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On Nano g**Λ- Contra Continuous Functions

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Abstract: *The aim of this paper is to give and discuss stronger form of nano continuity called nano contra continuity using nano $g^{**}\Lambda -$ closed sets*

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I. Introduction

Ganster and Reily [3] discussed LC Continuous functions. Dontchev[2] had given contra continuous functions. Lelli's Thivagar etal[4] introduced a nano topological space with respect to a subset X of an Universe which is defined in terms of lower and upper approximations of X. The elements of nano topological space are called nano open sets 'Nano' is a greeek word which means 'very small'. The topology studied here is given the name nano topology as it has atmost five elements. Certain weak forms of nano sets were studied by various authors.

Here we study a new form of nano contra continuity called nano $g^{**}\Lambda$- contra contiuity and its relation to other nano contra continuous fuctions.

II. Preliminaries

DEFINITION:2.1[4]

Let U be a non empty finite set of objects called the universe R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space . Let X ⊆ U

(1) The lower approximation of X with respect to R is the set of the all objects, which can be for certain classified as X with respect to R and is denoted by L_R(X). That is,L_R(X) = $\{\cup_{X\in U}\{R(X): R(X) \subseteq X\}\}$, where R(X) denotes the equivalence class determined by X.

(2) The upper approximations of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$=$\{ \cup_{X\in U}\{R(X): R(X) \cap X \neq \emptyset\}\}$

(3) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

DEFINITION:2.2[4]

Let U be the universe ; R an equivalence relation on U and $\tau_R(X)$={U,ϕ , $L_R(X)$, $U_R(X)$, $B_R(X)$} where X⊆U and $\tau_R(X)$satisfies the following axioms

(1) U and ϕ ∈ $\tau_R(X)$

(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology U called as the nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets . A set A is said to be nano closed if its complement is nano open.

DEFINITION:2.3[4]

If (U, $\tau_R(X)$) is a nano topological space with respect to X where X⊆U and if A⊆U , with respect to X where X ⊆U and if A⊆U, then nano interior of A is defined as the union of all nano open subsets contained in A and it is denoted by N int(A). That is, N int(A) is the largest nano open set contained in A.

The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by N cl(A). That is, N cl(A) is the smallest closed set containing A.

DEFINITION:2.4

A Subset A of a nano topological space (U, $\tau_R(X)$) as called

(1)Nano regular closed if A=Ncl N int (A).

(2)Nano generalized closed (nano g-closed) if Ncl A⊆U,U is nano open in (U, $\tau_R(X)$).

(3)Nano g* -closed if N cl A ⊆ U whenever A ⊆ U, U is nano g open in (U, $\tau_R(X)$).

(4) Nano λ – closed if A = C∩ D where C is nano Λ set and D is a nano closed sets.

(5) Nano g Λ- closed if N cl_λ(A) ⊆ U whenever A ⊆ U, U is nano open in (U, $\tau_R(X)$).

(6) Nano Λ g – closed if N cl_λ(A) ⊆ U whenever A ⊆ U, U is nano λ open in (U, $\tau_R(X)$).

(7) Nano g*Λ – closed if N cl_λ(A) ⊆ U whenever A ⊆ U, U is nano g open in (U, $\tau_R(X)$).

(8) Nano g**Λ – closed if N cl_λ(A) ⊆ U whenever A ⊆ U, U is nano g* open in (U, $\tau_R(X)$).

The complements of the above metioned nano closed sets are respectively nano open sets .

A Λ set is a set A which is equal to its kernel, that is, the intersection of all open super sets of A.

The intersection of all λ – closed sets containing A is called the λ – closure of A and it is denoted by cl_λ(A).

DEFINITION:2.5

Let (U, $\tau_R(X)$) and (V, $\tau_R(X)$) be nano topological spaces. Then the mapping f: (U, $\tau_R(X)$) → (V, $\tau_R(X)$) is called

(1)Nano contra continuous if the inverse image of every nano open set in V is nano closed in U [4].

(2)Nano λ contra continuous if the inverse image of every nano open set in V is nano λ closed in U.

(3)Nano rg contra continuous if the inverse image of every nano open set in V is nano rg closed in U.

(4)Nano g* contra continuous of the inverse image of every nano open set in V is nano g* closed in U.

(5)Nano gΛ contra continuous if the inverse image of every nano open set in V is nano gΛ closed in U.

III. NANO G**Λ Contra Continuous Functions.

DEFINITION :3.1:

Let (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) be nano topological spaces. A mapping f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is called nano g**Λ - contra continuous if the inverse image of every nano open set in V is nano g**Λ - closed in U.

DEFINITION:3.2:

A mapping f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is said to be

(1)Strongly nano g**Λ - continuous if the inverse image of every nano subset of V is nano g**Λ -open in U.

(2)Perfectly nano g**Λ -continuous if the inverse image of every nano open set in V is nano g**Λ - cl open in U.

THEOREM:3.3:

(1)Every nano λ contra continuous map is nano g**Λ - contra continuous.

(2)Every nano contra continuous map is nano g**Λ - contra continuous map.

(3)Every nano contra regular closed map is nano g**Λ - contra continuous map.

(4) Every nano g* contra continuous map is nano g**Λ - contra continuous map.

(5) Every nano g**Λ continuous map is nano gΛ contra continuous map.

(6) Every strongly nano g**Λ - continuous map is nano g**Λ - contra continuous map.

(7)Every perfectly nano g**Λ - continuous map is nano g**Λ - contra continuous map.

PROOF: Obvious

The converse of the above statements need not to be true can be seen from the following examples.

EXAMPLE:3.4:

Let U = { a, b,c,d} =V

U/R ={{a},{c},{b,d}} , X = {a,b}

$\tau_R(X)$ = { U, ϕ, {a}, {a,b,d}, {b,d}} = $\tau_R(Y)$

Define f : (U, $\tau_R(X)$) → (V, $\tau_R(Y)$) by f(a) = a, f(b) = b,f(c)=d ,f(d) = c

f is nano g**Λ - contra continuous function but not nano λ contra continuous as

f^{-1}({a,b,d}) = {a,b,c} is not nano λ closed.

EXAMPLE:3.5 Refer example – 3.4

f is nano g**Λ - contra continuous but not nano contra continuous as f^{-1}({b,d}) = {b,c} is not nano closed.

EXAMPLE:3.6: Refer example- 3.4

f is nano g**Λ - contra continuous but not nano contra regular closed as $f^{-1}(\{a\}) = \{a\}$ is not nano regular closed.

EXAMPLE:3.7: Refer example- 3.4
f is nano g**Λ - contra continuous but not nano g* contra contiuous as $f^{-1}(\{a\}) = \{a\}$ is not nano g* -closed.

EXAMPLE:3.8: Take (U, $\tau_R(X)$) and (V, $\tau_R(Y)$) as in example-3.4. Define f : (U, $\tau_R(X)$) → (V, $\tau_R(Y)$) by f(a) = b , f(b) = a , f(c) = c, f(d) = d f is nano gΛ - contra continuous but not nano g**Λ - continuous as $f^{-1}(\{a\}) = \{b\}$ is not nano g**Λ - closed.

EXAMPLE:3.9: Refer example – 3.4
f is nano g**Λ - contra continuous but not strongly nano g**Λ- continuous as $f^{-1}(\{b\}) = \{b\}$ is not nano g**Λ - closed.

EXAMPLE:3.10: Refer example – 3.4
f is nano g**Λ - contra continuous but not perfectly nano g**Λ- continuous as $f^{-1}(\{a,b,d\}) = \{a,b,c\}$ is not nano g**Λ - cl open.

THEOREM:3.11:
Let arbitrary union of nano g**Λ open sets be nano g**Λ - open . Then the following statements are equivalent for a map f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$)
(1)f is nano g**Λ - contra continuous.
(2)For every nano closed set F of V, $f^{-1}(F)$ is nano g**Λ - open in X.
(3)For each x ϵ X and each nano closed set F of V containing f(x) , there exists nano g**Λ - open A containing x such that f(A) ⊆ F.
(4)For each x ϵ X and each open set B of V not containing x, there exists nano g**Λ - closed set K containing x such that $f^{-1}(B)$ ⊆K.

PROOF:
(1) **)**⟺ (2) obvious
(2) **)**⇒ (3) Let F be nano closed in V containing f(x). Hence x ϵ f^{-1} (F). By (2) f^{-1} (F) is nano g**Λ - open in U containing x. Let A = f^{-1} (F). This implies A is nano g**Λ - open in U containing x and f(A) ⊆ F. so (3) holds.
(3)) ⇒(2) Let F be nano closed of V containing f(x). So x ϵ f^{-1} (F). By (3), there exists nano g**Λ - open set A_x of U containing x such that f(A_x) ⊆ F. so f^{-1} (F) =$\cup\{A_x : x \in f^{-1}(F)\}$ This is a union of nano g**Λ - open sets and hence it is nano g**Λ - open. Hence (2) holds .
(3)**)**⇒ (4)4) Let B be nano open in V not containing f(x). ThenV–B is nano closed in V containing f(x). By (3), there exists nano g**Λ - open set A in U containing x such that f(A) ⊆ V-B. This implies A ⊆ f^{-1}(V-B) = U – f^{-1}(B). Hence f^{-1}(B) ⊆ U – A. Let K = X – A. K is nano g**Λ - closed in U not containing x such that f^{-1}(B) ⊆ K. Hence (4) holds.
(4) ⇒ (3) Let F be nano closed in V containing f(x). Then V – F is nano open in V not containing f(x). From (4), there exists nano g**Λ - closed K in U not containing x such that f^{-1}(V - F) ⊆ K. This implies U - f^{-1}(F) ⊆ K. Hence U – K ⊆ f^{-1}(F). That is f(U – K) ⊆ F. Let A = U – K. Then A is nano g**Λ - open in U containing x such that f(A) ⊆ F. So (3) holds.

DEFINITION:3.12:
A map f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is called almost nano g**Λ - contra continuous if f^{-1}(W) is nano g**Λ - closed for every nano regular open set W of V.

THEOREM:313:
The following are equivalent for a map f : (U, $\tau_R(X)$) → (V, $\tau_R(Y)$)
(1)f is almost nano g**Λ - contra continuous.
(2)f^{-1} (N int (N cl (G))) is nano g**Λ - closed in U for every nano closed set G of V.
(3)f^{-1} (N cl (N int (F))) is nano g**Λ - open in U for every nano closed set F of V.
PROOF:
(1)⟺ (2) Let G be nano open in V. Then N int (N cl(G)) is nano regular open in V.Hence f^{-1} (N int (N cl (G))) is nano g**Λ - closed in U.
(2) ⟺ (1) obvious
(1)⟺ (3) Let F be nano closed in V. Then N cl (N int (F)) is nano regular closed in V. Hence f^{-1} (N cl (N int (F))) is nano g**Λ - open in U.

(3)⟺ (1) is obvious.

DEFINITION:3.14:

A map f : (U, $\tau_R(X)$) → (V, $\tau_{R^I}(Y)$) is said to be nano R- map if f^{-1}(A) is nano regular open in U for each nano regular open set A of V.

THEOREM:3.15:

For two mappings f : (U, $\tau_R(X)$) → (V, τ_{R^I}(Y)) and g : (V, $\tau_{R^I}(Y)$) → (W, $\tau_{R^{II}}(Z)$), then for g ∘ f : (U, $\tau_R(X)$) → (W, $\tau_{R^{II}}(z)$) , the following properties hold:

(1)If f is almost nano g**Λ - contra continuous and g is nano R map, then g ∘ f is almost nano g**Λ - contra continuous.

(2)If f is almost nano g**Λ - contra continuous and g is perfectly nano continuous , then g ∘ f is almost nano g**Λ -contra continuous and atmost nano g**Λ -continuous.

PROOF:

(1) Obvious

(2)Let A be nano regular open in W. g^{-1}(A) is nano cl open in V and hence nano regular open and nano regular closed. $f^{-1}(g^{-1}$(A)) = (g ∘ f)$^{-1}$(A) is nano g**Λ - open and nano g**Λ - closed in U, as f is atmost nano g**Λ - contra continuous . So g ∘ f is atmost nano g**Λ -contra continuous and atmost nano g**Λ - continuous.

DEFINITION:3.16:

A nano topological space (U, $\tau_R(X)$) is called nano $T_{g**\Lambda}$ space if every nano g**Λ -open set is nano open.

THEOREM:3.17:

Let f : (U, $\tau_R(X)$) → (V, τ_{R^I}(Y)) be a nano g**Λ - contra continuous map and g : (V, $\tau_{R^I}(Y)$) → (W, $\tau_{R^{II}}(Z)$) be nano g**Λ - continuous . If (V, $\tau_{R^I}(Y)$) is nano $T_{g**\Lambda}$ space then g ∘ f : (U, $\tau_R(X)$) → (W, $\tau_{R^{II}}(z)$) is atmost nano g**Λ - contra continuous map.

PROOF:

Let A be nano regular open in W. g^{-1} (A) is nano g**Λ - open in V. As V is nano $T_{g**\Lambda}$ space , g^{-1} (A) is nano open in V. $f^{-1}(g^{-1}$(A)) = (g ∘ f)$^{-1}$(A) is nano g**Λ - closed in U. Hence g ∘ f is atmost nano g**Λ - contra continuous map.

DEFINITION:3.18:

A map f : (U, $\tau_R(X)$) → (V, $\tau_{R^I}(Y)$) is called strongly nano g**Λ - open (strongly nano g**Λ - closed) if f(A) is nano g**Λ - open (nano g**Λ - closed) in V for every nano g**Λ - open (nano g**Λ - closed) set A of V.

THEOREM:3.19:

If f : (U, $\tau_R(X)$) → (V, τ_{R^I}(Y)) is surjective strongly nano g**Λ - open (strongly nano g**Λ - closed) map and g : (V, $\tau_{R^I}(Y)$) → (W, $\tau_{R^{II}}(Z)$) is a map such that g ∘ f : (U, $\tau_R(X)$) → (W, $\tau_{R^{II}}(z)$) is almost nano g**Λ - contra continuous,,then g is almost g**Λ contra continuous

PROOF:

Let A be nano regular closed (nano regular open) set in W . As g ∘ f is atmost nano g**Λ - contra continuous , (g ∘ f)$^{-1}$(A) = $f^{-1}(g^{-1}$(A)) is nano g**Λ - open (nano g**Λ - closed) in U. Since f is surjective and strongly nano g**Λ -open (strongly nano g**Λ - closed), f($f^{-1}(g^{-1}$(A))) = g^{-1} (A) is nano g**Λ - open (nano g**Λ - closed) in V. Hence g is atmost nano g**Λ - contra continuous.

DEFINITION:3.20:

A map f : (U, $\tau_R(X)$) → (V, τ_{R^I}(Y)) is called weakly nano g**Λ - continuous if for each x ∈ X and each nano open set of V containing f(x), there exists nano g**Λ - open set A of U such that f(A) ⊆ N cl B.

THEOREM:3.21:

If a map f : (U, $\tau_R(X)$) → (V, τ_{R^I}(Y)) is nano g**Λ -contra continuous , then f is weakly nano g**Λ - continuous map.

PROOF:

Let x ∈ X and B be nano open set in V containing f(x). N cl (V) is nano closed in V containing f(x). N cl (V) is nano closed inV containing f(x). Since f is nano g**Λ - contra continuous , f^{-1} (N cl (V)) is nano g**Λ - open in U. Let A = f^{-1}(Ncl B). f(A) ⊆ Ncl B so f is weakly nano g**Λ - continuous.

DEFINITION:3.22:
A nano topological space (U, $\tau_R(X)$) is called locally nano g**Λ - indiscrete if every nano g**Λ - open sets is nano closed in U.

THEOREM:3.23:
If a map f : (U, $\tau_R(X)$) → $(V, \tau_{R'}$(Y)) is almost nano g**Λ - contra continuous and U is locally nano g**Λ - indiscrete, then f is almost nano continuous.
PROOF:
Let B be nano regular open in V. As f is almost nano g**Λ - contra continuous, f^{-1}(B) is nano g**Λ - closed in U. Since U is locally nano g**Λ -indiscrete space, f^{-1}(B) is nano open in U. Hence, f is almost nano continuous.

DEFINITION:3.24;
A nano topological space (U, $\tau_R(X)$) is called strongly nano –S - g**Λ closed if and only if every nano g**Λ - closed cover of U has a finite subcover.

THEOREM:3.25:
Nano g**Λ - contra continuous images of strongly nano S - g**Λ closed spaces are nano compact.
PROOF:
Let f : (U, $\tau_R(X)$) → $(V, \tau_{R'}$(Y)) be nano g**Λ - contra continuous function and onto. Let U be strongly nano S - g**Λ closed. Let $(B_i)_{i\in I}$ be a nano open cover of V. Then (f^{-1} $(B_i))_{i\in I}$ is a nano g**Λ closed cover of U, since f is nano g**Λ - contra continuous. Then for some finite J ⊂ I, we have U = $\cup_{i\epsilon J} f^{-1}(B_c)$. As f is onto,
V =$\cup_{0\in J} B_c$. That is V is nano compact.

DEFINITION:3.26:
A map f : (U, $\tau_R(X)$) → $(V, \tau_{R'}$(Y)) is called nano g**Λ - irresolute if and only if the inverse image of every nano g**Λ - closed set in V is nano g**Λ - closed in U.

REMARK:3.27:
f is nano g**Λ irresolute ⟺ the inverse image of every nano g**Λ- open set under f is nano g**Λ- open.

THEOREM:3.28:
Let f : (U, $\tau_R(X)$) → $(V, \tau_{R'}$(Y)), g : (V, $\tau_{R'}(Y)$) → (W, $\tau_{R''}(Z)$) be functions. Then,
(1)If f is nano g**Λ- contiuous and g is nano continuous, then g ∘ f is nano g**Λ- continuous.
(2)If f and g are nano g**Λ - irresolute , then g ∘ f is nano g**Λ - irresolute.
(3)If f is nano g**Λ - irresolute and g is nano g**Λ - continuous then g ∘ f is nano g**Λ - continuous.
PROOF:
Obvious.
DEFINITION:3.29:
A nano topological space (U, $\tau_R(X)$) is said to be nano g**Λ - Hausdroff if and only if x and y are distinct points of U, there exists disjoint nano g**Λ - open sets A and B containing x and y respectively.

THEOREM:3.30:
Let (U, $\tau_R(X)$) be a nano topological space and $(V, \tau_{R'}$(Y)) be a nano Hausdroff space. If
f : (U, $\tau_R(X)$) → $(V, \tau_{R'}$(Y)) is injective and nano g**Λ - continuous, then (U, $\tau_R(X)$)is a nano g**Λ - Hausdroff space.
PROOF:
Let x and y be distinct points of U. Since f is injective f(x) and f(y) are distinct points of V. As V is nano Hausdroff, there exists disjoint nano open sets A and B containing f(x) and f(y) respectively. Since f is nano g**Λ -continuous and A and B are disjoint; f^{-1}(A) and f^{-1} (B) are disjoint nano g**Λ -open sets. So U is nano g**Λ -Hausdroff:
Following the same lines, we can prove the following .

THEOREM:3.31:
Let (U, $\tau_R(X)$) be a nano topological space and $(V, \tau_{R'}$(Y)) be a nano g**Λ - Hausdroff . If

f : (U, $\tau_R(X)) \rightarrow (V, \tau_{R^I}$(Y)) is injective and nano g**Λ - irresolute , then (U, $\tau_R(X)$) is nano g**Λ - Hausdroff .

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