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To Solve Inventory Model with Algebraic Approach

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Abstract *We solve an open question proposed by Chang, Chuang, and Chen that was published in the International Journal of Production Economics, entitled "Short comments on technical note – The EOQ and EPQ models with shortages derived without derivatives". Our solution approach has a unique character in that we find the minimum value before we derive the minimum point. Our results will be useful for researchers to develop their solution process from an algebraic point of view.*

Key words: *Economic Production Quantity; Economic Order Quantity; Shortage; Inventory Model*

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I. Introduction

We tried to solve an open question proposed by Chang et al. [2] to solve the minimum problem of an inventory model by the algebraic method under a pre-designed route. Chang et al. [2] had been cited 73 times. However, none of them paid attention to the open question proposed by Chang et al. [2]. There are several related papers: Deng et al. [3], Deng [4], Deng et al. [5], Jung et al. [6], Lan et al. [7], Lin, and Hopscotch [8], Tang et al. [10], and Yang et al. [11], that have worked on improving solution procedures for inventory models. We follow their trend to respond to the open question proposed by Chang et al. [2].

II. Notation and Assumptions

To be compatible with Chang et al. [2], we adopt the same notation and assumptions as theirs.

B maximum backorder level,
b backorder cost per unit per unit time,
c cost of production per unit
D demand rate,
h holding cost per unit per unit time,
K setup cost,
P production rate, with $P > D$, for EPQ models,
Q maximum inventory level,
$\rho = (P - D)/P$,
$C(Q, B)$ the average cost per unit time.

The goal is to solve the minimum problems of the Economic Production Quantity (EPQ) model, $C(Q, B)$, under the restrictions $Q > 0$ and $B > 0$, from a purely algebraic point of view, without referring to calculus.

III. Review of previous results

For an EPQ inventory model discussed in Càrdenas-Barrón [1] with its cost function $C(Q, B)$ as

$$C(Q, B) = \frac{b + h}{2\rho Q}B^2 - hB + \frac{h\rho}{2}Q + \frac{KD}{Q} + cD \ . \tag{1}$$

Ronald et al. [9] mentioned that the algebraic approach proposed by Càrdenas-Barrón [1] was developed under the knowledge of the optimal solution. Hence, Ronald et al. [9] derived a complicated method without knowing the optimal solution in advance. Chang et al. [2] provided a simplification for Ronald et al. [9] to solve this problem as follows,

$$C(Q,B)=\frac{b+h}{2\rho Q}\left(B-\frac{h\rho}{b+h}Q\right)^2+\frac{bh}{2(b+h)}Q+\frac{KD}{Q}+cD \tag{2}$$

to derive the relation between the optimal solution, Q^* and B^* as

$$B^*=\frac{h\rho}{b+h}Q^*. \tag{3}$$

They assume a new expression, $B(Q)=\frac{h\rho}{b+h}Q$ to change the problem from $C(Q,B)$ to $C(Q,B(Q))$ as a function in one variable Q to yield that

$$C(Q,B(Q))=\frac{bh}{2(b+h)}Q+\frac{KD}{Q}+cD$$

$$=\left(\sqrt{\frac{bh}{2(b+h)}Q}-\sqrt{\frac{KD}{Q}}\right)^2+\sqrt{2\rho\frac{bh}{b+h}KD}+cD \tag{4}$$

to obtain that

$$Q^*=\sqrt{\frac{2(b+h)}{bh\rho}KD}, \tag{5}$$

$$B^*=B(Q^*)=\sqrt{\frac{2h\rho}{b(b+h)}KD}, \tag{6}$$

and

$$C(Q^*,B^*)=\sqrt{2\rho\frac{bh}{b+h}KD}+cD. \tag{7}$$

Chang et al. [2] provided an alternative approach to derive that

$$C(Q,B(Q))=\frac{h\rho}{2}Q+\left(KD+\frac{b+h}{2\rho}B^2\right)\frac{1}{Q}-hB+cD$$

$$=\left[\sqrt{\frac{h\rho}{2}Q}-\sqrt{\left(KD+\frac{b+h}{2\rho}B^2\right)\frac{1}{Q}}\right]^2+\sqrt{2h\rho}\sqrt{KD+\frac{b+h}{2\rho}B^2}-hB+cD \tag{8}$$

to find another relation between the optimal solution, Q^* and B^* as

$$\frac{h\rho}{2}(Q^*)^2=KD+\frac{b+h}{2\rho}(B^*)^2 \tag{9}$$

such that Chang et al. [2] assumed another new expression

$$Q=Q(B)=\sqrt{\frac{2}{h\rho}\left(KD+\frac{b+h}{2\rho}B^2\right)} \tag{10}$$

to convert $C(Q,B)$ to $C(Q(B),B)$ as a function in one variable B to yield that

$$C(Q(B),B)=cD+h\left(\sqrt{(1+\alpha)B^2+\beta}-B\right) \tag{11}$$

where $\alpha=b/h$ and $\beta=2\rho KD/h$.

Motivated by Equation (11), Chang et al. [2] raise the following open question:
To solve the minimum problem of

$$\sqrt{(1+\alpha)B^2+\beta}-B \tag{12}$$

without referring to calculus, with $\alpha>0$ and $\beta>0$.

IV. Our solution approach

Motivated by Equation (12), we will find the minimum for $f(B)$, where

$$f(B) = \sqrt{(1+\alpha)B^2 + \beta} - B \tag{13}$$

for $B > 0$, with $\alpha > 0$ and $\beta > 0$, by algebraic method, without referring to calculus. From Equation (13), we know that

$$(1+\alpha)B^2 + \beta = (B + f(B))^2, \tag{14}$$

and then we arrange the expression of Equation (14) in decreasing order of B. For the moment, we treat $f(B)$ as a constant term to yield that

$$\alpha B^2 - 2f(B)B + \beta - (f(B))^2 = 0. \tag{15}$$

We complete the square for B to imply that

$$\alpha(B - (f(B)/\alpha))^2 + \beta = (1+\alpha)(f(B))^2/\alpha. \tag{16}$$

Since $\alpha > 0$, to attain the minimum, we should have $B^* = f(B^*)/\alpha$ and then we find that

$$f(B^*) = \sqrt{\alpha\beta/(1+\alpha)}, \tag{17}$$

and then we derive that

$$B^* = \sqrt{\beta/\alpha(1+\alpha)}. \tag{18}$$

V. The distinct character of our solution approach

Through our approach, we first find $f(B^*) = \sqrt{\alpha\beta/(1+\alpha)}$ and then we derive $B^* = \sqrt{\beta/\alpha(1+\alpha)}$.

To the best our knowledge, in the past researcher must solve the optimal solution of B^* and then plug B^* into the original problem to derive $f(B^*)$.

Our solution approach took an inverse route to find $f(B^*)$ first and then to locate B^* second.

We can predict that our approach will have the significant advantage to save the compute effort and time for the following problems:

(i) The focus is concentrated on the optimal value, $f(B^*)$ and the optimal point, B^* is not important.

(ii) For those problems, plugging B^* into $f(B)$ to evaluate $f(B^*)$ becomes tedious work.

VI. Conclusion

We solve the open question proposed by Chang et al. [2] through a purely algebraic approach. Our procedure may provide a new method to solve many operational research problems with those practitioners who are not familiar with calculus.

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Appendix The calculus approach

To solve $f'(B)=0$, we find that

$$\left(\frac{1}{2}\right)\frac{2(1+\alpha)B}{\sqrt{(1+\alpha)B^2+\beta}}=1 \tag{19}$$

to imply that $(1+\alpha)B=\sqrt{(1+\alpha)B^2+\beta}$ and then

$$B^*=\sqrt{\beta/\alpha(1+\alpha)}. \tag{20}$$

Consequently, after we find B^*, we compute $f(B^*)$ as

$$f\left(\sqrt{\frac{\beta}{\alpha(1+\alpha)}}\right)=\sqrt{\frac{\beta}{\alpha}+\beta}-\sqrt{\frac{\beta}{\alpha(1+\alpha)}}$$

$$=\sqrt{\frac{\beta}{\alpha(1+\alpha)}}((1+\alpha)-1)=\sqrt{\frac{\alpha\beta}{1+\alpha}}\ . \tag{21}$$

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