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Higher Order Thinking Skills of Undana FKIP Mathematics Students in Linear Algebra

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Abstract: *The material structure of Linear Algebra generally consists of 2 categories of material, namely matters relating to technical mathematics and abstract mathematics. For this reason, it is very necessary for students to have higher order thinking skills (HOTS) on the theoretical and applicable meaning of material such as the solution of a Linear Equation System, the skills to prove abstract problems such as General Vector Space, Linear Free, Stretching, Bases, and Dimensions.*

This study was conducted with the aim of analyzing the extent to which the Higher Order Thinking Skills of Mathematics students of FKIP Undana in Linear Algebra. Data obtained through tests and interviews. The test was given to 86 students of the Mathematics Education Study Program FKIP Undana who were programming Linear Algebra courses in the even semester of the 2020/2021 academic year. From the test results, 9 students were selected as research subjects, each of which consisted of 3 people in the High, Medium, and Low categories. Data analysis was carried out in a qualitative descriptive manner.

The results showed that: a) In general, the ability of the mathematics students of FKIP Undana in Linear Algebra was still relatively low. The test results show that more than 50% of students are in the category of low scores. Students' abilities are still dominant in technical problems of mathematical calculations and real concepts, but they are still unable to understand more deeply related to abstract mathematical problems. b) Higher order thinking skills of FKIP Undana mathematics students are still at a low level. HOTS ability is still around C4 where only a small number of students are able to reach C5 level. The ability to prove a general vector space by applying 10 axioms has been done well by some students. Axioms that are still the main obstacle for students in proving vector spaces are closed axioms on addition and closed on multiplication as well as axioms of identity addition and identity of multiplication. c) The combination and relationship of students' understanding regarding the concept of spanning, linear independent, Base, and Dimension is still not comprehensive.

Keywords*: HOTS, Linear Algebra*

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I. Introduction

Linear Algebra is a Mathematics course which is the basis for studying several other subjects such as Linear Programming, Operations Research, Spatial Geometry, and other similar subjects. The main materials studied in Linear Algebra include: Matrices, Systems of Linear Equations, Vectors in Spaces 2 and 3, General Vector Spaces, and Eigen Values [4], [3]. As with other mathematics learning structures, Linear Algebra generally consists of 2 categories of material, namely matters relating to technical mathematical calculations and material that is abstract.

In studying Linear Algebra, Higher Order Thinking Skills (HOTS) are needed which are related to: how to have a deep understanding of the theoretical and applicable meaning of the solution of a System of Linear Equations, namely single solutions, multiple solutions, and no solutions . In addition, skills in proving abstract problems such as: general vector space proving by applying 10 axioms of vector space, sub-space proving by applying 2 axioms. Besides that, it is very necessary to have the ability to interpret geometric and algebraic problems on Combination of Linear, Linear-Free, Span, Base, and Dimensional in a vector space. Furthermore, abstract problems such as Inner Multiply Space and Transformation and Eigen Value problems of a matrix also need to be understood better. Furthermore, the understanding relationship between elements such as Linear Combination, Linear Free, Span, Base, and Dimension is also very important in studying and developing an understanding of Inner Product Spaces, Transformations to Matrix Eigen Values.

For this reason, this study will analyze the extent to which the Higher Order Thinking Skill (HOTS) of mathematics students of FKIP Undana in mathematics, especially in linear algebra. Have the students' HOTS levels reached C5 and C6 or are they still at a lower level? HOTS ability is very important needed and known because with HOTS, one can solve a mathematical problem not only theoretically but can develop it in various forms, both applications and in the form of a higher representation. In relation to the ability to think, Mustaji (2012) also defines critical thinking as thinking rationally and reflectively by emphasizing making decisions about what to believe or do [2]. The following are examples of critical thinking skills, for example (1) comparing and distinguishing, (2) making categories, (2) examining small parts and the whole, (3) explaining causes, (4) making sequences, (5) determine reliable sources, and (6) make predictions. King, Goodson & Rohani, (2012) describe Higher Order Thinking as the ability to solve problems that involve critical, logical, reflective thinking, metacognition, reasoning, and creative and abstract [6]. This is in line with what was stated by Jailani (2018) which presents higher order thinking as a problemsolving activity but provides more complete details of what abilities are priority in problem solving as higher order thinking [5].

II. Research Method

This research is a qualitative descriptive study which is intended to describe and analyze the extent of Higher Order Thinking Skill (HOTS) in Linear Algebra for students of the Mathematics Education Study Program, Faculty of Teacher Training and Education, Nusa Cendana University. The research will be carried out at the Mathematics Education Study Program, Faculty of Teacher Training and Education, Nusa Cendana University, Kupang. The research is focused on students who program Linear Algebra courses in the fifth semester of the 2021/2022 academic year. One of the data needed in this research is the test results of Linear Algebra learning outcomes for students. In this study, a test was given to all Undana Mathematics Education students who were taking Linear Algebra courses in the fifth semester of the 2021/2022 academic year. The test is subjective because with the test it can be seen how the student's process of working on the questions given so that it can be known and students' understanding and how the thinking process is especially related to students' Higher Order Thinking Skills. Some parts of Linear Algebra material that need to be analyzed and included in the test include (1) the theoretical and applicable meaning of the solutions of a Linear Equation System such as single solutions, multiple solutions, and no solutions; (2) proving abstract problems such as proving the general vector space and sub-spaces; (3) geometric and algebraic interpretation of the Combination of Linear, Linear-Free, Span, Base, and Dimensional in a vector space; (4) understanding of Inner Multiplication Space; (5) understanding of Linear Transformation; and (6) understanding of the eigenvalues of a matrix. After the test results are checked, then tabulation is carried out according to the scores obtained by the students. From this tabulated data, the students were further categorized into 3 categories, namely high category, medium category, and low category. From each category group, 3 people were selected and used as research subjects to be interviewed. Interviews were conducted on research subjects. Interviews are based on student work results. The questions given are tracking the understanding of what has been done previously. In the interview process, additional questions or tracking questions can also be given to be reworked by the research subject. This is intended to be able to find out in more detail the understanding and ability of students' Higher Order Thinking towards Linear Algebra. The interview focused on how students think and thinking skills, namely students' Higher Order Thinking. In this case, it will be sought to find out further how the level of thinking of students has reached levels C4, C5, and C6 [1]. The research data will be validated through 2 techniques, namely triangulation techniques and extension of subject participation. Triangulation is done by comparing test result data and interview data, whether the questions that have been done are really understood and whether the questions that have not been completed are really not understood by students. Likewise, whether the questions that were not completed were actually not understood or were it due to other factors that hindered the process of working on the existing questions. While the extension of participation is participation in the research setting to obtain more valid data for a more accurate interpretation [7].

Furthermore, the interpretation of the students' HOTS abilities in Linear Algebra will be carried out. This interpretation deals with representation and HOTS but also how students can use their initial abilities and mathematical skills in solving Linear Algebra problems.

III. Result And Discussion

The test results show that the average grade obtained is 60.54. Then the scores from the initial test are divided into 3 categories, namely the category of students with high ability (T), the category of students with moderate ability (S) and the category of students with low ability (R) as shown in table 1 below:

Table 1. Categorial Distinction Based on Students' Ability

No	Value Range	Category	Number of Students
1	$x \geq 80$	High	12
2	$60 \leq x < 80$	Medium	36
3	$x \leq 60$	Low	67
Total			115

x = student grades

From table 1 it can be seen that only 12 or 10.43% of students are in the category with high ability and there are still 58.62% of students who are in the category with low thinking ability. This shows that there are still more than 50% of class abilities related to Higher Order Thinking Skills that are still in the low category. This figure also indicates that students' understanding of Linear Algebra is still in the low category. If it is associated with the graduation rate with a minimum score of >=60, it is said that the class completeness has not reached 50%.

From the range of values in table 1, 9 students were taken as research subjects consisting of 3 students in each category which were then coded T1, T2, and T3 for the three high group students, then S1, S2, and S3 for medium group students. While R1, R2, and R3 for students in the Low group. The subjects of this study were then interviewed.

Results of Interviews with Subjects T1

1. T1 is actually able to correctly answer question number 1 where it can be seen that T1 is able to explain well the proof of the 10 axioms of vector space according to the definition.
2. T1 correctly answers question number 2 which is indicated by the ability to explain the proof of sub-vector space according to the given problem
3. T1 can correctly answer question number 3. The results of the work show that they are able to explain using theories about linear dependence to find the value of T1 works by making the vector u as a linear combination of the vectors v and w, and gets the SPL for u = k1v + k2w. then T1 changes the SPL to an enlarged matrix, then it is solved to get the matrix form $\begin{pmatrix} 1 & 1 & -2\alpha \\ 0 & 1 & -(2\alpha-1) \\ 0 & 0 & \frac{4\alpha^2-2\alpha-2}{2} \end{pmatrix}$.
4. T1 wants the SPL to have a solution so that T1 makes 4α^2-2α-2=0, and we get =1 or =½, so T1 concludes that when =1 or =-½, the vector set u, v and w is not linearly independent.
5. T1 is able to correctly answer question number 4 which can understand well that if {u, v, w} is a basis then it must satisfy linear independence and span in R3.
6. T1 cannot work on problem number 5 because it understands that if {v1, v2, v3} is a basis then {v1, v2, v3} is linearly independent and spans across V. But when there is a set {u1, u2, u3} where u1= v1, v2=v1+v2, u3=v1+v2+v3, T1 has difficulty showing that {u1,u2, and u3} are bases. Next, T1 is faced with a logical question that if {v1,v2,v3} base, then what information is obtained? T1 is limited to thinking that {v1,v2,v3} is linearly independent and spans in V. Furthermore, in terms of understanding what information is obtained if {v1, v2, v3} is linearly independent and spans across V, T1 understands that this This can be obtained by simply finding the determinant of the coefficient matrix formed from {v1,v2,v3} which is not zero. Furthermore, T1 does not understand any further about linear and span-free problems.

Results of Interviews with Subjects T2:

1. T2 correctly answers question number 1, namely the proof of the General Vector Space. This is indicated by the ability and understanding in explaining well the application of the 10 axioms of vector space according to the definition.
2. T2 is also capable of performing technical calculations for the addition of two vectors u+v and scalar multiplication of ku in a vector space. But when making a decision that whether u+v and ku are elements of the set W, where W={(a, b, c)┃b=a+c+1}, T2 still makes an error. T2 assumes that u+v is an element of W and ku is also an element of W. So T2 concludes that W is a sub-vector space of R3. The results of the interview show that T2 is sure of the answer given, he immediately corrects his answer and gives the correct answer and reason that u+v is not a W element, because b1+b2 = (a1+a2) + (c1+c2) + 2 and ku =ka+kb+k, where ka is any real number, so it could be that k is not 1. Then T2 concludes that W is not a sub-vector space of R3. This shows that T2 is able to understand well the identity elements in addition operations and multiplication operations in a vector space where the identity of addition is not only the number 0 and the identity of multiplication is not only the number 1 but any element, whether any real number, or other objects can be used as as an identity in a vector space.
3. T2 correctly answered question number 3 which is related to linear independent problem. This is shown by T2 where the person concerned understands that if u, v, and w are not linearly independent then the solution form of k1u+k2v+k3v=0, is a multiple solution, which means k1 or k2 or k3 is not 0. In this case it is explained that when obtaining a homogeneous Linear Equation System formed from k1u+k2v+k3v=0, T2 then converts it into a matrix form AX=B, where A is a coefficient matrix, X is a variable matrix and B is a constant matrix, then T2 makes the determinant of matrix A=0. The purpose of making det A=0, is so that the solution of the homogeneous SPL is not only a trivial solution (k1=k2=k3=0) but there is a nontrivial solution as well so as to guarantee that u, v, and w are not linearly independent. And obtained an equation in that is 3–¾α–¼α=0. Then solve and get =1 or =-½.

4. T2 is able to correctly answer question number 4. He understands well that if {u, v, w} is a basis then it must satisfy linear independence and span in R3.
5. T2 cannot do problem number 5. Similarly, T1, T2 knows that if {v1,v2,v3} is a basis then {v1,v2,v3} is linearly independent and spans across V. But when there is a set {u1, u2, u3} where u1=v1, v2=v1+v2, u3=v1+v2+v3, then T1 has difficulty showing that {u1,u2, and u3} are a basis. Next in terms of what information is obtained for a condition where if {v1, v2, v3}? T2 simply repeats the answer that {v1,v2,v3} is linearly independent and spans at V. Furthermore, in terms of what information is obtained if {v1,v2,v3} is linearly independent and spans at V? T2 answers that it is enough to find the determinant of the coefficient matrix formed from {v1, v2, v3} which is not zero. The interviewer tries to ask again and then from that information what can you do? T2 also tries to assume v1= $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, v2= $\begin{pmatrix} d \\ e \\ f \end{pmatrix}$, dan v3= $\begin{pmatrix} g \\ h \\ i \end{pmatrix}$, so that the determinant $\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$ not zero.

This means u1 = v1 = $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, sou2 = v1 + v2 = $\begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$,

u3 =v1 + v2 +v3= $\begin{pmatrix} a+d+g \\ b+e+h \\ c+f+i \end{pmatrix}$

6. T2 continues that it is sufficient to show that the determinant $\begin{pmatrix} a & a+d & a+d+g \\ b & b+e & b+e+h \\ c & c+f & c+f+i \end{pmatrix}$ also not zero. But T2 is not able to show this. The interviewer asked again, can T1 use the properties of the determinant function to solve the determinant above. T1 answered no.

Results of Interviews with Subjects T3

1. T3 correctly answered question number 1 where T3 was able to explain well the proof of 10 axioms of vector space according to the given definition.
2. T3 correctly answered question number 2. This is shown by the ability to explain the proof of sub-vector space by involving two axioms.
3. T3 understands that if u, v, and w are not linearly independent then the form of the solution of k1u+k2v+k3v=0, is a multiple solution, which means that k1 or k2 or k3 is not 0. So, when faced with a homogeneous SPL formed from k1u+ k2v+k3v=0, T2 converts it into a matrix form AX=B, where A is a coefficient matrix, X is a variable matrix, and B is a constant matrix. Furthermore, T2 makes a determinant of the matrix A=0 with the aim that the solution of the homogeneous SPL is not only a trivial solution (k1=k2=k3=0) but also a non-trivial solution so as to guarantee that u, v, and w are not linearly independent. However, when T3 is faced with the equation in, namely $3-\frac{3}{4}\alpha-\frac{1}{4}\alpha=0$, T3 cannot solve it. This is because T3 does not understand how to find a solution to a polynomial equation with degree 3. However, overall T3 is able to use the information or theory that has been learned to solve problem number 3, which is about theory and linear independent problems.
4. T3 is able to correctly answer questions about the Base problem. T3 understands well that if {u, v, w} is a basis then it must satisfy linear independence and span across R3.
5. T3 cannot do problem number 5, where T3 knows that if {v1, v2, v} is a basis then {v1, v2, v3} is linearly independent and spans V. But when there is a set {u1, u2, u3} in where u1=v1, v2=v1+v2, u3=v1+v2+v3, T3 has difficulty showing that {u1, u2, and u3} are bases. Furthermore, if T3 is faced with the condition that if {v1, v2, v3} basis then what information is obtained? In this regard, T3 can only understand that {v1, v2, v3} is linearly independent and spans at V. The same applies to what information is obtained if {v1, v2, v3} is linearly independent and spans at V, T3 understands that it is enough to find the determinant of the coefficient matrix formed from {v1, v2, v3} is not zero.

Results of Interviews with Subjects S1:

1. S1 correctly answered question number 1.
2. Able to explain the proof of 10 axioms of vector space according to the definition given properly
3. S1 correctly answered question number 2, S1
4. Able to explain proof of sub-vector space according to the given problem
5. S1 could not do the questions given. Then the interviewer asks what S1 understands that u, v, and w are not linearly independent. S1 answered that he only understands about linear independence and is confused if it is

not linearly independent. Then the interviewer asked again what S1 understood about linear independence. S1 answers that u, v, w are linearly independent if k1u+k2v+k3w=0 where k1=k2=k3=0. Then the interviewer asked again if one of the ki is not zero, what about it? Then S1 answered that it was not linearly independent. Then the interviewer asked S1 to solve it, and S1 was able to solve the problem as T3, and had the same reason because he forgot how to solve a polynomial of degree 3. However, S1 used the try end error method to get the value of and got the value =1.

6. S1 is able to answer this question correctly. He understands well that if {u, v, w} is a basis then it must satisfy linear independence and span at R3.

7. S1 cannot do problem number 5. S1 is not much different from T1, T2 and T3, knowing that if {v1, v2, v} is a basis then {v1, v2, v3} is linearly independent and spans in V. But when there is a set {u1, u2, u3} where u1=v1, v2=v1+v2, u3=v1+v2+v3, S1 has difficulty showing that {u1, u2, and u3} are bases. The interviewer tried to ask T3, if {v1, v2, v3} what information base was obtained? T3 only repeats the answer that {v1, v2, v3} is linearly independent and spans at V. The interviewer tries to ask again, what kind of information is obtained if {v1, v2, v3} is linearly independent and spans at V? S3 doesn't answer.

Results of Interviews with Subjects S2:

1. S2 menjawabdenganbenarsoalnomor 1.
2. Able to explain the proof of 10 axioms of vector space according to the definition given properly
3. S2 answered that the set W is not a subspace of R3, but S2 still has an error in proving the axiom 2 vector space. S2 has proven and explained well the results of u + v but to me, S2 is procedurally capable of working, but gives the wrong conclusion. The result of ku is (ka, ka+kc+k, kc) and S2 answers that ku belongs to the set W, where W = {(a, b, c)┃b=a+c+1}. S2 thought that k could be 1, so I was included in the set W. But when the interviewer asked what k was? S2 is able to answer that it is a real number scalar. Then the interviewer asked whether k is only equal to 1, S2 answered no. Then the interviewer asked again then what about k which is not equal to 1? T1 is able to answer does not meet. Then the interviewer asked whether the elements of the set W can apply to any value of k. T1 answered no and then gave a good conclusion.
4. S2 has the same case as S1 which is not being able to work on the questions given. Then the interviewer asks what S1 understands that u, v, and w are not linearly independent. S2 already understands that if it is not linearly independent, then vector 0 does not form a linear combination of u, v, and w. But S2 doesn't understand how to solve it. Then the interviewer asks what S2 does if {u, v, w} is linearly independent. S2 explains that S2 will solve the problem by showing the determinant of the coefficient matrix formed from the SPL obtained is not zero, in order to ensure that k1=k2=k3=0. Then the interviewer asks what if the coefficient matrix det is zero? S1 answered that the SPL did not have a solution. The interviewer asked again, if you look at the SPL formed, it turns out that the right side of the SPL is 0, then what does it mean? S2 answered that the SPL is homogeneous, so the possibility is that if the determinant of the coefficient matrix is zero then the SPL has a non-trivial solution. Then the interviewer asked S2 to solve the problem and S2 was able to complete it until it was finished.
5. S2 is able to answer this question correctly. He understands well that if {u, v, w} is a basis then it must satisfy linear independence and span at R3.
6. For question number 5, the work pattern of the S2 subject is the same as that of the S3 subject.

Results of Interviews with Subjects S3:

1. From the results of the S3 work, it can be seen that the person concerned was wrong in doing the questions because he could not apply axioms 2, 7 and 8, but after being confirmed again, the S3 subject was able to correct the mistakes that had been made properly.
2. S3 correctly answered question number 2
3. Able to explain proof of sub-vector space according to the given problem
4. S3 works on problem number 3 using the theory that if {u, v, w} is not linearly independent, then one of the vectors in the set is a linear combination of the other 2 vectors. S3 explains if u is a linear combination of u and v, then, it can be written:

u = k1v + k2w.

$$\begin{pmatrix} \alpha \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = k1 \begin{pmatrix} -\frac{1}{2} \\ \alpha \\ -\frac{1}{2} \end{pmatrix} + k2 \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \alpha \end{pmatrix}$$

S3 then explains that it means that it can make k1 = 1 and k2 = 0, because that means that {u, v, and w} are not linearly independent. Until he gets

$$\begin{pmatrix} \alpha \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = 1 \begin{pmatrix} -\frac{1}{2} \\ \alpha \\ -\frac{1}{2} \end{pmatrix}$$

And we get α=-1.
Then the interviewer asked what about k1 and other k2 that met? S3 answered what he understood if that was the case. then the interviewer asked again whether the possible value of is only -1/2. After thinking for a long time, S3 then answered that it could also be =1. The interviewer asks again why? It can be if k1 = -1 and k2 = -1 so that we get

$$\begin{pmatrix} \alpha \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\alpha \\ \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\alpha \end{pmatrix}$$

then α=1. And the interviewer asked again, if k1=1 and k2=1, what is the value of α? He said could not. Because

$$\begin{pmatrix} \alpha \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \alpha \\ -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \alpha \end{pmatrix}$$

In the first line I getα =-1 but not with the other lines, so we can't use k1=1 and k2=1.
Even though the answer from doctoral doctor is different from other friends, he is already able to use the information or theory that he has learned well.
5. S3 is able to correctly answer this question. He understands well that if {u, v, w} is a basis then it must satisfy linear independence and span at R3.
6. Number 5 Equal to S3

Results of Interviews with Subjects R1
1. The proofs of axioms 2, 8, 9 and 10 are still wrong, but the other proofs of axioms are correct. When asked to see the error, the subject has not been able to know where the error is. When led to proof of axioms 2 and 8, the subject can correct errors in axioms 9 and 10. After that, the subject is able to conclude that the set with the given operation belongs to a vector space.
2. R1 correctly answered question number 2
3. Able to explain proof of sub-vector space according to the given problem
4. R1 spelled question number 3 but did not finish. From R1's worksheet, it is known that R1 immediately writes

α – ½ - ½ = 0
α = ½ + ½
α = 1

the interviewer asked why R1 wrote down like that? R1 answered that as far he understood {u, v, w} are not linear free, then k1u+k2v+k3w=**0** so we get if the first components of u, v, and w are added up is α–½-½=0.
The interviewer asked again what do you mean {u, v, w} is not linearly independent k1u+k2v+k3w=0? R1 did not answer. The interviewer asked again, what about the values of k1, k2, k3 if they are not linearly independent? R1 repliedk1≠k2≠k3. R3 continues if it substitutes the value of α=1, then k1=1, k2=-2/4 and k3=0. Because k1≠k2≠k3 then {u, v, w} are not linear free. The interviewer asked again, if what is obtained is k1=k2=k3=2, does it mean that it is linearly independent? He answered no, linearly independent if k1=k2=k3=0.
5. R1 is able to answer this question correctly. He understands well that if {u, v, w} is a basis then it must satisfy linear independence and span at R3.
6. R1 can't do problem number 5. R2 is not much different from other research subjects, knowing that if {v1, v2, v} is a basis then {v1, v2, v3} is linearly independent and spans in V. But when there is a set {u1, u2, u3} where u1 = v1, v2=v1+v2, u3=v1+v2+v3, S1 has difficulty showing that {u1,u2, and u3} are bases. The interviewer tries to ask R2, if {v1,v2,v3} what information base is obtained? R2 did not answer don't know.

Results of Interviews with Subjects R2:
1. R2 Unable to work on question number 1, but when guided the subject is able to work on and explain the 10 axioms well.
2. Subject R2 answered that the set W is not a subspace of R3, but s2 there is still an error in proving the axiom 2 vector space. R2 has proven and explained well that u + v is not closed, but for me, R2 is procedurally capable of working, but gives the wrong conclusion. The result of ku is (ka, ka+kc+k, kc) and R2 answers that

ku belongs to the set W, where W = {(a, b, c) | b=a+c+1}. R2 thinks that k can be 1, so I fall into the set W. But when the interviewer asks what is k? R2 is able to answer that it is a real number scalar. Then the interviewer asked whether k is only equal to 1, R2 answered yes. Then the interviewer asks again and then k is any real number, then why is k only equal to 1, then R2 answers that I am a member of the set w. The interviewer asks again, why in the proof section of u + v, T2 can conclude that it is not a set W, and T2 answers because the second component of the result u + v does not match the second component that should have a set W. The second component of u+v is (a1+a2)+(c1+c2)+2 while the second component of the set W is a+c+1 so u+v is not a W element. Then the interviewer reiterated my point, we can't choose only 1 k, but must hold for every k real numbers. R2 finally understands that and changes his conclusion that ku is not an element of the set W, because it does not hold for every real number scalar.

3. R2 only wrote this on the work sheet $\begin{vmatrix} \alpha & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \alpha & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \alpha \end{vmatrix}$. Then the interviewer asked, what did R2 write? R2 answers the determinant of the coefficient matrix. The interviewer asked again, why did he write that down, R2 answered that if what he understood was to show linear independence, then a is sufficient to show that the coefficient matrix formed from the set {u, v, w} is not zero. So he thought that if it was not linearly independent, he might have to find the determinant of the coefficient = 0. The interviewer asked again, why is that? R2 did not answer. Then the interviewer asked again, can he solve it if the coefficient determinant = 0? R2 answered no. Because he's having a hard time performing the surgery that's still thereα. The interviewer asked again, can you give it a try? R2 tried it and he got$8\alpha^3$-6α-2=0. Then, he answered α=1. The interviewer asked again why α=1, he answered because if α=1 is substituted to $8\alpha^3$-6α–2 the result is 0.

4. R2 is able to answer this question correctly. He understands well that if {u, v, w} is a basis then it must satisfy linear independence and span at R3.

5. R2 can't do problem number 5. R2 is not much different from other research subjects, knowing that if {v1, v2, v} is a basis then {v1, v2, v3} is linearly independent and spans in V. But when there is a set {u1, u2, u3} where u1=v1, v2=v1+v2, u3=v1+v2+v3, S1 has difficulty showing that {u1, u2, and u3} are bases. The interviewer tries to ask R2, if {v1, v2, v3} what information base is obtained? R2 doesn't answer.

Results of Interviews with Subjects R3:

1. R3 Can't work on question number 1, the question is in R+ but the subject is working on R2. The subject is not able to process the information provided by the interviewer, that the question is in R+. so that it is still difficult to prove the 10 axioms correctly by the subject.

2. Subject R3 can work on question number 2, but the subject works without seeing the condition that b=a+c+1. So, we conclude that the set W is a vector subspace of R3. When the interviewer asked the subject to look at the conditions for the set W, the subject was confused. Finally, the interviewer asked the subject to see that there was a condition b=a+c+1. The subject then understood, but when the u+v and ku operations were performed, the subject still did not pay attention to the conditions for the set W. So, the subject still concluded that u + v and ku are members of the set W.

3. R3 did not work on question number 3, and when asked by the interviewer what information R3 got from question number 3, R3 did not answer. The interviewer asked again, Does R3 know what is meant by linearly dependent? R1 answered no. Then R3 asked again, if it's linear, does R3 know? R3 knows, that is, if it satisfies k1u+k2v+k3w=0. The interviewer asked again, what does R3 mean by fulfilling k1u+k2v+k3w=0.? R3 did not answer.

4. R3 is able to correctly answer this question. He understands well that if {u, v, w} is a basis then it must satisfy linear independence and span at R3.

5. R3 was not asked because R3 did not do the question and was unable to answer the c4 level question.

IV. Discussion

From the test results on 115 students, there were 12 students or 10.34% who were in the High ability category, 36 or 31.30% of the students were in the medium category, while 67 or 58.26% of the students were in the low category. In general, it shows that there are still around 50% of students who are still in the low level category. However, in the interview process it was still seen that some students or subjects in the intermediate category were able to demonstrate their ability to answer high-level questions which at the time of the test they could not do it. This shows that the ability of a person shown in the written test is not necessarily in accordance with what was shown at the time of the interview. This condition especially occurs in questions related to high-level logic where students cannot show it in writing but can think more critically in an oral atmosphere or vice versa.

Specifically, students' understanding in the high ability category is very good, especially with regard to proving the General Vector Space by applying 10 axioms. The same understanding also occurs for the proof of sub-vector space which only involves 2 axioms. By successfully proving vector space and vector subspace, it means that students have a good understanding of identity and inverse elements in a vector space, both for addition and multiplication operations. Likewise, the implication is that students' understanding in the high category is said to be good, especially in terms of closed properties problems for vector addition and multiplication in vector space. With this understanding relationship, the student actually understands the concepts and techniques related to general vector spaces. Thus, according to the level of difficulty of the questions given, the group of students is already at level C4. Furthermore, students' abilities in relation to the concepts of Linear Combination, Stretching, Linear-Free, Base, and Dimensional are still not fully and continuously mastered, especially relationships and variations or combinations of problems related to these elements. In general, all students have not been able to work on varied questions involving all elements such as spanning, linear independence, basis, and dimensions. This shows that the C5 level of Higher Order Thinking has begun to appear on a limited scale or it can be said that it is still lacking, while the C6 level of thinking for students has not been achieved at all.

For students in the intermediate category, in general, they already have skills in proving a general vector space by applying 10 axioms. Likewise, students can prove Sub-Vector Spaces by applying 2 axioms, namely closed axioms on addition and closed on scalar multiplication. This skill also actually shows that the understanding of the General Vector Space is quite good and is at the C4 thinking level, but the C5 and C6 thinking levels cannot be carried out by the middle group students.

The analysis of students in the low group shows that only a small number of students can work on problems about proving a general vector space by involving 10 axioms of vector space. The main weakness of the students is in the application of axioms related to closed properties, both closed to addition and closed to multiplication and the application of the inverse property of a vector. If someone is not able to prove a vector space, then the next work is only understanding technical problems where the student does not understand philosophically from a vector space. With this understanding, it can be said that the thinking level of the low group students is still at the C3 level and below.

From all students in the research class, it can be said that the dominant students have Higher Order Thinking Skills at the C3 and C4 levels. There are only a small number of students who managed to reach level C5, while there were no students who were able to have HOTS level C6. In general, students' ability to prove General Vector Spaces by applying 10 axioms has been done well. Likewise in proving vector subspaces by involving 2 axioms. However, the understanding relationship between Linear Combination, Linear Free, Base, and Dimension has not been mastered well.

V. Conclusion

From the results of research and discussion in previous chapters, it can be concluded several things as follows:

a. a. In general, the ability of FKIP Undana mathematics students in Linear Algebra is still relatively low. This is indicated by the test results where more than 50% of students are in the category of low score acquisition. This also shows that students' abilities are still dominant in technical problems of mathematical calculations and real concepts, but they are still unable to understand more deeply related to abstract mathematical problems.

b. b. Higher order thinking skills of FKIP Undana mathematics students are still at a low level. HOTS ability is still around C4 where only a small number of students are able to reach C5 level. The ability to prove a general vector space by applying 10 axioms has been done well by some students. However, there are still many students who, even though they have succeeded in proving general vector spaces, still make mistakes in proving subspaces using only 2 axioms. This proves that the ability to general vector space is not perfect. The axioms that are still the main obstacle for students in proving vector spaces are closed axioms on addition and closed on multiplication. Likewise, the axioms related to the identity of addition and identity of multiplication. Error models like this show that understanding of vector space in general is not very good, most still think that a is still limited as in vector space n.

c. c. The combination and relationship of students' understanding of the concept of spanning, linear independent, Base, and Dimension is still not comprehensive. In general, all students have not been able to work on variation questions that involve all these elements. This shows that students' Higher Order Thinking has not yet reached levels 5 and 6.

Refrences

[1]. Bloom, B. S. (1956). *Taxonomy of educational objectives: The classification of educational goal*s. London: Ann Arbor, MI: Addison Wesley Publisher.

[2]. Garak, S.S. & Samo, Damianus. 2020. *Developing Higher Order Thinking Skills of mathematics Education Students, A Grounded Theory. International Journal of Innovation, Creativity and Change*. www.ijicc.net. Volume 14, Issue 5, 2020

[3]. Garak, S.S. 2017. *Teori dan soalAljabar Linier*. Universitas Nusa Cendana.
[4]. Howard, Anton. 1987. *Aljabar Linear ElementerEdisikelima*. Jakarta: Erlangga
[5]. Jailani., dkk. 2018. *Desain PembelajaranMatematikauntukMelatih Higher Order Thinking Skil*l. Yogyakarta: UNY Press.
[6]. King, F.J., Goodson, L., & Rohani, F. (2012). *Higher Order Thinking Skill: Definition, Teaching Strategies, Assessment*. Tersedia di: http://www.cala.fsu.edu/files/higher_order_thinking_skills.pdf.
[7]. Moleong, L.J. 2008.*MetodologiPenelitianKualitatif*, Bandung: PT. Remaja Aksara.

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