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TOTALLY $G_{(b*g)*}$ Continuous Function in Grill Topological Spaces

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Abstract : *The purpose of this paper is to introduce and study a new class of totally continuous function called totally $G_{(b*g)*}$, continuous function, Its relation to othercontinuous function are obtained.*

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I. Introduction

It is found from literature that during the recent years many topologists are interested in the study of generalized type of closed sets. For instance, a certain form of generalized closed sets was initiated by Levine [7]. Following this trend, we have introduced and investigated a kind of generalized closed sets the definition being formulated in terms of grills. The concept of grill was first introduced by Choquet [2] in the year 1947. From subsequent investigations it is revealed that grills can be used as an extremely useful device for investigation of a number of topological Problems.

Many topologists have put forth various types of generalizations of continuity T. M Nour[8] introduced and studied totally semi continuous function. In this paper we study totally $G_{(b*g)*}$continuous function.

II. Preliminaries

Definition 2.1: A nonempty collection G of non-empty subsets of a topological space X is called a grill if (i) $A \in G$ and $A \subseteq B \subseteq X \implies B \in G$ and (ii) $A, B \subseteq X$ and $A \cup B \in G \implies A \in G$ or $B \in G$.
Let G be a grill topological space (X, τ) In an operator $\Phi : P(x) \longrightarrow P(x)$ was defined by$\Phi(A) = \{ X \epsilon X / U \cap A \in G, \forall U \in \tau(x)\}$, $\tau(x)$ denotes the neighborhood of x. Also the map $\Psi: P(X) \to P(X)$, given by $\Psi(A) = \{A \cup \Phi(A)$ for all $A \in P(X)$. Corresponding to a grill G on a topological space (X, τ) there exists a unique topology τ_G on X given by $\tau_G = \{U \subseteq X / \Psi(X - U) = X - U\}$ where for any $A \subseteq X$, $\Psi(A) = A \cup \Phi(A) = \tau_{G-}cl(A)$.

Thus a subset A of X is τ_G – closed (resp. τ_G – dense in itself) if $\psi(A) = A$ or equivalently if $\Phi(A) \subseteq A$ (resp. $A \subseteq \Phi(A)$).
In the next section, we introduce and analyze a new class of generalized continuity, called totally $G_{(b*g)*}$ continuity. Throughout the paper, by a space X, we always mean a topological space(X,τ) with no separation properties assumed. If $A \subseteq X$, we shall adopt the usual notations int(A) and cl(A) respectively for the interior of A and closure of A in (X,τ). Similarly, whenever we say that a subset A of a space X is open (or closed), it will mean that A is open (or closed) in (X, τ).For open and closed set with respect to any other topology on X, eg. τ_G we shall writeτ_G – open τ_G – closed. The collection of all open neighborhoods of a point x in(X, τ) will be denoted $\tau_{(x).}$ (X, τ, G) denotes a topological space(X, τ) with a grill G.
Definition 2.2: A subset A of a topological space (X,τ) is called
1. b open if $A \subseteq \text{int}\, cl(A) \cup cl(\text{int}(A))$
2. b^*g closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is b open
3. (b*g)*closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is b^*g open
4. θclosed if $A = \theta cl(A)$ where $\theta cl(A) = \{ x \in X : cl(U) \cap, A \neq \theta\ \forall\ U \in \tau$ and $X \epsilon U\}$
5. δclosed if $A = \delta\, cl(A)$ where$\delta\, cl(A) = \{ x \epsilon U: \text{int}(cl(U) \cap A \neq \theta, \forall\, U \in \tau$ and$x \epsilon U\}$.

Definition 2.3:A function f: (X, τ)→ (y, σ) is called totally continuous if $f^{-1}(V)$ is clopen in X, for every open set V of Y.

3. TOTALLY $G_{(b*g)*}$CONTINUOUS FUNCTION

Definition 3.1: A subset A of a (X,τ,G) is called $G_{(b*g)*}$ closed if Φ(A) ⊆ U whenever A⊆ U and U is b^*g open in X.

Definition3.2:A function f: (X,τ,G) → (Y, σ) is called totally $G_{(b*g)*}$ continuous of $f^{-1}(V)$ is $G_{(b*g)*}$clopen in X, for every open set V of Y.

Theorem 3.3: The following statements are equivalent for a function f: (X, τ, G)→ (Y, σ).

1. f is totally $G_{(b*g)*}$ continuous
2. For each xϵX and for each open set V of Y containing f(x) there exists a $G_{(b*g)*}$ clopen set U of X such that f(U) ⊆ V.

Proof : (i) => (ii)

Let xϵXand V be open in y containing f(x). Then x∈f^{-1}(V) which is $G_{(b*g)*}$clopen in X. $f(f^{-1}(V)) \subseteq V$

(ii) => (i)

Let V be open in Y and x∈ $f^{-1}(V)$. Then f(x)∈ V. There exists a $G_{(b*g)*}$clopen set U_x of X such that $f(U_x)$⊆V. Hence U_x⊆ $f^{-1}(V)$. $f^{-1}(V) = \cup_{x\in f^{-1}(V)} U_x$, which is $G_{(b*g)*}$clopen in X.

Remark3.4: It is clear that every totally $G_{(b*g)*}$ continuous function is $G_{(b*g)*}$ continuous. But the converse need not be true can be seen from the follow example.

Example3.5: Let X={a,b,c},$\tau = \{\Phi, \{a\}, X\}$ $G = \{\{a,\}, \{b\}, \{c\}, \{a,c\}, \{a,c\}, \{b,c\}, X\}$Define $f: (X, \tau, G) \to (X, \tau)$ to be the identity function f is $G_{(b*g)*}$ continuous but not totally $G_{(b*g)*}$ continuous as$f^{-1}(\{a\})$={a} is not $G_{(b*g)*}$ closed.

Remark3.6: It is clear that totally continuous function is totally $G_{(b*g)*}$ continuous. But the converse need not be true can be seen from the following example.

Example 3.7: Let X={a,b,c}$\tau = \{\Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\ X\}$G={{b, c} , X}Define $f: (X, \tau, G) \to (X, \tau)$ by f(a)=a, f(b)= a, f(c)=b

f is totally $G_{(b*g)*}$ continuous but not totally continuous as f^{-1}({a}) = {a, b} is not closed.

Definition3.8:A space(X, τ, G) is said to be $T_{G(b*g)*}$space if every $G_{(b*g)*}$ open set of X is open in X.

Theorem3.9:A function f: (X, τ, G) → (Y, σ) is totally $G_{(b*g)*}$ continuous and X is $T_{G(b*g)*}$space then f is totally continuous

Proof : Let V be open in Y. Then f^{-1}(V) is $T_{G(b*g)*}$clopen in X. As X is a $T_{G(b*g)*}$ space $f^{-1}(V)$ is clopen.

Definition3.10:A topological space X is said to be $G_{(b*g)*}$ connected if it cannot be written as the union of two nonempty disjoint $G_{(b*g)*}$ open sets.

Theorem 3.11: If f is a totally $G_{(b*g)*}$ continuous function from a $G_{(b*g)*}$ connected space X onto any space Y, then Y is an indiscrete space.

Proof : If possible let Y be not indiscrete. Let A be a proper nonempty open subset of Y. Then $f^{-1}(A)$ is a proper nonempty $G_{(b*g)*}$ clopen subset of X, which is a contradiction to the fact that X is$G_{(b*g)*}$ connected.

Theorem 3.12 A topologicalspace (X,τ, G)is $G_{(b*g)*}$ connected if and only if every totally $G_{(b*g)*}$ continuous function from a space (X,τ, G) into any T_0 space (Y,σ)is constant.

Proof: Let X be not $G_{(b*g)*}$ connected. Let every totally $G_{(b*g)*}$ continuous function from (X,τ, G) to (Y, σ) be constant. Since (X,τ, G) is not $G_{(b*g)*}$ connected there exists a proper nonempty $G_{(b*g)*}$ clopen subset A of X. Let Y = {a, b}, $\sigma = \{\Phi, \{a\}, \{b\}, Y\}$ be a topology on Y. Let f: (X,τ, G) → (Y, σ) be a function such that f(A)= {a}, f(Y-A)={b}. Then f is non constant and totally $G_{(b*g)*}$ continuous such that Y is T_0 which is a contradiction. Hence X must be $G_{(b*g)*}$ connected.

Conversely, let X be $G_{(b*g)*}$ connected. Let f: X → Y be totally $G_{(b*g)*}$ continuous. Let a, b be distinct points of X such that $f(a) = \alpha \neq \beta = f(b), \alpha, \beta \in Y$ and they are distinct. As Y is T_0 there exists open set U containing α but not β. So U is a proper open subset of Y. $f^{-1}(U)$ is a proper $G_{(b*g)*}$ clopen subset of X, which contradicts X is $G_{(b*g)*}$ connected. Hence f must be constant.

Theorem 3.13: Let f: (X, τ, G) → (Y, σ) be a totally $G_{(b*g)*}$ continuous function and Y is T_1 space. If A is a nonempty $G_{(b*g)*}$ connected subset of X, then f(A) is a single point.

Proof: Obvious.

Lemma 3.14: If A $\in G_{(b*g)*}O(X)$ and B $\in G_{(b*g)*}$ O(Y) then A x B $\in G_{(b*g)*}$O(X x Y).

Theorem 3.15: If the function f_i: $X_i \rightarrow Y_i$is totally $G_{(b*g)*}$ continuous function for each i=1, 2 then f_1 x f_2 : X_1 x $X_2 \rightarrow Y_1$ x Y_2 defined by (f_1 x f_2)(x_1 ; x_2)=($f_1(x_1)$, $f_2(x_2)$), for each $x_1 \in X_1$, $x_2 \in X_2$ is totally $G_{(b*g)*}$ continuous.

Proof: Let V_1 x V_2 $\in$O(Y_1 x Y_2) then $V_1 \in O(Y_1)$, $V_2 \in O(Y_2)$.f_1^{-1} (V_1)$\in G_{(b*g)*}$ CO(X1), $f_2^{-1}(V_2) \in G_{(b*g)*}$ CO(X_2) (f_1 x f_2)$^{-1}$ (V_1 x V_2) = (f_1^{-1} (V_1),$f_2^{-1}(V_2)$) = (f_1^{-1} (V_1) x $f_2^{-1}(V_2)$) $\in G_{(b*g)*}$ CO(X_1 x X_2). Hence f_1 x f_2 is totally $G_{(b*g)*}$ continuous.

Definition 3.16: Let (X,τ, G) be a topological space. Then the set of all points y in X such that x and y cannot be separated by $G_{(b*g)*}$ separation of X is said to be the quasi $G_{(b*g)*}$ component of X.

Theorem 3.17: Let f : (X,τ, G) → (Y, σ) be a totally $G_{(b*g)*}$ continuous function from a grill topological space X into a T_1 space Y. Then f is constant on each quasi $G_{(b*g)*}$component.

Proof: Let x, y ∈ X that lie in the same quasi $G_{(b*g)*}$ component of X. Let f(x) = α ≠ β = f(y). Since Y is T_1, {α} is closed in Y and Y − {α} is open in Y. Since f is totally $G_{(b*g)*}$ continuous f^{-1}({α}) and f^{-1}(Y − {α}) are disjoint $G_{(b*g)*}$ clopen subsets of X. Further x ∈f^{-1}({α}) and y ∈ f^{-1}(Y – { α}) which is a contradiction to the fact that x and y belongs to the same quasi $G_{(b*g)*}$ component of X. Hence the Theorem.

Definition 3.18:A $G_{(b*g)*}$ frontier of a subset A of X is $G_{(b*g)*}$ (Fr(A) = $G_{(b*g)*}$(cl(A))∩ $G_{(b*g)*}$(cl(X – A)).

Theorem 3.19: The set of all points x ∈ X in which a function f :(X,τ, G) → (Y, σ) is not totally $G_{(b*g)*}$ continuous is the union of $G_{(b*g)*}$ frontier of the inverse images of open sets containing f(x) if arbirtrary union of $G_{(b*g)*}$ clopen sets in X is $G_{(b*g)*}$ clopen in X.

Proof: Let A = { x ∈ X : f is not totally $G_{(b*g)*}$continuous at x}. Let B be the union of $G_{(b*g)*}$ frontier of the inverse images of open sets containing f(x). Let x ∈ A. Then there exists an open set V of Y containing f(x) such that f(U) is not contained in V for each U ∈ $G_{(b*g)*}O(X)$ containing x. Hence x ∈ $G_{(b*g)*}$cl(X – f^{-1}(V)). On the other hand x ∈ f^{-1}(V) ⊂ $G_{(b*g)*}$ cl(f^{-1}(V). So x ∈ $G_{(b*g)*}$ Fr(f^{-1}(V). Hence A ⊆ B. Conversely, let f be totally $G_{(b*g)*}$ continuous at x ∈ X. Let V be open in Y containing f(x). Then there exists U ∈ $G_{(b*g)*}$ CO(X) containing x such that f(U) ⊆ V. That is U ⊆ f^{-1}(V). Hence x ∈ $G_{(b*g)*}$int(f^{-1}(V). x ∉ $G_{(b*g)*}$ cl(X – f^{-1}(V). Hence x ∉ $G_{(b*g)*}$ Fr f^{-1}(V). So x ∉ A implies x ∉ B. Hence x B ⊆ A.

Theorem3.20: Let {X_λ : λ∈A} be any family of topological spaces. If f: X → IIX_λ is a totally $G_{(b*g)*}$ continuous function, then P_λ:X → X_λ is totally $G_{(b*g)*}$ continuous function for each λ ∈ A, where P_λ is the projection of IIX_λ onto X_λ.

Proof: We shall consider a fixed λ ∈ A. Suppose U_λ is an arbitrary open set in X_λ. Then$P_\lambda^{-1}(U_\lambda)$ is open in IIX_λ. Since f is totally $G_{(b*g)*}$ continuous, we have by $f^{-1}(P_\lambda^{-1}(U_\lambda))$ = (P_λo f)$^{-1}$(U_λ) is $G_{(b*g)*}$ clopen in X. Hence the assertion.

Definition 3.21: i) A filter base A is said to be $G_{(b*g)*}$ co-convergent to a point x ∊ X, if for any U ∊$G_{(b*g)*}$ CO(X) containing x, there exists B ∊A such that B ⊆ U.

ii) A filter base A is said to be convergent to a point x ∊X of for any U ∊O(X) containing x, there exists B ∊Asuch that B ⊆ U.

Theorem3.22: If a function f: (X,τ, G) → (Y, σ) is totally $G_{(b*g)*}$ continuous, then for each point x ∊X and each filter base A in X , $G_{(b*g)*}$ co-convergent to x, the filter base f(A) is convergent to f(x) .

Proof: Let x ∊X and A be any filter base in X,$G_{(b*g)*}$ co-convergent to x. Since f is totally $G_{(b*g)*}$ continuous then for any V ∊O(Y) containing f(x) there exists a U ∊$G_{(b*g)*}$ CO(X) containing x such that f(U) ⊆ V. Since A is $G_{(b*g)*}$ co-convergent to x, there exists a B ∊A such that B ⊆ U. This implies f(B) ⊆ V. Hence the filter base f(A) converges to f(x).

4 Covering Properties

Definition 4.1:A space (X, τ, G) is said to be $G_{(b*g)*}T_2$ if for any two distinct points x and y of X, there exists disjoint $G_{(b*g)*}$ open sets U and V such that x∊Uand y∊V.

Theorem 4.2: If arbitrary intersection of $G_{(b*g)*}$ closed sets is $G_{(b*g)*}$ closed in a grill topological space X, then X is $G_{(b*g)*}$ T_2 if and only if for any two distinct points x and y of X, there exists a $G_{(b*g)*}$ neighborhood N_y of y such that x ∉ $G_{(b*g)*}N_y$.

Proof : Let X be $G_{(b*g)*}$ T_2. Let x and y be distinct points of X. Then there exists $G_{(b*g)*}$ open sets U and V such that $x \in U, y \in V$ and $U \cap V = \Phi$. But $U \cap V = \Phi$ implies $V \subseteq X - U$ so $y \in V \subseteq X - U$. Put X – U = N_y.We have$G_{(b*g)*}clN_y$ = $G_{(b*g)*}$ cl(X – U) = X – U = N_y as X – U is $G_{(b*g)*}$ closed. N_y is a $G_{(b*g)*}$ neighbourhood of y such that $X \notin G_{(b*g)*}$ cl N_y.
Conversely, let X be a grill topological space such that for any two distinct points x and y of X, there exists $G_{(b*g)*}$ neighbourhood N_y of y such that x$\notin$ $G_{(b*g)*}$ cl N_y. $G_{(b*g)*}$ cl N_y is also $G_{(b*g)*}$ neighbourhood of y. Since $G_{(b*g)*}$ cl N_y *is*$G_{(b*g)*}$closed, X – $G_{(b*g)*}$ cl N_y is $G_{(b*g)*}$ open. x$\notin$ $G_{(b*g)*}$ cl $N_y$implies x$\in$ X – $G_{(b*g)*}$ cl N_y. As N_y is a $G_{(b*g)*}$ neighbourhood of y, there exists a $G_{(b*g)*}$ open set U that $y \in U$ and (X – $G_{(b*g)*}$ cl N_y) $\cap$ U = Φ. Hence X is $G_{(b*g)*}T_2$.

Theorem 4.3: If arbitrary intersection of $G_{(b*g)*}$ closed sets is $G_{(b*g)*}$ closed in a grill topological space X, then X is $G_{(b*g)*}T_2$ if for any two distinct points x and y of X, there exists a $G_{(b*g)*}$ open sets U and V such that x $\in$ Uy $\in$ V and $G_{(b*g)*}$ clU$\cap$ $G_{(b*g)*}$clV = Φ
Proof: Let X be a grill topological space. Let x and y be distinct points of X. Then there exists $G_{(b*g)*}$ open sets U and V such that $x \in U$, $y \in V$ and $G_{(b*g)*}$ clU$\cap$ $G_{(b*g)*}$clV = Φ. V is a $G_{(b*g)*}$ neighbourhood of y such that x $\notin$ $G_{(b*g)*}$ cl V, as x $\in$ $G_{(b*g)*}$ cl U. Hence by the above theorem X is $G_{(b*g)*}T_2$.

Lemma 4.4: Let arbitrary intersection of $G_{(b*g)*}$ closed sets be $G_{(b*g)*}$ closed in a grill topological space X and let f : $(X, \tau, G) \to (Y, \sigma)$ be a totally $G_{(b*g)*}$ continuous injective function. If Y is T_0, then X is$G_{(b*g)*}T_2$.

Proof : Let x and y be any pair of distinct points of X. Then f(x) $\neq$ f(y). Since Y is T_0 there exists an open sets U containing f(x) but not f(y). Then x $\in f^{-1}$(U) and y $\notin f^{-1}$(U). As f is totally $G_{(b*g)*}$ continuous, f^{-1}(U) is $G_{(b*g)*}$ clopen in X. Also x $\in f^{-1}$(U) and y $\in$x – f^{-1}(U). By the above theorem X is $G_{(b*g)*}T_2$.

Definition 4.5:A grill topological space X is said to be $G_{(b*g)*}$ compact if every $G_{(b*g)*}$ open cover of X has a finite subcover.

Definition 4.6:A subset A of grill topological a space X is said to be $G_{(b*g)*}$ cocompact relative to X if every cover of A by $G_{(b*g)*}$ clopen sets of X has a finite subcover.

Definition 4.7: A subset A of a grill topological space X is said to be $G_{(b*g)*}$ cocompact if the subspace A is $G_{(b*g)*}$ cocompact.

Theorem 4.8: If arbitrary union of $G_{(b*g)*}$ clopen sets is $G_{(b*g)*}$ clopen for a grill topological space X and a function f : $(X, \tau, G) \to (Y, \sigma)$ is totally $G_{(b*g)*}$ continuous and A is $G_{(b*g)*}$ cocompact relative X, then f(A) is compact in Y.

Proof : Let {H_α: $\alpha \epsilon I$} be any cover of f(A) by open sets of the subspace f(A). For each $\alpha \epsilon I$, there exists an open set $A_\alpha$of Y such that $H_\alpha = A_\alpha \cap f(A)$. For each x$\epsilon$A there exists $\alpha_x \epsilon I$ such that f(x) $\epsilon A_{\alpha x}$and there exists $U_x \in G_{(b*g)*}$ CO(X) containing x such that $f(U_x) \subset A_{\alpha x}$. Since the family {$U_x$: x $\in$ A} is a cover of A by $G_{(b*g)*}$ clopen sets of X, there exists a finite subset A_0 of A such that A $\subset$ U{U_x : x $\in A_0$}.
Therefore we obtain f(A) $\subseteq$ U{f(U_x) : x $\in A_0$} which is a subset of {$A_{\alpha x}$: xϵA}. Thus f(A) = U {$A_{\alpha x} \cap$ f(A): x $\in A_0$} = U{ $H_{\alpha x}$: xϵA_0}. Hence f(A) is compact.

Corollary 4.9: If arbitrary union of $G_{(b*g)*}$ clopen sets is $G_{(b*g)*}$ clopen in grill topological space X and if f : $(X, \tau, G) \to (Y, \sigma)$ is totally $G_{(b*g)*}$ continuous surjective function and X is $G_{(b*g)*}$ cocompact, the Y is compact.

Proof: Follows from the above theorem

Definition 4.10:A grill topological space X is said to be
I) Countably $G_{(b*g)*}$ cocompact if every $G_{(b*g)*}$ clopen countable cover of X has a finite subcover
II) $G_{(b*g)*}$ co-Lindelof if every $G_{(b*g)*}$ clopen cover of X has a countable subcover.
Theorem 4.11: Let f : f : $(X, \tau, G) \to (Y, \sigma)$ be a totally $G_{(b*g)*}$ continuous surjective function. Then the following statements hold:
I) If X is $G_{(b*g)*}$ co-Lindelof, the Y is Lindelof

II) If X is countably $G_{(b*g)*}$ cocompact, the Y is countably compact

Proof: i) Let $\{V_\alpha : \alpha\epsilon I\}$ be an open cover of Y. Since f is totally $G_{(b*g)*}$ continuous then $\{f^{-1}(V_\alpha) : \alpha\epsilon I\}$ is a $G_{(b*g)*}$ clopen cover of X. Since X is $G_{(b*g)*}$ co-Lindelof, there exists a countable subset I_0 of I such that X = $\cup\{f^{-1}(V_\alpha) : \alpha\epsilon I\}$. Then Y = $\cup\{V_\alpha : \alpha\epsilon I\}$ and hence Y is Lindelof.

ii) similar to (i)

Definition 4.12:A grill topological space X is said to be

I) $G_{(b*g)*}$ coT_1, if for each pair of distinct points x and y of X, there exist $G_{(b*g)*}$ clopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$;

II) $G_{(b*g)*}$ coT_2, if for each pair of distinct points x and y of X, there exist disjoint $G_{(b*g)*}$ clopen sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 4.13: If $f : (X, \tau, G) \to (Y, \sigma)$ is a totally $G_{(b*g)*}$ continuous injective function and Y is T_1, then X is $G_{(b*g)*}$ coT_1.

Proof: Suppose Y is T_1 for any distinct points x and y in X, there exists V, WϵO(Y) such thatf(x) $\epsilon V, f(y) \notin V$ and f(y) $\epsilon W, f(x) \notin W$. Since f is totally $G_{(b*g)*}$continuous $f^{-1}(V)$ and $f^{-1}(W)$ are $G_{(b*g)*}$ clopen subsets of (X, τ, G) such that x $\epsilon f^{-1}(V)$ y $\notin f^{-1}(V)$ and y $\epsilon f^{-1}(W)$, x $\notin$ $f^{-1}(W)$. This shows X is $G_{(b*g)*}$ co T_1.

Theorem 4.14: If $f : (X, \tau, G) \to (Y, \sigma)$ is a totally $G_{(b*g)*}$ continuous injective function and Y is T_2, then X is $G_{(b*g)*}$ coT_2.

Proof: For any pair of distinct points x and y in X, there exists disjoint open sets U and V in Y such that f(x) $\epsilon U, f(y)\epsilon V$. Since f is totally $G_{(b*g)*}$ continuous $f^{-1}(U)$ and $f^{-1}(V)$ are $G_{(b*g)*}$ clopen in X containing x and y respectively.$f^{-1}(U) \cap f^{-1}(V) = \Phi$because $U \cap V = \Phi$. This shows X is $G_{(b*g)*}$ co T_2.

Definition 4.15:A grill topological space X is called $G_{(b*g)*}$ coregular if for each $G_{(b*g)*}$ *clopen* set F and each point $x \notin F$, there exists disjoint open sets U and V such that $F \subset U$ and $x \in V$.

Definition 4.16:A grill topological space X is said to be $G_{(b*g)*}$ conormal if for any pair of distinct $G_{(b*g)*}$clopen sets F_1 and F_2, there exists disjoint open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Definition 4.17: If f is totally $G_{(b*g)*}$ continuous injective open function from a $G_{(b*g)*}$ coregular space X onto a space Y, the Y is regular.

Proof: Let F be a closed set of Y and $y \notin F$. Take y = f(x) sincef is totally $G_{(b*g)*}$continuous $f^{-1}(F)$ is a $G_{(b*g)*}$ clopen set. Take G = $f^{-1}(F)$ we have $x \notin G$. Since X is $G_{(b*g)*}$ coregular, there exists disjoint open sets U and V such that $G \subseteq U$ and $x \in V$. We obtain that F = f(G) $\subset$ f(U) and y = f(x) $\in$ f(V) such that f(U) and f(V) are disjoint open sets. Hence Y is regular.

Theorem 4.18: If f is totally $G_{(b*g)*}$ continuous injective open function from a $G_{(b*g)*}$ conormal space X onto a space Y, then Y is normal.

Proof: Similar to the above proof.

Definition 4.19: For a function $f : (X, \tau, G) \to (Y, \sigma)$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by G(f).

Definition 4.20:A graph G(f) of a function $f : (X, \tau, G) \to (Y, \sigma)$ is said to be strongly $G_{(b*g)*}$ co-closed if for each (x , y) $\in$ (XxY) $-$ G(f), there exist U $\in G_{(b*g)*}$ CO(X) containing x and V $\in$ O(Y) containing y such that (U x V) $\cap$ G(f) $= \Phi$.

Lemma 4.21:A graph G(f) of a function $f : (X, \tau, G) \to (Y, \sigma)$ is strongly $G_{(b*g)*}$co-closed in X x Y if and only if for each (x, y) ϵ (X x Y) $-$ G(f), there exist U$\epsilon G_{(b*g)*}$ CO(X) containing x and VϵO(Y) containing y such that f(U) $\cap$ V $= \Phi$.

Proof: Let G(f) be strongly $G_{(b*g)*}$- co-closed. Let (x, y) ϵ (X x Y) – G(f). Then there exist $G_{(b*g)*}$ clopen set U containing x and V ϵ O(Y) containing y such that (U ∩ V) x G(f) = Φ. That is V ∩ f(X) = Φ. That is V ∩ f(U) = Φ.
Conversely, let for each (x, y) ϵ (X x Y) – G(f), there exist U ϵ$G_{(b*g)*}$ CO(X) containing x and V ϵ O(Y) containing y such that f(U) ∩ V = Φ.Let yϵ V. y ϵ Y − f(X). That is y ≠ f(x) for any x. That is V ∩ f(U) = Φ. This implies (U x V) ∩ (X x f(X)) = Φ.
That is (U x V) ∩ G(f) = Φ.

Theorem 4.22: Let f : (X, τ, G) → (Y, σ) has a strongly $G_{(b*g)*}$ co-closed graph G(f). If f is injective, then X is $G_{(b*g)*}$ Co T_2.

Proof: Let x and y be any two distinct points of X.
Then, we have (x, f(y)) ϵ (X x Y) – G(f).
By the above Lemma there exist $G_{(b*g)*}$ clopen set U of X and V ϵ O(Y) such that (x, f(y)) ϵ (U x V) and f(U) ∩ V ≠ Φ.
Hence U ∩ f^{-1}(V) = Φ, xϵ U and y ϵ f^{-1}(V).
Hence X is $G_{(b*g)*}$ co T_2.

Theorem 4.23: If arbitrary union of $G_{(b*g)*}$ clopen sets is $G_{(b*g)*}$ clopen in a grill topological space X and f : (X, τ, G) → (Y, σ) is a totally $G_{(b*g)*}$ continuous and Y is T_2, then G(f) is strongly $G_{(b*g)*}$ co-closed in the product space X x Y.

Proof: Let (x , y) ∈(X x Y) − G(f). Then y ≠ f(x) and there exist open sets V_1 and V_2 such that f(x) ∈ V_1 y ∈ V_2 and $V_1 \cap V_2$ ≠ Φ. From hypothesis, there exists U ∈ $G_{(b*g)*}$ CO(X, x) such that f(U) ⊆ V_1. Therefore we obtain f(U) ∩ V_2= Φ. So G(f) is strongly $G_{(b*g)*}$ co-closed graph.

Definition 4.24: A function f : (X, τ, G) → (Y, σ) is said to be:
I) Totally $G_{(b*g)*}$ irresolute if the preimage of $G_{(b*g)*}$ clopen subset of Y is $G_{(b*g)*}$ clopen in X
II) Totally pre $G_{(b*g)*}$ clopen if the image of every $G_{(b*g)*}$ clopen subset of X is $G_{(b*g)*}$ clopen in Y

Theorem 4.25: Letf : (X, τ, G) → (Y, σ)be surjective and totally $G_{(b*g)*}$ irresolute and totally pre $G_{(b*g)*}$ clopen and g : (Y, σ) → (Z, η)be any function. Then g o f : (X, τ, G) → (Z, η)is totally $G_{(b*g)*}$ continuous if and only if g is totally $G_{(b*g)*}$ continuous.

Proof: Let g be totally $G_{(b*g)*}$ continuous. Let V be open in Z. g^{-1}(V) is $G_{(b*g)*}$ clopen in Y. f^{-1}((g^{-1}(V)) is $G_{(b*g)*}$ clopen in X. Hence g o f is totally $G_{(b*g)*}$ continuous.
Conversely, let g o f :(X, τ, G) → (Z, η) be totally $G_{(b*g)*}$ continuous. Let V be open in Z. Then (g o f)$^{-1}$(V) is $G_{(b*g)*}$ clopen in X. That if f^{-1}((g^{-1}(V)) is $G_{(b*g)*}$ clopen. Since f is totally pre $G_{(b*g)*}$ clopen, f(f^{-1}((g^{-1}(V))) is $G_{(b*g)*}$ clopen in Y. That is g^{-1}(V) is $G_{(b*g)*}$ clopen in Y. Hence g is totally $G_{(b*g)*}$ continuous.

Theorem 4.26: Let f : (X, τ, G) → (Y, σ)has a strongly $G_{(b*g)*}$ co-closed graph G(f). If f is surjective totally pre $G_{(b*g)*}$ clopen function, then Y is $G_{(b*g)*}T_2$ space.

Proof: Let y_1 and y_2 be distinct points of Y. Since f is surjective f(x) = y_1, for some x ∈ X. (x, y_2) ∈ (X x Y) – G(f). There exist U ∈ $G_{(b*g)*}$ CO(X) and V ∈ O(Y) such that (x, y_2) ∈U x Y and (U x Y) ∩ G(f) = Φ. Then we have f(U) ∩ V = Φ. Since f is totally pre $G_{(b*g)*}$clopen f(U) is $G_{(b*g)*}$clopen such that f(x) = y_1 ∈ f(U). Hence Y is $G_{(b*g)*}T_2$.

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