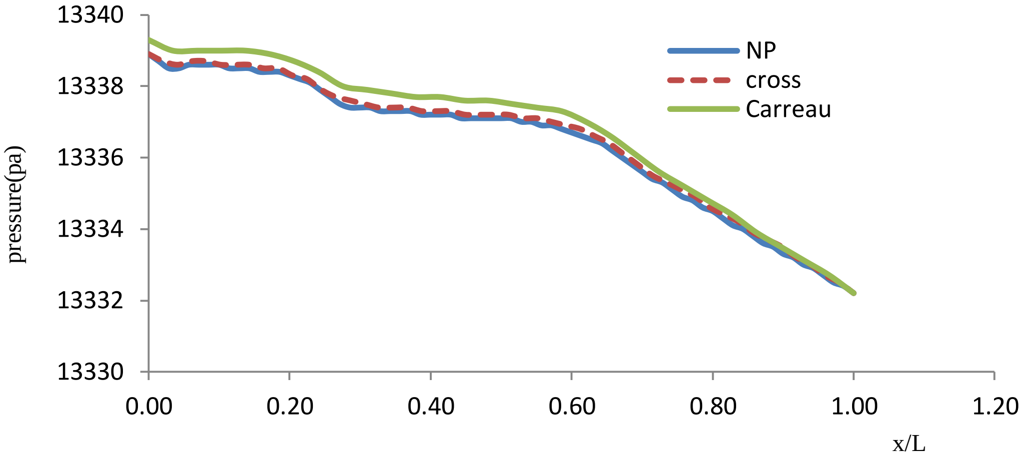


Figure 6: Pressure distribution at centerline for Newtonian, Cross and Carreau model at early systole.

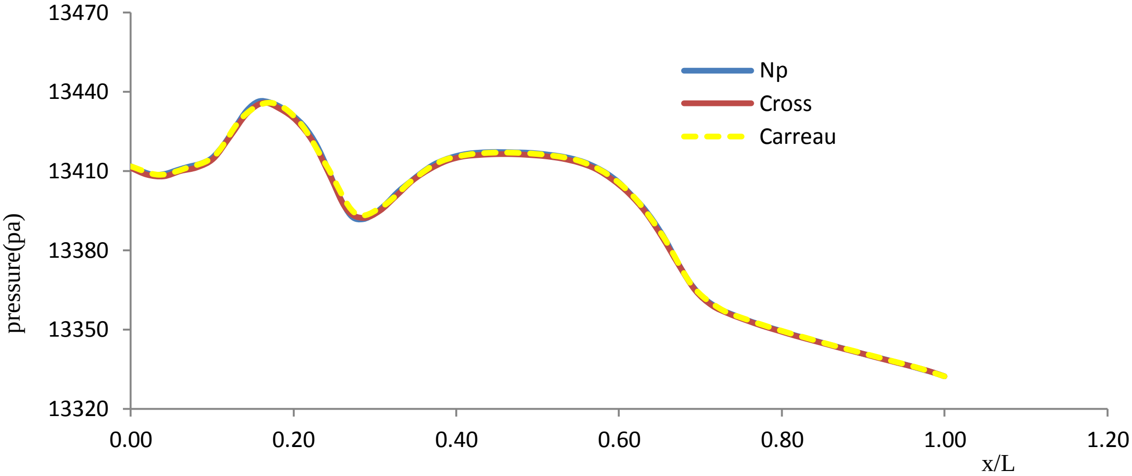


Figure 7: Pressure distribution at centerline for Newtonian, Cross and Carreau model at peak systole.

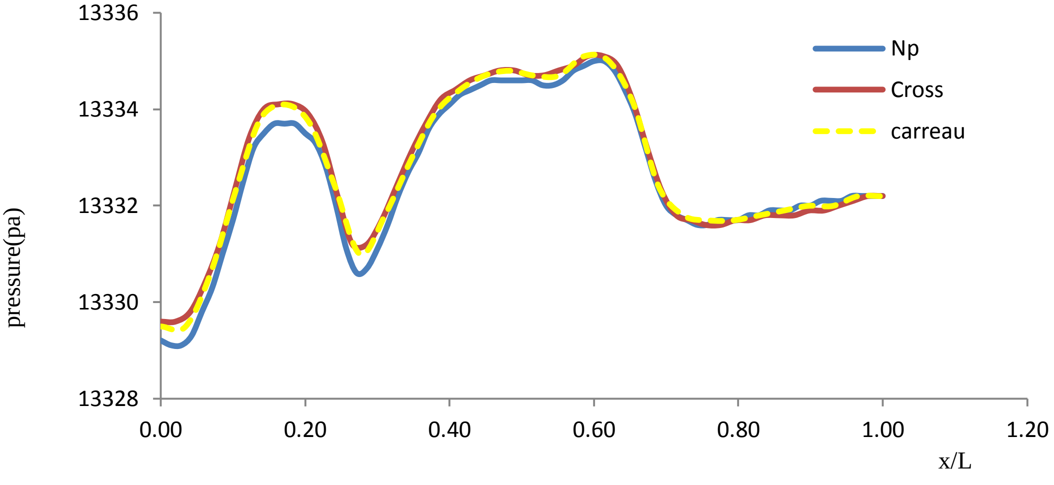


Figure 8: Pressure distribution at centerline for Newtonian, Cross and Carreau model at diastole

IV. Conclusion

In this study I want to present significant changes of flow behavior to pulsatile Newtonian and Non-Newtonian blood flow through abdominal aortic aneurysm. This is to provide a basic understanding of atherosclerosis. The blood is assumed to be incompressible, homogeneous and Newtonian or non-Newtonian, while artery is assumed to be a rigid wall. The transient analysis is performed using ANSYS-14.5. Since the blood flow to be fully developed at inlet region, so inlet velocity profile is taken as parabolic. Again, physiological velocity profile has been imposed at inlet for unsteady condition. The investigation has a Reynolds number range of 77 to 896 for physiological blood flow. As the heat transfer in the blood flow process is not considered, therefore energy equation is not solved. Pressure based solver, and finite volume method are used for calculations. Due to enlargement blood passes through the pre and post aneurysm region with high velocity, the flow velocity at aneurysm region decreases but pressure of that increases. So, moderate and severe abdominal aneurysm can produce highly disturbed flow characteristics. So, in the investigation of non-Newtonian modeling a AAA of 400% severity (by area) is taken to compare the results of two non-Newtonian models, (a) Carreau and (b) Cross models, with the results of Newtonian model..

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