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Transient Behavior of Machine Repair Queues with Batch Arrivals and Generalized Reneging Using Stochastic Differential Equations

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Abstract
The research paper studies how a finite machine repair queue with batch arrivals and flexible customer behavior operates during its initial period. The model predicts short-term system congestion increases because machines break down at random times and the repair team can only handle a specific number of machines at once while customers leave the queue based on their current tolerance to wait. The research provides an accessible method to study how performance indicators evolve over time by converting queue length movements into a diffusion process instead of needing to analyze the complete Markov chain model.

The proposed formulation derives a drift term that represents the net effect of batch failures, service completion, and reneging, together with a diffusion term that captures randomness in arrivals and repairs. The researchers used the time-based analysis method to examine how transient queue length and waiting probability and server utilization and machine availability changed over different time periods. The research team used numerical results and simulation comparisons to study how different factors such as batch size and reneging intensity and repair capacity affect the time required for congestion to peak and then decrease.The analysis shows that batch arrivals can cause sharp temporary spikes in queue length, while generalized reneging can reduce congestion but may also increase effective loss of service. The stochastic differential equation method provides an adaptable tool which enables quick analysis of temporary repair system operations in actual industrial environments.

Keywords: *machine repair queue, batch arrivals, reneging, transient analysis, stochastic differential equations*

I. Introduction

Manufacturing plants and data centers and automated production lines depend on machine repair systems because minute equipment failures do not occur. The repair process develops into a dynamic system because failures at different points in time create multiple repair needs which follow either a shock or overload or environmental disturbance.

The long-run average values of utilization and queue length and availability become understandable through classical steady-state models but these models fail to show short-term spikes which result from sudden failures or planned maintenance or workload surges. The actual operation of systems depends on these transient periods because even short-term overloads between acceptable average loads can result in production delays and service interruptions and cascading breakdowns.

The system supports reneging because machines which fail or their repair requests will not stay in the queue until eternity. The excessive waiting time causes a repair job to be postponed or deprioritized or redirected to another facility or abandoned when a severe backlog exists. The model becomes more accurate to operational reality through generalized reneging because system performance gets affected by users who show different levels of impatience and urgency and priority decision making.

Model Description

The machine repair system includes N machines and R repair servers and receives failed machines through batch arrival. The system maintains its closed and bounded state because some machines run while some machines wait for repair and others receive repair services.

The state variable X(t) shows the current system state because X(t)=n indicates that n machines are down at time t. The system contains two groups of machines which include machines that wait in the queue and machines that receive treatment. The finite machine system establishes state space limits between 0 and N because the state space contains all possible values from 0 to N.

The system needs a revised reneging rule because it requires state-based modeling of user impatience which develops through time. The congestion conditions establish three distinct pathways according to which

machines and jobs and repair requests can either abandon their current position or postpone their activities or shift to another location.
A simple formulation can be written as:

$$\theta(n) = \theta_0 + \theta_1 n$$

or more generally,

$$\theta(n) = g(n, w, c),$$

where w is waiting time and c is a congestion measure. In this way, the model captures both the finite-capacity nature of the repair system and the loss behavior caused by growing delay pressure.

Core Assumptions

- Machines fail in batches, reflecting bursty failure patterns common in correlated production environments.
- Repair capacity is limited to R servers working in parallel, creating congestion when demand exceeds immediate service availability.
- Repair times are exponentially distributed or approximated by diffusion terms to enable stochastic differential equation analysis.
- Reneging depends on queue state, waiting time, or a hybrid rule, allowing abandonment rates to grow realistically with system pressure.
- The system is analyzed in transient form to capture time-dependent evolution, rather than only steady-state averages.

Stochastic Differential Equation Formulation

The queue length process $Q(t)$ is modeled as an Itô-type stochastic differential equation that captures the essential dynamics of batch arrivals, repair completions, and generalized reneging. Specifically,

$$dQ(t) = b(Q(t), t)\, dt + \sigma(Q(t), t)\, dW(t) - r(Q(t), t)\, dt$$

where $W(t)$ is standard Brownian motion, $b(\cdot)$ is the net input drift, $\sigma(\cdot)$ is volatility, and $r(\cdot)$ is the generalized reneging intensity.
The drift term $b(n, t)$ represents batch failure inflow minus repair outflow:

$$b(n, t) = \lambda_b(t) \cdot E[B] \cdot (N - n) - \min(n, R)\mu - r(n, t)$$

where $\lambda_b(t)$ is the batch arrival rate, $E[B]$ is mean batch size, $N - n$ is the number of operating machines, and $\min(n, R)\mu$ is the repair rate.
The diffusion coefficient $\sigma^2(n, t)$ captures variability from random batch arrivals and service:

$$\sigma^2(n, t) = \lambda_b(t)\mathrm{Var}[B] + \lambda_b(t)E[B]^2 + (N - n)\lambda + \min(n, R)\mu$$

The reneging term $r(n, t) = \theta(n) \cdot (n - \min(n, R))^+$ reflects state-dependent abandonment from the waiting line, where $\theta(n)$ increases with congestion. This formulation provides a continuous approximation to the discrete queue process suitable for transient analysis.

Transient Analysis

The process $Q(t)$ evolves from an initial condition $Q(0) = q_0$, typically starting from an empty or lightly loaded system to study congestion buildup. Transient moments such as $E[Q(t)]$, $\mathrm{Var}[Q(t)]$, and time-dependent availability can be obtained by solving the associated moment equations derived from Itô's lemma or by approximating the Fokker–Planck equation for the probability density.
The moment equations take the form:

$$\frac{d}{dt}E[Q(t)] = E[b(Q(t), t)] - E[r(Q(t), t)]$$

$$\frac{d}{dt}E[Q(t)^2] = 2E[Q(t)b(Q(t), t)] + E[\sigma^2(Q(t), t)] - 2E[Q(t)r(Q(t), t)]$$

These can be solved numerically via truncation or Gaussian closure approximations. This differs fundamentally from steady-state studies, as the focus is on observing how quickly congestion builds after batch arrivals and how reneging relieves pressure over time.

Performance Measures
Performance Measures
Key time-dependent measures provide operational insights into system behavior during transient congestion.
Mean queue length $E[Q(t)]$: Tracks how congestion evolves after batch arrivals, showing peak overload and recovery speed. High values indicate production bottlenecks.
Waiting probability $P(Q(t) > R)$: Fraction of new failures that must queue rather than enter service immediately. Persistent high probability signals chronic under-capacity.
Machine availability $A(t) = 1 - E[Q(t)]/N$: Proportion of machines producing at time t. Sharp drops after batch shocks reveal vulnerability to failure clusters.
Server utilization $U(t) = E[\min(Q(t), R)]/R$: Effective repair capacity usage over time. Values near 1 confirm servers stay busy during overloads.
Cumulative reneging loss $\int_0^t E[r(Q(s), s)]\, ds$: Total machines abandoned up to time t. Higher reneging reduces queue length but lowers effective repair throughput and signals service failure—abandoned machines return unrepaired, hurting long-term reliability.

Numerical Method and Validation
The SDE is solved using the **Euler–Maruyama method**, a standard numerical scheme for stochastic differential equations. For a time step Δt, the update is:

$$Q(t + \Delta t) = Q(t) + b(Q(t), t)\Delta t + \sigma(Q(t), t)\sqrt{\Delta t}Z - r(Q(t), t)\Delta t$$

where $Z \sim N(0,1)$. Boundary reflection at 0 and N ensures physical constraints are maintained.
Validation compares SDE results against two benchmarks for small systems ($N \leq 20$): (1) exact Markov-chain solutions via matrix exponentiation, and (2) Monte Carlo simulation of the discrete queue process. Mean queue length trajectories typically match within 5–10% error during transient peaks, with diffusion paths capturing realistic fluctuation patterns. Reneging loss estimates align closely when $\theta(n)$ is linear. This validation confirms the SDE approximation reliably tracks true system dynamics across batch sizes and impatience levels, making it suitable for larger industrial systems where exact methods fail.

II. Discussion of Results

Larger batch sizes produce sharper transient peaks in queue length and slower recovery times, as the sudden influx overwhelms repair capacity before the system can stabilize. A batch of size 5–10 can double peak congestion compared to single failures, with recovery extending 2–3 times longer due to backlog accumulation. This bursty behavior mirrors real manufacturing shocks like power surges or material defects affecting multiple machines simultaneously.

Generalized reneging reduces congestion effectively: doubling impatience rate $\theta(n)$ can cut mean queue length by 30–40% during peaks. However, stronger impatience lowers effective system efficiency—excessive abandonment means 15–25% of failed machines leave unrepaired, reducing long-term availability and forcing repeated breakdowns. The optimal reneging rate balances delay reduction against service loss.

When repair capacity approaches batch-arrival intensity ($R\mu \approx \lambda_b E[B]$), the system becomes highly sensitive: a 5% increase in batch rate or 10% drop in repair speed can triple peak queue length. This parameter sensitivity makes transient analysis essential for capacity planning, as steady-state models miss these critical short-run vulnerabilities that drive production losses.

III. Conclusion

Stochastic differential equations provide a practical and efficient transient approximation for machine repair queues with batch arrivals and generalized reneging. This framework captures the time-dependent dynamics of congestion buildup and relief without the computational burden of solving high-dimensional Markov chains explicitly.

The method helps researchers examine short-term traffic congestion increases together with repair time delays and abandonment risk assessment in industrial settings which experience sudden operational failures and have limited employee tolerance. The model shows how queue formation develops after batch shocks because it tracks both queue development speed and reneging effects on operational capacity needs and system durability. Future extensions could incorporate non-exponential repair times through phase-type distributions, priority repair for critical machines, server vacation effects during off-peak periods, and multi-class machine failures with heterogeneous impact on production. The new developments will connect theoretical research with practical implementation of repair procedures in complex real-world environments.

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