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The Co-Infection Model of COVID19 and Bacterial Pneumoniaon Human Population.

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Abstract:
Corona Virus Disease is highly contagious respiratory disease that lead to the dangerous complications in the respiratory system. The virus weakens the body immunity making a way to some opportunistic diseases, pneumonia being highly common. The co-infection of COVID 19 and bacterial pneumonia is more dangerous because both attack the lungs blocking the respiratory system of the human body. The model was developed under the assumptions that human population is not constant, no vertical transmission of diseases and no vaccination of the human population. The basic reproduction numbers
were calculated and after the parameter values substitutions it was observed that RC = 0.02500 andR P = 0.0069 which indicate that one infected individual can infect on average less than one individual.
The equilibrium points are locally asymptotically stable for both disease free and endemic points. For the diseases impacts, it was observed that COVID 19 disease has positive impact on pneumonia infections. As COVID 19 infections increases the more the number of pneumonia infections according to the interpretationof the impacts basing on the basic reproduction numbers. Antibiotics, avoiding the congested areas, using sanitizers, washing hands with soap and running water and wearing mouth masks are recommended initiatives in controlling the co-infection.

Keywords: *Co-infection, COVID-19, Pneumonia, Endemic, Equilibrium Point, Reproduction number, Stability.*

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I. Introduction

A family of corona viruses causes the Coronavirus Disease 2019 (COVID 19) to human and some animals which cause respiratory infections that lead to different complications in the respiratory system. The disease is highly infectious caused by the virus which were known after the December 2019 outbreak in Wuhan City of China.(Guan et al., 2020) Before the virus were unknown in their existence. The spread of this disease is through small droplets from the nose or mouth dispersed by a person infected by the viruses during coughing, exhaling or sneezing. The viruses can exist on the surface of the object for a couple of hours. When a person touches such surface, the part of the body that touches the surface carries the virus (mostly hands). Then if the contaminatedpart of the body (hands mostly) comes into contact with the nose or mouth, the viruses get directly into therespiratory system of the human body. (Sohrabi et al., 2020) The incubation period for the coronaviruses is atmost fourteen (14) days. That, an infected person with the virus can show up the symptoms from the first day to two weeks of infection. Some precautions can be taken to reduce chances for coronavirus disease 2019 infections like; staying indoors, washing hands with alcohol-based hand rub, soap and water. Avoiding unnecessary touching of eyes, nose or mouth, wearing masks and covering the mouth using the bent elbow during cough or sneezing. People also should have the habit of searching new updates on coronavirus disease 2019 through several media. The symptoms of coronavirus disease 2019 include the development of fever, dry cough anddifficulties in breathing. When the symptoms are observed, the person should call in advance for the emergency from the medical services available in the nearest area. Pneumonia is a lung infection that causes inflammation inthe tinny air sacks called alveoli. The bacterial pneumonia is caused by streptococcus pneumoniae bacteria that attack the lungs causing the accumulation of pus after damaging the alveoli. The pus fills the alveoli covering thespace for air. This may lead to severe shortness of breath, cough, fever, chest pain, chills or fatigue. Bacterial pneumonia is deadly but treatable respiratory lung infectious disease. The mostly prescribed antibiotics at health centers is the Amoxicillin dispersible tablets. For severe cases where a patient has a compromised immune system, hospitalization is recommended with the prescription of Cotrimoxazole pneumonia patients with viral infections. (Organization et al., 2020) Apart from pneumonia vaccination, it can also be prevented by strengthening the body immune system by considering the adequate nutrition with strong diet. Also staying away from

environmental pollutants such as smoke and avoiding the crowded environment which contribute to a good environment for bacterial attack.

II. Literature Review

The respiratory system of human being is exposed to an air environment which may contain some organic particulates including virus and bacteria. The body immunity is said always to be unstable towards the virus because of their characteristics of mixing their genetic materials leading to them not being specific. The newly COVID19 outbreak has limited information regarding the risk factors for severe disease. It has been observed with currently available information that older adults and people under serious medical conditions might be at higher risk of being affected by COVID19. Most cases with the COVID19 infections were reported also to havePneumonia cases which made the complications in the recognition of the full extent of the disease and difficulties in handling the patients especially those aged ones (Wu et al., 2020). According to the study that wasconducted in Wuhan, China. It was observed that 4.35% of the COVID19 patients also got infected with the influenza virus that cause pneumonia. This is to explain about the situation of the co-infection existence and how it complicates the disease intensity (Ding et al., 2020). It is shown also that a high proportion of viral infections in patients with community acquired pneumonia (bacterial pneumonia). The virus related to coronavirus alone had no significant impact on the increase in mortality for the patients. The high risk of deaths was to the patients with co-infection (Bettenay et al., 1988). More complications to COVID 19 patients was mostly influenced by the presence of bacterial pneumonia where the use of antibiotics was prescribed for treatments. It was reported that more than 90% of the COVID 19 patients admitted in hospitals received antibiotics for initial treatment phase (Wang et al., 2021).

III. The Mathematical Model

The total human population N is subdivided into five compartments. The whole population is susceptible S to both COVID19 and pneumonia diseases with the recruitment rate π. It is assumed that individuals enter the susceptible compartment through birth at the rate π and become infected with COVID19 forming the compartment I_C at the rate λ_C. The susceptible individual also can be infected with pneumonia forming the compartment I at the rate of infection of λ_P. An individual in I_C can get Pneumonia infections at the rate $\psi\ 1\lambda_P$ and an individual in I_P can acquire coronavirus at $\psi_2\ \lambda_C$ both forming the compartment I_{CP} which indicates the co-infection compartment. The infected can be Recovered R. Thus, I_C, I_P and I_{CP} recover at therates κ, φ and ν respectively. Bacterial pneumonia can be treated from the co-infected individual at the rate σ andthe co-infected individual can recover naturally from COVID19 at the rate ρ. Recovered individuals also become susceptible at the rate τ. Individuals die naturally at the rate μ and also can die of the diseases at the rates δ_C, δ_P and δ_{CP}.

The table below indicates the summary of state variables;

State Variable	Description
S	Number of susceptible human individuals.
I_C	Number of human individuals infected with COVID19.
I_P	Number of human individuals infected with pneumonia.
I_{CP}	Number of human individuals with both COVID19 andPneumonia infections.
R	Number of human individuals fully recovered from infections.

Model Assumptions:

The model assumes that;

(i) The human population is not constant.

(ii) No vertical transmission of the diseases.

(iii) No vaccination of human population.

(iv) Individuals enter the susceptible compartment through birth and fully recovered.

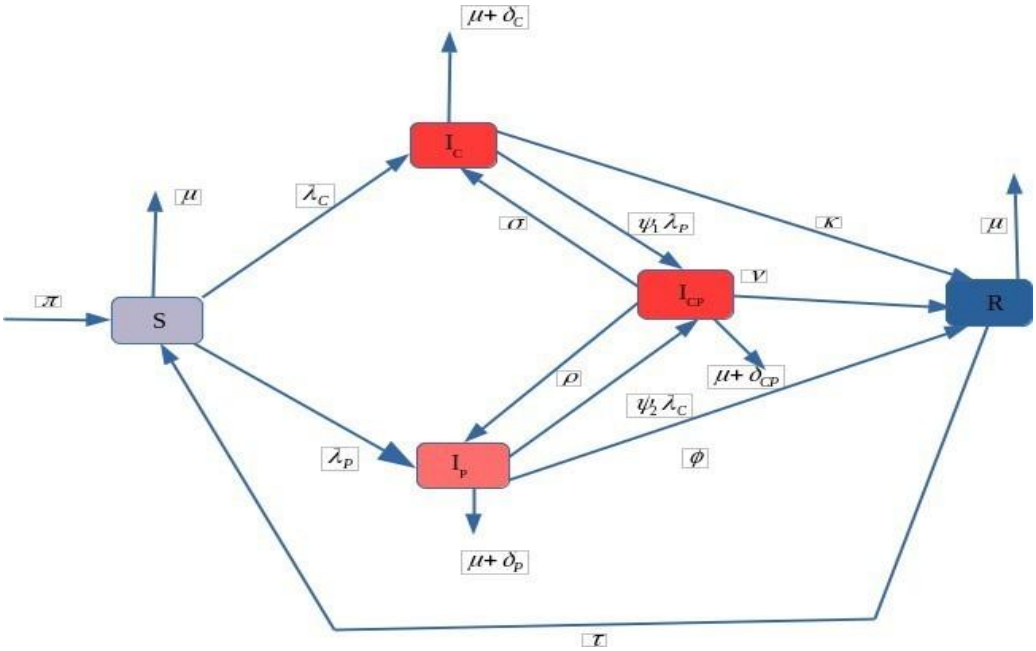

Figure 3.1: Flow diagram including COVID19 and pneumonia co-infection

The flow diagram in Figure 3.1 is used to express the diseases dynamics which assisted in obtaining themodel.

Parameter	Description
π	Recruitment rate to both diseases
μ	Natural death
δ_C	COVID19 induced mortality
δ_P	Pneumonia induced mortality
ψ_1	Proportion of Pneumonia infections to COVID19 infected individuals.
ψ_2	Proportion of COVID19 infections to Pneumonia infected individuals.
λ_P	Rate of pneumonia new-infections.
λ_C	Rate of COVID19 new-infections.
γ	Probability of acquiring pneumonia.
θ	Contact rate with Pneumonia infective.
ν	Rate of the co-infections recovery.
β	Contact rate with COVID19 infective.
α	Probability of acquiring COVID19.
φ	Recovery rate of Pneumonia.
κ	Recovery rate of COVID19.
τ	Rate at which the recovered progress to susceptible.
δ_{CP}	Co-infection induced mortality rate.
ρ	Pneumonia treatment rate of the co-infected.
σ	COVID19 treatment rate of the co-infected.

Table 3.1: Description of Parameters

From Figure 3.1 we obtain the system of deterministic model of differential equations.

$$\frac{dS}{dt} = \pi + \tau R - (\mu + \lambda_C + \lambda_P)S,$$

$$\frac{dI_C}{dt} = \lambda_C S + \sigma I_{CP} - (\mu + \delta_C + \psi_1\lambda_P + \kappa)I_C,$$
$$\frac{dI_P}{dt} = \lambda_P S + \rho I_{CP} - (\mu + \delta_P + \psi_2\lambda_P + \phi)I_P,$$
$$\frac{dI_{CP}}{dt} = \psi_1\lambda_P I_C + \psi_2\lambda_C I_P - (\mu + \delta_{CP} + \nu + \sigma + \rho)I_{CP},$$
$$\frac{dR}{dt} = \phi I_P + \nu I_{CP} + \kappa I_C - (\mu + \tau)R.$$

The force of infections considered by the model are;

$\lambda_C = \alpha\beta \boldsymbol{I}_C$ and $\lambda_P = \gamma\theta \boldsymbol{I}_P$.

Mathematical Model analysis

A. Equilibrium Points

1) Disease Free Equilibrium (DFE).

This is the assumed point where there are no infections in the population at a time *t*. It is computedthrough letting all infections equal to zero.

i . e
S^0, $I^0{}_P = 0$, $I^0{}_C = 0$, $I^0{}_{CP} = 0$, R^0 .

Solving the system and substituting the values of state variables;

$$0 = \pi + \tau R^0 - (\mu + \lambda_C + \lambda_P)S^0,$$
$$0 = \lambda_C S^0 + \sigma {I_{CP}}^0 - (\mu + \delta_C + \psi_1\lambda_P + \kappa){I_C}^0,$$
$$0 = \lambda_P S^0 + \rho {I_{CP}}^0 - (\mu + \delta_P + \psi_2\lambda_C + \phi){I_P}^0,$$
$$0 = \psi_1\lambda_P {I_C}^0 + \psi_2\lambda_C {I_P}^0 - (\mu + {\delta_{CP}}^0 + \sigma + \rho){I_{CP}}^0,$$
$$0 = \phi {I_P}^0 + \nu {I_{CP}}^0 + \kappa {I_C}^0 - (\mu + \tau)R^0.$$

$$\pi + \tau R^0 - \mu S^0 = 0,$$
$$-(\mu + \tau) R^0 = 0$$

$$R^0 = 0, \mu + \tau \neq 0,$$

$$\pi - \mu S^0 = 0,$$
$$\therefore S^0 = \frac{\pi}{\mu}.$$

Therefore, the disease free equilibrium point is given as; $E^0 = (\frac{\pi}{\mu}, 0, 0, 0, 0)$

Basic Reproduction Number (R_0).

The number of secondary cases of infections which one case would produce in a completely susceptible population is known as basic reproduction number.(Dietz, 1993)

The basic reproduction number is more useful in epidemiological studies because it is the key factor for determining whether the disease dies out or it persists in population (endemic). If the result of $R_0 < 1$, it means that each infected individual can infect on average less than one individual which in reality depicts no new infection. Hence the disease dies out of the population. When the result of $R_0 > 1$, it indicates that one infected individual can on average infect more than one individual in the susceptible population. Hence the disease prevails (endemic). The case where $R_0 = 1$ it means that one infectious individual can infect on average one individual in the population.

Through the use of next generation matrix in calculating the basic reproduction number, it was obtained that;

$R_C = \frac{\alpha\beta\pi}{(\delta_C+\kappa)\mu+\mu^2}$ and $R_P = \frac{\gamma\theta\pi}{(\delta_P+\phi)\mu+\mu^2}$.

Where R_C and R_P are the basic reproduction numbers of the two diseases forming the co-infection.

Impacts between Diseases Dynamics.

The basic reproduction number for the co-infection is indicated by the two individual disease basic reproduction numbers. In analyzing the co-infection, the impact of one disease infections on the other was considered. The process was to express one basic reproduction number in terms of the other.

2) Impact of COVID19 on Pneumonia Dynamics.

The pneumonia basic reproduction number R_P was expressed in terms of COVID19 basic reproduction number R_C. The result obtained was,

$$R_P = \frac{\gamma\theta}{\alpha\beta}\left(\frac{\kappa+\mu+\delta_C}{\phi+\mu+\delta_P}\right)R_C \quad (3.1.1)$$

Obtaining the partial derivative of R_P with respect to R_C we get, $\frac{\partial R_P}{\partial R_C} = \frac{\gamma\theta}{\alpha\beta}\left(\frac{\kappa+\mu+\delta_C}{\phi+\mu+\delta_P}\right)$

(3.1.2)

Due to (3.1.2) being strictly positive, implies that an increase in COVID19 cases results in an increase inpneumonia infections in the community.

Also, to see the impact of pneumonia in COVID19 dynamics we express R_C in terms of R_P. We obtain the following expression.

$$R_C = \frac{\alpha\beta}{\gamma\theta}\left(\frac{\phi+\mu+\delta_P}{\kappa+\mu+\delta_C}\right)R_P \quad (3.1.3)$$

Obtaining the partial derivative of R_C with respect to R_P we get,

$$\frac{\partial R_C}{\partial R_P} = \frac{\alpha\beta}{\gamma\theta}\left(\frac{\phi+\mu+\delta_P}{\phi+\mu+\delta_P}\right) \qquad (3.1.4)$$

Because (3.1.4) is positive, it shows that pneumonia cases in the co-infection group results in a positive impacton COVID19 cases in the community. Meaning that the increase in pneumonia infections increases the COVID19 complications on human population.

Impact of Pneumonia Recovery on COVID19

Here we differentiate partially (3.1.3) with respect to the parameter representing the rate at which patients receive pneumonia treatment. So, we have,

$$\frac{\partial R_C}{\partial \varphi} = \frac{\alpha\beta R_P}{\gamma\theta(\delta_C+\kappa+\mu)} \qquad (3.1.5)$$

Since (3.1.5) is positive, it indicates that the recovery of pneumonia will have a negative impact on thedynamics of pneumonia and COVID19 co-infection (Okosun et al., 2016)

From the above results, the analysis was summarized in the following lemma;

Lemma

Recovery of pneumonia only in the co-infection model, will have

- A positive impact on the pneumonia and COVID19 co-infection if (3.1.5) < 0.
- No impact on the pneumonia and COVID19 co-infection if (3.1.5) = 0.
- A negative impact on the pneumonia and COVID19 co-infection if (3.1.5) > 0.

Local Stability of Disease Free Equilibrium.

The Jacobian matrix obtained by differentiating each equation with respect to each state variable wasused in showing that the DFE point was locally asymptotically stable.

To simplify the system in (3),Let,

$$X_1 = \mu + \alpha\beta I_C + \gamma\vartheta I_P,$$
$$X_2 = \mu + \delta_C + \psi_1\gamma\vartheta I_P + \kappa,\ X_3 = \mu + \delta_P + \psi_2\alpha\beta I_C + \varphi, X_4 = \mu + \delta_{CP} + \nu + \sigma + \rho.$$

Then the Jacobian matrix of Disease Free was obtained as;

$$J(E) = \begin{pmatrix} -\mu & -\frac{\alpha\beta\pi}{\mu} & -\frac{\gamma\theta\pi}{\mu} & 0 & \tau \\ 0 & \frac{\alpha\beta\pi}{\mu} - \delta_C - \kappa - \mu & 0 & \sigma & 0 \\ 0 & 0 & \frac{\gamma\pi\theta}{\mu} - \delta_P - \mu - \phi & \rho & 0 \\ 0 & 0 & 0 & -(\delta_{CP} + \nu + \rho + \sigma + \mu) & 0 \\ 0 & \kappa & \phi & \nu & -(\mu + \tau) \end{pmatrix}$$

Leading to the eigen values;

$$\left[\frac{\gamma\pi\theta - \mu\delta_P - \mu^2 - \mu\phi}{\mu}, \frac{\alpha\beta\pi - (\delta_C + \kappa + \mu)\mu}{\mu}, -(\mu+\tau), -(\delta_{CP} + \mu + \nu + \rho + \sigma), -\mu\right]$$

After some manipulations, it was observed that the eigen values can be put in terms including R_0.

$[-(\delta_P + \varphi + \mu)(-R_P + 1), -(\delta_C + \kappa + \mu)(-R_C + 1), -\mu - \tau, -\delta_{CP} - \mu - \nu - \rho - \sigma, -\mu]$

The disease free equilibrium point is locally asymptotically stable. This is because in order for all eigenvalues to be negative, R_P and R_C should be less than a unit. Otherwise it is the endemic equilibrium point.

Local Stability of Endemic Equilibrium Point.

The Equilibrium point was obtained through considering each differential equation equal to zero. Specifically, (*EEP)* was obtained through the assumption that $\frac{dS}{dt} = \frac{dI_C}{dt} = \frac{dI_P}{dt} = \frac{dI_{CP}}{dt} = \frac{dR}{dt} = 0$.

$$0 = \pi + \tau R^* - (\mu + \lambda_C + \lambda_P)S^*,$$
$$0 = \lambda_C S^* + \sigma I_{CP}^* - (\mu + \delta_C + \psi_1\lambda_P + \kappa)I_C^*,$$
$$0 = \lambda_P S^* + \rho I_{CP}^* - (\mu + \delta_P + \psi_2\lambda_C + \varphi)I_P^*,$$
$$0 = \psi_1\lambda_P I_C^* + \psi_2\lambda_C I_P^* - (\mu + \delta_{CP}^* + \nu + \sigma + \rho)I_{CP}^*,$$
$$0 = \varphi I_P^* + \nu I_{CP}^* + \kappa I_C^* - (\mu + \tau)R^*$$

Now $(S^*, I_C^*, I_P^*, I_{CP}^*, R^*)$ is the endemic equilibrium point. Considering the modifying parameters *a* and *b* in the system.

For simplicity,

Let $(\mu+\lambda_C+\lambda_P) = a_1$, $(\mu+\delta_C+\psi_1\lambda_P+\kappa) = a_2$, $(\mu+\psi_2\lambda_C+\delta_P+\varphi) = a_3$, $(\mu+\delta_{CP}+\nu+\sigma+\rho) = a_4$, $(\mu+\tau = a_5$, $(\lambda_C S + \sigma I_{CP}) = b_1$, $(\lambda_P S + \rho I_{CP}) = b_2$, $(\psi_1\lambda_P I_C + \psi_2\lambda_C I_P) = b_3$.

$$S^* = \frac{a_2a_3a_4a_5\pi + (a_3a_4b_1\kappa + (a_3b_3\nu + a_4b_2\varphi)a_2)\tau}{a_1a_2a_3a_4a_5}.$$

$$I^*{}_P = \frac{b_2}{a_3}.$$

$$I^*{}_C = \frac{b_1}{a_2}.$$

$$I^*{}_{CP} = \frac{b_3}{a_4}.$$

$$R^* = \frac{a_3a_4b_1\kappa + (a_3b_3\nu + a_4b_2\varphi)a_2}{a_2a_3a_4a_5}.$$

Thus,

$$(S^*, I^*{}_C, I^*{}_P, I^*{}_{CP}, R^*) = \left(\frac{a_2a_3a_4a_5\pi + (a_3a_4b_1\kappa + (a_3b_3\nu + a_4b_2\varphi)a_2)\tau}{a_1a_2a_3a_4a_5}, \frac{b_1}{a_2}, \frac{b_2}{a_3}, \frac{b_3}{a_4}, \frac{a_3a_4b_1\kappa + (a_3b_3\nu + a_4b_2\varphi)a_2}{a_2a_3a_4a_5}\right).$$

The local stability of the EEP was displayed with the aid of a diagram.

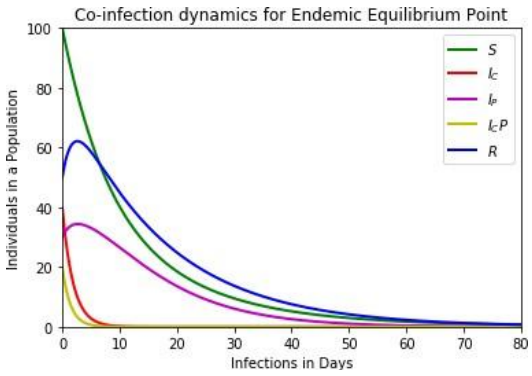

Figure 3.2: Local stability of the endemic equilibrium point.

Figure 3.2 confirms that the solution of the system converges to the equilibrium point after small perturbations. It was observed that the convergence was after 20 Days and the plot was plotted using the values $R_C = 3.3478$ and $R_P = 1.5863$ indicating that the endemic equilibrium point is locally asymptotically stable. In long run the co-infection is going to die out.

Numerical Model Analysis

Parameters Simulation.

The numerical simulation was used to depict the impacts of the co-infection of CoVID19 and bacterial pneumonia on human population. This paper is based on the impacts brought after varying the parameter related to the two diseases and their co-infection. Python programing language was used on the parameter values from table Table 4.1

Parameter	Description	Units	Value	Source
π	Recruitment rate	Per day	0.06	(Townsend et al., 2020)
δ_C	CoVID19 induced mortality	per day	0.0236	(for Disease Control et al., 2020)
τ	Rate at which the recovered progress to susceptible	per day	0.054	Assumed
ψ_1	Proportion of Pneumonia infections to COVID19 infected individuals	per day	0.060	(Townsend et al., 2020)
ψ_2	Proportion of COVID19 infections to Pneumonia infected individuals	per day	0.18	Assumed
ν	Coinfections recovery rate	per day	0.009	Assumed
β	Contact rate with COVID19 infective	per day	0.200	(Elie et al., 2020)
κ	Recovery rate of COVID19	per day	0.08	(Nthiiri et al., 2015)
ϑ	Contact rate with Pneumonia infec-Tive	per day	0.08	Nthiiri et al. (2015)
ρ	Pneumonia treatment rate of the co-infected	per day	0.023	(Onyinge et al., 2016)
φ	Recovery rate of Pneumonia	per day	0.05	(Tchuenche et al., 2011)
α	Probability of acquiring COVID19	per day	0.4964	(WHO)
γ	Probability of acquiring pneumonia	per day	0.0021	Assumed
μ	Natural death rate	per day	0.02	(Goncalves Mendes Neto et al., 2021)
δ_P	Pneumonia induced death rate	per day	0.0032	Assumed
σ	COVID19 treatment rate of the co-Infected	per day	0.0451	Assumed
δ_{CP}	Co-infection induced mortality rate	per day	0.004	Assumed

Table 4.1: Parameter estimates of the model obtained from different literature

IV. Discussion of Results and Recommendation.

The model was formulated basing on dividing the population into five compartments involving the Susceptible, Covid-19 infected, Pneumonia infected, Co-Infected and Recovered. The parameters were fixed to explain the rates of the diseases dynamics between the compartments. The model gives the full picture of the epidemiology of diseases in our communities. Mathematically some calculations were performed to obtain the helpful formula whose interpretations explain the real life situation on the co-infection of the two diseases.

It was observed that both equilibrium points were locally asymptotically stable. This implied that for small perturbations the co-infection dies out in less than twenty days after the infections. The recovery of pneumonia had a negative impact on the co-infection progress. This implies that as the number of individuals in the co-infection compartment recover from pneumonia, the number of the co-infections becomes less.

In controlling the co-infection, it is recommended to treat pneumonia first using the antibiotics which impacts on complications caused by the corona virus. The model also considers the population which is dynamic with no vertical transmission of the diseases, without vaccinated population as the assumptions. Bacterial pneumonia was seen more influential to the reduction of COVID 19 effects due to the use of the basic reproduction numbers. The result was that if we treat pneumonia using the antibiotics the co-infected cases were negatively affected. Thus, the complication risks of the co-infection decrease.

V. Conclusion

The information in this paper is more useful in formulation of various policies specifically for healthy practitioners for treatment and dealing with the co-infection. The importance of this model lies on the basic reproduction number (R_0) interpretation. The data used in this paper was obtained from various written sources which is the limitation for the real situation results.

This paper is the way for the other research related works. There are some aspects which were not included in this paper such as the involvement of the control measures such as regular hand washing, sanitizers, face masks wearing for the coinfection. Also, the impact of vaccination can be considered in the co-infection as the future works.

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