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Adomian Decomposition Method for Solving Coupled Nonlinear System of Klein-Gordon Equations

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Abstract; *In this paper, we apply Adomian Decomposition Method(ADM) for solving coupled nonlinear Klein-Gordon equations (CNLKGE) which arise in particle physics, wave theory and other physical phenomena of linear and nonlinear nature. The numerical solutions of CNLKGE have been compared to the exact solutions and presented graphically. The numerical results are in good agreement with exact solutions which shows the efficiency and reliability of the proposed algorithm.*

Keywords; *Adomian Decomposition Method ;Adomian Polynomials; Coupled nonlinear Klein-Gordon Equations; Recursive algorithms.*

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I. Introduction

A wide variety of physically significant problems such as Coupled Nonlinear system of Klein-Gordon Equations (CNLKGE) , has been the focus of extensive studies for the last decades. In order to better understand these nonlinear behaviour, many researchers developed more accurate approaches to the solutions. The Nonlinear coupled Klein-Gordon equation was first studied by [1,2,3]. Recently, [6,7,8] have solved CNLKGE by using ADM and their modified and extended versions.

In this paper, Our aim is to develop ADM for CNLKGE and compare the solutions with the exact solutions given by [1]. In 1994, the Adomian Decomposition Method (ADM) was introduced by George Adomian [5].The method does not require any transformation, discretization, perturbation or any restrictive assumptions but it utilises Adomian polynomials to handle nonlinear terms.

The structure of the paper is as follows; Section II gives the basic concepts of ADM and how to generate the Adomian Polynomials. Section III develops the recursive algorithms for the CNLKGE. Section IV tabulates the results for various orders and compares them and their convergence. Section V concludes the study.

The system to be solved is defined by;

$$F_1(u,v,u_{xx},u_{tt}) = u_{xx} - u_{tt} - u + 2u^3 + 2uv = (\Box + 1)u - 2u^3 - 2uv = 0,$$

$$x \in [0,L], L > 0, t \in [0,t_f], t_f > 0 \qquad (1)$$

$$F_2(u,u_t,v_x,v_t) = v_x - v_t - 4uu_t = 0, x \in [0,L], L > 0, t \in [0,t_f], t_f > 0 \qquad (2)$$

Subject to the initial conditions

$$u(x,0) = \psi_1(x) = \sqrt{\frac{(1+c)}{(1-c)}}\sec h\left(\frac{x}{\sqrt{1-c^2}}\right), \qquad x \in [0,L], L > 0 \qquad (3)$$

$$u_t(x,0) = \psi_2(x) = \left(\frac{c}{1-c}\right)\sec h\left(\frac{x}{\sqrt{1-c^2}}\right)\tanh h\left(\frac{x}{\sqrt{1-c^2}}\right), x \in [0,L], L > 0 \qquad (4)$$

$$v(x,0) = \psi_3(x) = \left(\frac{-2c}{1-c}\right)\sec h^2\left(\frac{x}{\sqrt{1-c^2}}\right), \qquad x \in [0,L], L > 0 \qquad (5)$$

Where $c^2 < 1$

II. Adomian Decomposition Method (ADM)

2.1 . Concepts of Adomian Decomposition Method (ADM)

To illustrate the basic concepts of ADM, we consider the following general nonlinear system.
According to [4,5], to use ADM, the system is expressed in an operator form.

$$L_t u + L_x v + N_1(u,v) = g_1$$
$$L_t v + L_x u + N_2(u,v) = g_2 \qquad (6)$$

with initial data

$$u(x,0) = f_1(x)$$
$$v(x,0) = f_2(x) \qquad (7)$$

where L_t and L_x are first order partial differential operators, N_1 and N_2 are nonlinear operators and g_1 and g_2 are source terms. Operating with the integral operator the system of equations (6) and using (7) we get

$$u(x,t) = f_1(x) + L_t^{-1} g_1 - L_t^{-1} L_x v - L_t^{-1} N_1(u,v)$$
$$v(x,t) = f_2(x) + L_t^{-1} g_2 - L_t^{-1} L_x u - L_t^{-1} N_2(u,v) \qquad (8)$$

The linear unknowns $u(x,t)$ and $v(x,t)$ are decomposed into infinite series as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$$
$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) \qquad (9)$$

while the nonlinear operators N_1 and N_2 are represented using Adomian polynomials
A_n and B_n as

$$N_1(u,v) = \sum_{n=0}^{\infty} A_n$$
$$N_2(u,v) = \sum_{n=0}^{\infty} B_n \qquad (10)$$

Using equations (9) and (10) in (8), yields

$$\sum_{n=0}^{\infty} u_n(x,t) = f_1(x) + L_t^{-1} g_1 - L_t^{-1} L_x \left(\sum_{n=0}^{\infty} v_n(x,t) \right) - L_t^{-1} \left(\sum_{n=0}^{\infty} A_n \right)$$
$$\sum_{n=0}^{\infty} v_n(x,t) = f_2(x) + L_t^{-1} g_2 - L_t^{-1} L_x \left(\sum_{n=0}^{\infty} u_n(x,t) \right) - L_t^{-1} \left(\sum_{n=0}^{\infty} B_n \right) \qquad (11)$$

Recursive algorithms are now constructed from equation (11) as follows;

$$u_0(x,t) = f_1(x) + L_t^{-1} g_1,$$
$$u_{k+1}(x,t) = -L_t^{-1}\left(L_x v_k\right) - L_t^{-1}\left(A_k\right), \quad k \geq 0 \qquad (12)$$

and

$$v_0(x,t) = f_2(x) + L_t^{-1} g_2,$$
$$v_{k+1}(x,t) = -L_t^{-1}\left(L_x u_k\right) - L_t^{-1}\left(B_k\right), k \geq 0 \qquad (13)$$

Finally, using equation (9) gives the required solutions.

2.2 Adomian Polynomials

According to Adomian [5], these polynomials for the nonlinear terms are evaluated by using the expression

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}, n = 0,1,2,\ldots \quad (14)$$

It can be simplified as follows:

$$A_0 = F(u_0)$$
$$A_1 = u_1 F'(u_0)$$
$$A_2 = u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0) \quad (15)$$
$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0)$$
$$A_4 = u_4 F'(u_0) + \left(\frac{1}{2!}u_2^2 + u_1 u_3\right)F''(u_0) + \frac{1}{2!}u_1^2 u_2 F'''(u_0) + \frac{1}{4!}u_1^4 F^{(4)}(u_0)$$

2.3 Computation of Adomian Polynomials

(i) Nonlinear polynomials

Case 1: $F(u) = u^2$

$$A_0 = F(u_0) = u_0^2$$
$$A_1 = u_1 F'(u_0) = 2u_0 u_1$$
$$A_2 = u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0) = 2u_0 u_2 + u_1^2 \quad (16)$$
$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0) = 2u_0 u_3 + 2u_1 u_2$$

Case 2: $F(u) = u^3$

$$A_0 = F(u_0) = u_0^3$$
$$A_1 = u_1 F'(u_0) = 3u_0^2 u_1$$
$$A_2 = u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0) = 3u_0^2 u_2 + 3u_0 u_1^2 \quad (17)$$
$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0) = 3u_0^2 u_3 + 6u_0 u_1 u_2 + u_1^3$$

(ii) Function of two variables

Case 3: $F(u,v) = uv$

$$A_0 = u_0 v_0$$
$$A_1 = u_1 v_0 + u_0 v_1$$
$$A_2 = u_2 v_0 + u_1 v_1 + u_0 v_2 \quad (18)$$
$$A_3 = u_3 v_0 + u_2 v_1 + u_1 v_2 + u_0 v_3$$

(iii) Nonlinear derivatives

Case 4: $F(u) = uu_t$

$$A_0 = u_0 u_{0t}$$
$$A_1 = u_1 u_{0t} + u_0 u_{1t}$$
$$A_2 = u_2 u_{0t} + u_1 u_{1t} + u_0 u_{2t} \quad (19)$$
$$A_3 = u_3 u_{0t} + u_2 u_{1t} + u_1 u_{2t} + u_0 u_{3t}$$

III. NUMERICAL SOLUTION OF COUPLED KLEIN-GORDON EQUATIONS (CNLKGE) USING ADM

3.1 Solution of CNLKGE Using ADM

Re- arranging our system of equations (1) and (2) such that we have u_{tt} and v_t on the left hand side, we obtained,

$$u_{tt} = u_{xx} - u + 2u^3 + 2uv \tag{20}$$

$$v_t = v_x - 4uu_t \tag{21}$$

Writing the system of equations (20) and (21) in operator form we have

$$L_{1t}(u) = L_{1x}(u) - u + 2u^3 + 2uv \tag{22}$$

$$L_{2t}(v) = L_{2x}(v) - 4uu_t \tag{23}$$

Where

$$L_{1t} = \frac{\partial^2}{\partial t^2}, L_{1x} = \frac{\partial^2}{\partial x^2} \tag{24}$$

$$L_{2t} = \frac{\partial}{\partial t}, L_{2x} = \frac{\partial}{\partial x} \tag{25}$$

are partial differential operators.

Consequently, the integral operators if they exist can be regarded as

$$L_{1t}^{-1} = \int_0^t \int_0^t (\cdot)\, dt\, dt \tag{26}$$

$$L_{1x}^{-1} = \int_0^x \int_0^x (\cdot)\, dx\, dx \tag{27}$$

$$L_{2t}^{-1} = \int_0^t (\cdot)\, dt \tag{28}$$

and

$$L_{2x}^{-1} = \int_0^x (\cdot)\, dx \tag{29}$$

This means that

$$L_{1t}^{-1} L_{1t}(u(x,t)) = u(x,t) - tu_t(x,0) - u(x,0) \tag{30}$$

$$L_{1x}^{-1} L_{1x}(u(x,t)) = u(x,t) - xu_t(0,t) - u(0,t) \tag{31}$$

$$L_{2t}^{-1} L_{2t}(u(x,t)) = u(x,t) - u(x,0) \tag{32}$$

$$L_{2x}^{-1} L_{2x}(u(x,t)) = u(x,t) - u(0,t) \tag{33}$$

Applying

$L_{1t}^{-1} = \int_0^t \int_0^t (\cdot)\, dt\, dt$ to both sides of equation (22) and $L_{2t}^{-1} = \int_0^t (\cdot)\, dt$ to equation (23)

We obtained,

$$u(x,t) = u(x,0) + tu_t(x,0) + L_{1t}^{-1}\left(L_{1x}\left(u(x,t)\right)\right) + L_{1t}^{-1}\left(-u(x,t) + 2u^3(x,t) + 2u(x,t)v(x,t)\right) \tag{34}$$

$$v(x,t) = v(x,0) + L_{2t}^{-1}\left(L_{2x}\left(v(x,t)\right) - 4u(x,t)u_t(x,t)\right) \tag{35}$$

Substituting the linear terms $u(x,t)$ and $v(x,t)$ by the Adomian decomposition series

$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t)$ and $v(x,t)=\sum_{n=0}^{\infty}v_n(x,t)$ and the nonlinear terms $u^3(x,t)$, $u(x,t)v(x,t)$ and $u(x,t)u_t(x,t)$ by Adomian polynomials we obtained together with initial conditions equations (3),(4) and (5) and assuming $c=0.5$, , yielded

$$\sum_{n=0}^{\infty}u_n(x,t)=\sqrt{3}\sec h(\frac{2x}{\sqrt{3}})+t\sec h(\frac{2x}{\sqrt{3}})\tan h(\frac{2x}{\sqrt{3}})+L_{1t}^{-1}\left(L_{1x}\left(\sum_{n=0}^{\infty}u_n(x,t)\right)\right)+$$
$$L_{1t}^{-1}\left(-\sum_{n=0}^{\infty}u_n(x,t)+2\sum_{n=0}^{\infty}B_n+2\sum_{n=0}^{\infty}C_n\right) \quad (36)$$

$$\sum_{n=0}^{\infty}v_n(x,t)=-2\sec h^2(\frac{2x}{\sqrt{3}})+L_{2t}^{-1}\left(L_{2x}\left(\sum_{n=0}^{\infty}v_n(x,t)\right)-4\sum_{n=0}^{\infty}A_n\right) \quad (37)$$

The recursive algorithms can be constructed from equations (36) and (37) as follows;

$$u_0(x,t)=\sqrt{3}\sec h(\frac{2x}{\sqrt{3}})+t\sec h(\frac{2x}{\sqrt{3}})\tan h(\frac{2x}{\sqrt{3}}) \quad (38)$$

$$u_{k+1}(x,t)=L_{1t}^{-1}\left(L_{1x}\left(u_k\right)\right)+L_{1t}^{-1}\left(-u_k+2B_k+2C_k\right),k=0,1,2,... \quad (39)$$

Where

$$u^3=\sum_{n=0}^{\infty}B_n \text{ and } uv=\sum_{n=0}^{\infty}C_n$$

In a similar manner we have

$$v_0(x,t)=-2\sec h^2(\frac{2x}{\sqrt{3}}) \quad (40)$$

$$v_{k+1}(x,t)=L_{2t}^{-1}\left(L_{2x}\left(v_k\right)-4A_k\right),k=0,1,2,... \quad (41)$$

Where

$$uu_t=\sum_{n=0}^{\infty}A_n,$$

Using the zeroth components (38) and (40) and taking $k=0,1,2$ we obtained from equations (39) and (41) the following;

$$u_0(x,t) = \sqrt{3}\sec h(\frac{2x}{\sqrt{3}}) + t\sec h(\frac{2x}{\sqrt{3}})\tan h(\frac{2x}{\sqrt{3}}),$$

$$v_0(x,t) = -2\sec h^2(\frac{2x}{\sqrt{3}}),$$

$$u_1(x,t) = L_{1t}^{-1}\left(L_{1x}\left(u_0\right)\right) + L_{1t}^{-1}\left(-u_0 + 2B_0 + 2C_0\right);\ B_0 = u_0{}^3; C_0 = u_0v_0$$

$$= L_{1t}^{-1}\left(L_{1x}\left(u_0\right)\right) + L_{1t}^{-1}\left(-u_0 + 2u_0{}^3 + 2u_0v_0\right), \qquad (by\ 39)$$

$$v_1(x,t) = L_{2t}^{-1}\left(L_{2x}\left(v_0\right) - 4A_0\right); A_0 = u_0u_{0t}$$

$$= L_{2t}^{-1}\left(L_{2x}\left(v_0\right) - 4u_0u_{0t}\right), \qquad (by\ 41)$$

$$u_2(x,t) = L_{1t}^{-1}\left(L_{1x}\left(u_1\right)\right) + L_{1t}^{-1}\left(-u_1 + 2B_1 + 2C_1\right); B_1 = 3u_0{}^2u_1; C_1 = u_1v_0 + u_0v_1$$

$$= L_{1t}^{-1}\left(L_{1x}\left(u_1\right)\right) + L_{1t}^{-1}\left(-u_1 + 6u_0{}^2u_1 + 2u_1v_0 + 2u_0v_1\right), \qquad (by\ 39)$$

$$v_2(x,t) = L_{2t}^{-1}\left(L_{2x}\left(v_1\right) - 4A_1\right); A_1 = u_1u_{0t} + u_0u_{1t}$$

$$= L_{2t}^{-1}\left(L_{2x}\left(v_1\right) - 4u_1u_{0t} - 4u_0u_{1t}\right), \qquad (by\ 41)$$

$$u_3(x,t) = L_{1t}^{-1}\left(L_{1x}\left(u_2\right)\right) + L_{1t}^{-1}\left(-u_2 + 2B_2 + 2C_2\right); B_2 = 3u_0{}^2u_2 + 3u_1{}^2u_0; C_2 = u_2v_0 + u_1v_1 + u_0v_2$$

$$= L_{1t}^{-1}\left(L_{1x}\left(u_2\right)\right) + L_{1t}^{-1}\left(-u_2 + 6u_0{}^2u_2 + 6u_1{}^2u_0 + 2u_2v_0 + 2u_1v_1 + 2u_0v_2\right), \quad (by\ 39)$$

$$v_3(x,t) = L_{2t}^{-1}\left(L_{2x}\left(v_2\right) - 4A_2\right); A_2 = u_2u_{0t} + u_1u_{1t} + u_0u_{2t}$$

$$= L_{2t}^{-1}\left(L_{2x}\left(v_2\right) - 4u_2u_{0t} - 4u_1u_{1t} - 4u_0u_{2t}\right), \qquad (by\ 41)$$

Using the components $u_0, v_0, u_1, v_1, u_2, v_2, u_3, v_3$, we obtained the solutions as follows;

$$u(x,t) = \sum_{n=0}^{3} u_n(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) \qquad (42)$$

$$v(x,t) = \sum_{n=0}^{3} v_n(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + v_3(x,t) \qquad (43)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

4.1 Solution of CNLKGE and Numerical Convergence Using ADM

4.1.1 First Two Terms of ADM

Table 1: Values of $u(x,t)$ using first two terms of ADM; $c = 0.5, t = 0.1$.

From $u(x,t) = \sum_{n=0}^{1} u_n(x,t) = u_0(x,t) + u_1(x,t)$,we generated;

x	u (two terms)	u (exact)	Absolute error
0	1.729164056	1.72916806	4.00395E-06
0.1	1.729315686	1.72916806	0.000147626
0.2	1.706671583	1.706390891	0.000280692
0.3	1.662684418	1.662305739	0.000378679
0.4	1.600060559	1.599626806	0.000433754

0.5	1.522373316	1.521926123	0.000447193
0.6	1.433607129	1.433180153	0.000426976
0.7	1.337730409	1.33734623	0.000384179
0.8	1.23836548	1.238035736	0.000329744
0.9	1.138583593	1.138311112	0.000272481
1	1.040818342	1.040599983	0.000218358

Table 2: Values of $v(x,t)$ using first two terms of ADM; $c = 0.5, t = 0.1$

From $v(x,t) = \sum_{n=0}^{1} v_n(x,t) = v_0(x,t) + v_1(x,t)$, we generated;

x	v (two terms)	v (exact)	Absolute error
0	-2	-1.99334812	0.00665188
0.1	-2.000027329	-1.99334812	0.006679209
0.2	-1.94769774	-1.941179914	0.006517826
0.3	-1.848349568	-1.84217358	0.006175988
0.4	-1.711556276	-1.705870612	0.005685664
0.5	-1.549267667	-1.54417275	0.005094917
0.6	-1.373793506	-1.369336901	0.004456605
0.7	-1.196148064	-1.192329959	0.003818105
0.8	-1.025036757	-1.021821655	0.003215101
0.9	-0.866504698	-0.863834792	0.002669906
1	-0.7240919	-0.721898883	0.002193017

4.1.2 First Three Terms of ADM

Table 3: Values of $u(x,t)$ using first three terms of ADM; $c = 0.5, t = 0.1$.

From $u(x,t) = \sum_{n=0}^{2} u_n(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t)$,we generated;

x	u (three terms)	u (exact)	Absolute error
0	1.729148898	1.72916806	1.91626E-05
0.1	1.729148821	1.72916806	1.92394E-05
0.2	1.706372445	1.706390891	1.84456E-05
0.3	1.662288899	1.662305739	1.68405E-05
0.4	1.599612161	1.599626806	1.46446E-05
0.5	1.521913959	1.521926123	1.21647E-05
0.6	1.433170454	1.433180153	9.69864E-06
0.7	1.337338764	1.33734623	7.46598E-06
0.8	1.238030151	1.238035736	5.58467E-06
0.9	1.138307027	1.138311112	4.08461E-06
1	1.040597045	1.040599983	2.93777E-06

Table 4: Values of $v(x,t)$ using first three terms of ADM; $c = 0.5, t = 0.1$.

From $v(x,t) = \sum_{n=0}^{2} v_n(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t)$,we generated;

x	v (three terms)	v (exact)	Absolute error
0	-1.993333333	-1.99334812	1.47869E-05
0.1	-1.99432371	-1.99334812	0.00097559
0.2	-1.942993774	-1.941179914	0.00181386
0.3	-1.844541159	-1.84217358	0.002367579
0.4	-1.708460441	-1.705870612	0.002589829
0.5	-1.546689726	-1.54417275	0.002516976
0.6	-1.371574136	-1.369336901	0.002237235
0.7	-1.194181552	-1.192329959	0.001851593
0.8	-1.02326645	-1.021821655	0.001444795
0.9	-0.864907505	-0.863834792	0.001072713
1	-0.722661839	-0.721898883	0.000762956

4.1.3 First Four Terms of ADM

Table 5: Values of $u(x,t)$ using first four terms of ADM; $c = 0.5, t = 0.1$.

From equation (42), we generated;

x	u (four terms)	u (exact)	Absolute error
0	1.729168033	1.72916806	2.76041E-08
0.1	1.729166327	1.72916806	1.73334E-06
0.2	1.706387772	1.706390891	3.11815E-06
0.3	1.662301811	1.662305739	3.92852E-06
0.4	1.599622696	1.599626806	4.1096E-06
0.5	1.521922341	1.521926123	3.78219E-06
0.6	1.433176999	1.433180153	3.15433E-06
0.7	1.337343801	1.33734623	2.42934E-06
0.8	1.238033984	1.238035736	1.7514E-06
0.9	1.138309918	1.138311112	1.19418E-06
1	1.040599207	1.040599983	7.76124E-07

Table 6: Values of $v(x,t)$ using first four terms of ADM; $c = 0.5, t = 0.1$.

From equation (43), we generated;

x	v (four terms)	v (exact)	Absolute error
0	-1.9934672	-1.99334812	0.00011908
0.1	-1.993447368	-1.99334812	9.92475E-05
0.2	-1.941226629	-1.941179914	4.67144E-05
0.3	-1.842153021	-1.84217358	2.05589E-05
0.4	-1.705788119	-1.705870612	8.2493E-05
0.5	-1.544047695	-1.54417275	0.000125055
0.6	-1.369193297	-1.369336901	0.000143604
0.7	-1.192188653	-1.192329959	0.000141306
0.8	-1.021696441	-1.021821655	0.000125214
0.9	-0.863732128	-0.863834792	0.000102663

1	-0.721819642	-0.721898883	7.92418E-05

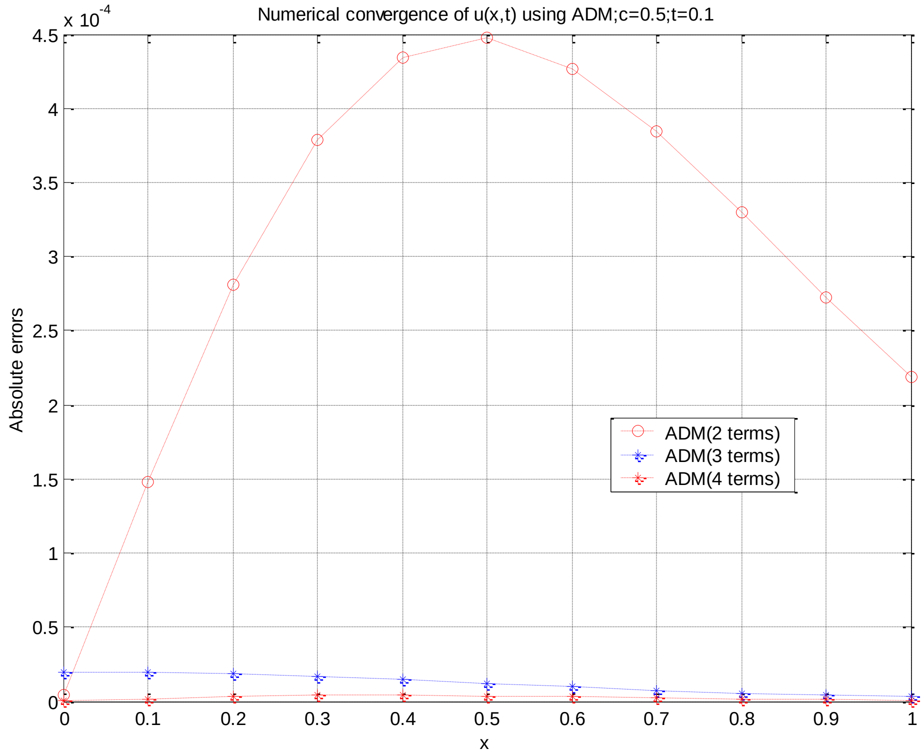

Fig. 1: Numerical convergence of $u(x,t)$ using ADM

From Fig. 1, the convergence of ADM is very good. The errors are in the order of 1E-4. As the number of terms increases, we can see from Tables 1-6 that the absolute errors decreases. The ADM (four terms) have an absolute error close to zero.

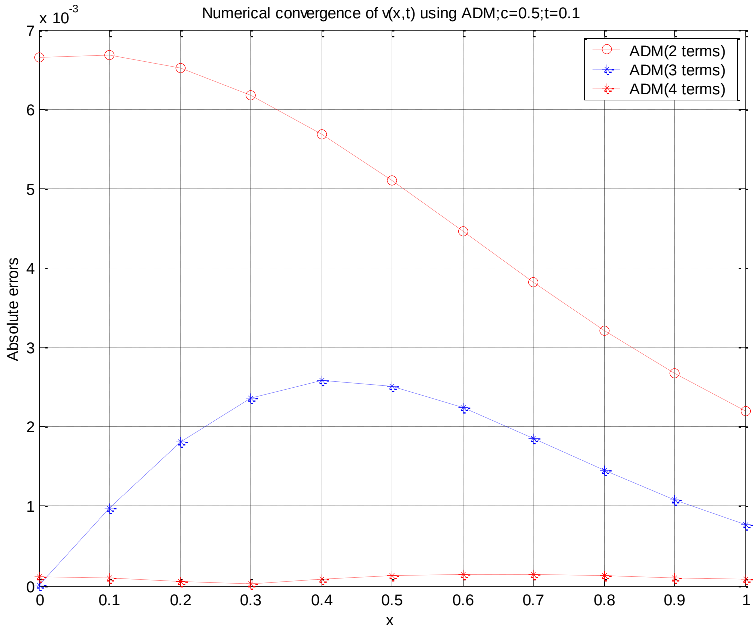

Fig. 2: Numerical convergence of $v(x,t)$ using ADM

From Fig. 2, the convergence is very good. Comparing this with Fig..1, we noted that $u(x,t)$ converged faster than $v(x,t)$. Convergence is achieved with fewer terms.

V. Conclusions

In this work, Adomian Decomposition Method (ADM) has been successfully applied for finding the approximate analytical solution of the coupled nonlinear Klein-Gordon equations. The numerical results obtained by ADM were observed to be in very close t agreement with the exact results obtained by [1]. This indicates that the method is very efficient, reliable and of high accuracy. In ADM we do not need discretization of temporal and spatial variables. The method is able to solve this nonlinear problems effectively as depicted by the numerical rapid convergence. Based on this accuracy and elegance, ADM has been used to solve a wide range of linear and non-linear practical problems occurring in several disciplines .As the number of terms are increased, the accuracy is improved as can be observed fro Tables 1-6, Fig. 1 and Fig.2.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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Bii Albert, et. al. "Adomian Decomposition Method for Solving Coupled Nonlinear System of Klein-Gordon Equations." *IOSR Journal of Mathematics (IOSR-JM)*, 18(1), (2022): pp. 49-58.