

A Remark on Squaring the Circle

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Abstract:

Following the wonderful geometrical construction of Ramanujan we propose a geometrical magnitude that solve the old problem –find a square whose area is equal to that of a given circle.

Key Word: pi, chord, circle, square.

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I. Introduction

Construct a square that has an area equal to that of an arbitrary given circle, is called a quadrature of the circle. Square the circle according to the ancient Greeks is a problem no yet solved. The first quadrature is due to Hippocrates. In our days this is possible with the Gayatri $\pi = (14 - \sqrt{2})/4$. [1,2,3]

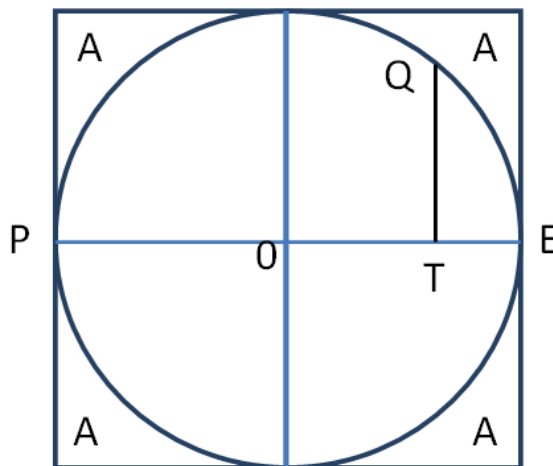
However with the official $\pi = 3.141592654 \dots$ this seems to be imposible after F. Lindemann’s work.

In 1913 SrinivasaRamanujan [4] published a very interesting geometrical construction.

In Ramanujan geometrical construction the fundamental geometric magnitude is $TQ = (\sqrt{5/3})R$ what is used to construct the chord RS. In our work we find a geometrical length very close to that given by Ramanujan, from purely geometric consideration.

II. Procedure

Let R be the radius of the given circle, D its diameter and center O. Draw a square of side D that circumscribe the circle.



$$R=OP$$

$$D=PE$$

$TQ = (\sqrt{5/3})R$ is the Ramanujan geometrical magnitude.

From the obvious:

$A = \left(1 - \frac{\pi}{4}\right)R^2$ is the area between the circle and the square.

From this very simple geometrical construction:

$$D^2 - 16A = (\pi - 3)D^2 = (\pi - 3)(2R)^2 = (\pi - 3)\left(\frac{6}{3}R\right)^2$$

$$\sqrt{D^2 - 16A} = \frac{\sqrt{36\pi - 108}}{3}R$$

$\sqrt{D^2 - 16A} = \frac{\sqrt{5.097335529\dots}}{3}R$ this number is very close to TQ.

If we divide R into 22 equal parts, the geometrical length

$R\cos 45^\circ + \frac{R}{22} = 0.752561327\dots R$ is very close to:

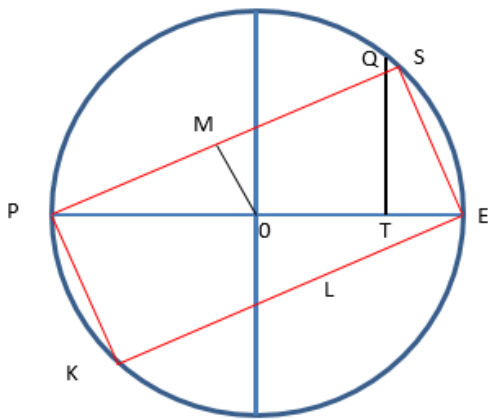
$$\frac{\sqrt{5.097335529\dots}}{3}R = 0.752575986\dots R$$

Now place $ES = \frac{\sqrt{5.097335529\dots}}{3}R$

And following Ramanujan geometrical construction. Join P and S and draw OM parallel to ES. $PM = \frac{1}{2}PS$.

Place a chord PK=PM. Join E and K. If $\alpha = \text{angle EPS}$. Then:

$$\sin \alpha = \frac{\sqrt{5.097335529\dots}}{6}; \quad \cos \alpha = \sqrt{\frac{30.902664471\dots}{36}}$$



$$PK = PM = \frac{D}{2} \cos \alpha$$

$$PK^2 = \frac{30.902664471\dots}{144} D^2$$

For the right triangle PEK:

$$D^2 = EK^2 + PK^2$$

$$EK^2 = D^2 - PK^2$$

$$EK^2 = D^2 - \frac{30.902664471 \dots}{144} D^2$$

$$EK^2 = \frac{36\pi}{144} D^2 = \frac{\pi}{4} D^2$$

If EK=L the side of the square searched

$$L^2 = \pi R^2 \quad \text{Exactly}$$

III. Conclusion

If we place $ES = \frac{\sqrt{5.097335529 \dots}}{3} R$ the quadrature is perfect.

This work would not be possible without the enlightenment of the Ramanujan's geometrical construction.

The same result will be obtained if we place $PK^2 = 4A$.

References

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