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Continuous-Time Markov Chain Model of Repairable Machine: Application to Asalaya Sugar Factory

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***Abstract**:*
This study aims at dealing with continuous time Markov chain model application on the fault time of two machines (Mill troup -Boiler), important machine in Asalaya Sugar Factory in season (January/2019–Decmber/2019), which affiliated to Sudanese Sugar Company.The study conduces that the failure time of machines follows Exponential distribution estimation fault distribution.Failure time represent transition matrix in the Continuous time Markov chain. The probability of the machine in operating state is greater than the probability of the machine in a fail state.The high probability of the machine in operating state and the mean time of a machine stay estimated by 4 hours in state (1) (operating state) meanwhile the machine stay in state (0) (fail state) estimated by one hour which indicates the efficiency of the maintenance unit, it is clear that, the probability of available time to repair machines when it fault approximately (0.80), this indicates that the machines has high availability.

***Keywords**: Continuous time Markov chain, Generator Matrix, Repairable Machine, Availability*

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I. Introduction:

The use of quantitative and mathematical methods provide an insight into the actual reality of the operational state of machinery and equipment. In addition, they are powerful tools in the final evaluation (*Norman, 2012*). Maintenance is one of the main factors that help in maintaining machines and prolonging their life. For this reason, most companies strive for maintenance management to have a high degree of planning, organization, and control. Maintenance aims to ensure that all production machinery and equipment are kept in good condition for optimum operational. And stand on the actual operational condition of the machines. There are many quantitative and mathematical models that help the industrial facility to achieve this.

The objective of the study is to apply Continuous time Markov chain (CTMC) Model monthly failure time for the season (January/2019–December/2019) of two machines (Mill troup -Boiler) important machine in the Asalaya Sugar Factory, which affiliated to Sudanese Sugar Company to measure the transitions probability machines from an operational state to other the probabilities of the machine can be one of two states state (1) machine operating state and state (0) machine is a fail state. In addition to the availability machines.

The data of this study have been collocated for monthly failure time for the season (January/2019–December/2019) of two machines (Mill troup -Boiler), and important machine in the Asalaya Sugar Factory. Based on mechanical faults; the study seeks to achieve the following hypotheses:

- The failure time distribution pattern is exponential distribution.
- The failure time transition matrices represented the continuous time Markov chain.
- The machines have high availability.

II. Continuous Time Markov Chain Process of Repairable Machine

Continuous Time Markov Chain Process is used to represent and calculate the transition probabilities of the machine from being in operating to being down at specific points in time. In addition to calculating the long run (stationary) probability for a machine to be state operating / state down, the mean sojourn time in state operating / state down.

2.1 The Model

Consider the machine has two possible states, state (1) a machine is operating , state (0) a machine is down .Assume a machine status Markov Chain $[X(t), t \geq 0]$ with the state machine taken $S = [0,1]$. A transition matrix is defined as:

$$P(t) = \begin{pmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{pmatrix} \quad (1)$$

$P(t)$ Probabilities, $0 \le P(t) \le 1$, we have $P_{ij} = (X(t) = j / X(0) = i)$ for all $t \ge 0\ i, j \in x$ is the probability for a machine to move from state i to state j at time . The sum of each row in the matrix $P_{ij}(t)$ is therefore equal to 1:

$$\sum_{j=0}^{1} P_{ij} = 1 \quad (2)$$

2.2 Stationary Distribution of Markov chain:

In any process that must be assigned stationary or stationary in the long term, the probability of the machine existence between different stages can be calculated after period of time units (stationary probabilities). The vector V is a stochastic vector that expresses the probabilities of the machine existence between different stages after the passage of a certain a period of time units. That means:

$$V = [V_1 \quad V_2 \quad V_3 \quad \dots] \quad (3)$$

By using vectors:

$$V_j = \lim_{n \to \infty} P_j^{(n)} \quad (4)$$

The vector element are obtained by solving the below equation :

$$V = VP_{ij} \quad (5)$$

Where P_{ij} transition Probability Matrix

2.3 The Exponential Distribution:

The first step to estimating a Continuous time Markov chain (CTMC) to test the pattern distribution of the data is exponential distribution. If t random variables represents an exponential distribution with rate parameter, if t random variables has a continuous distribution a with probability density function f given by:

$$f(t) = \lambda e^{-\lambda t} \quad \text{for } t \ge 0 \quad (6)$$

With mean ($1/\lambda$) and variance ($1/\lambda^2$).

The Cumulative distribution function F is given by:

$$F(t) = P(T > t) = e^{-\lambda t} \quad \text{for } t \ge 0 \quad (7)$$

The importance role of the exponential distribution in continuous time Markov chain. Let that j be the time that we have to wait for the jump achieves the lack of memory property, if the wait for t time units are done and the jump does not occur, then the remaining time has the same distribution like the original waiting time and this property is called constant failure rate property.

2.3 The Generator Matrix

The generator matrix contains all information about the transitions of the Continuous time Markov chain. Its property all diagonal the entries are negative , the sum over entries of each row is zero, and all diagonal entries are positive .Usually denoted by Q. The probability of transition state i is approximately for this reason, λ is often called the transition rate out of state i , we calculate positive constants (μ = total no of repairing time / no of failures and) , λ which is a rate of exponential distribution. Then we get the generator matrix for the continuous Markov chain:

$$Q = \begin{bmatrix} -\mu & \mu \\ \lambda & -\lambda \end{bmatrix} \quad (8)$$

We use Q-matrix (8) to estimate the transition probability (Marvin Rausand, Arnljot Hoyland,2003, p.314) we get.

$$P_{00} = \frac{\lambda}{\mu + \lambda} + \frac{\mu}{\mu + \lambda} e^{-(\mu + \lambda)t} \quad (9)$$

$$P_{01} = \frac{\mu}{\mu + \lambda} - \frac{\mu}{\mu + \lambda} e^{-(\mu+\lambda)t} \quad (10)$$

$$P_{10} = \frac{\lambda}{\mu + \lambda} - \frac{\lambda}{\mu + \lambda} e^{-(\mu+\lambda)t} \quad (11)$$

$$P_{10} = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\mu+\lambda)t} \quad (12)$$

2.4 Stationary -state probabilities

The Stationary -state probabilities -state probabilities machine stays in the long run in state 0 (operating) and state 1(fail) equal to:

$$V_0 = \frac{\lambda}{\mu + \lambda} \quad (14)$$

$$V_1 = \frac{\mu}{\mu + \lambda} \quad (15)$$

The stationary vector:

$$V = \left[\frac{\lambda}{\mu + \lambda} \quad , \frac{\mu}{\mu + \lambda} \right] \quad (16)$$

The mean time of machines stays in state 1 = $\frac{1}{\lambda}$, and the mean time of machines stay in state 0 = $\frac{1}{\mu}$.

III. Availability

Availability is the efficiency of the machine to perform its function. The interrelated factors of reliability and maintainability under certain conditions during a certain period of time are improving(*Hoang Pham,2006)*, and the indicator value should be the highest possible and depends on MTTF and MDT .The well-known formula to calculating is:

$$A(t) = \frac{MTTF}{MTTF + MDT} \quad (17)$$

IV. Application:

Transition probabilities of machines from state (0) to 1 are estimated .Firstly, from the failure time of the machines described data by using some descriptive measures. Secondly, using EasyFit software to test a failure time following the exponential distribution. Thirdly, Mathcad2000 software was used to calculate the generator matrix and Continuous-time Markov chain matrix after testing conditions. Above steps were applied to each machine separately, then the two machines.

4.1: Data of Study:

Table 1
Failure time for Repairable Machines during season (1/2019–12/2019)

Month	Machine			
	Mill troup		Boiler	
	Repair no	Failure time	Repair no	Failure time
1	3	2.1	2	2.8
2	6	4.8	3	4.2
3	9	5.7	6	9.7
4	6	6.3	5	4.8
5	9	8.5	4	5.1
6	6	3.9	5	3.9
7	2	3.5	3	1.8
8	2	2.3	2	1.2
9	5	5.5	3	2.8
10	1	5.8	4	1.8

11	2	1.3	8	7.8
12	6	3.2	6	2.9

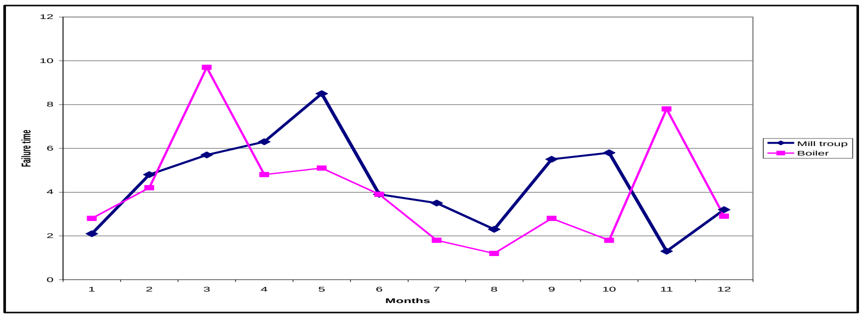

Figure 1: Shows the failure time for machines with time (month)

The above figure shows, the failure time of the machine is increasing and stationary in a certain period as well as decreasing in other periods. This indicates the similarity of fault behavior of machines during the study period

4.1.1 Description of Data of Study:

Table 2
Mean and standard deviation of failure times for both machines

Machine	Total failure no	Total failure time	Mean (hr)	Std. (hr)	95% Confidence Interval for Mean	
					Lower Bound	Upper Bound
Machine (Mill troup)	61	52.900	4.4083	2.0725	3.0915	5.7252
Machine (Boiler)	51	48.800	4.0667	2.5296	2.4594	5.6937
Both machines	112	101.7	4.2376	2.2683	3.2779	5.1953

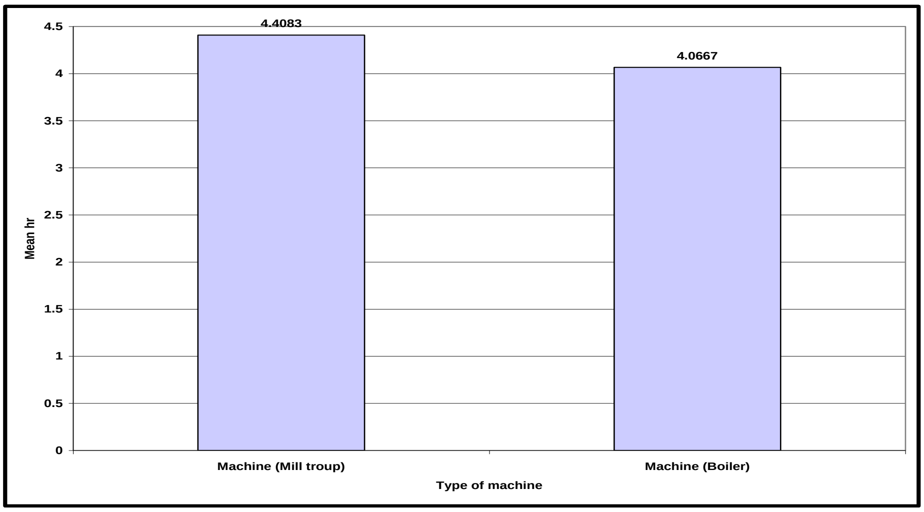

Figure 2: Shows the mean of failure time for machines

The above table 2 and figuer 2, show the mean values for the each machines , There is a convergence between the mean failure of the two machines, as the mean failure of machine (Mill troup) is (4.4083) hours, while the mean failure of machine (Boiler) is (4.0667) during the year.

4.2 Continuous Time. Markov Chain Model of machine (Mill troup):
4.2.1 Test of Failure Time Fit Exponential Distribution:
To test whether the failure time of a machine (Mill troup) follows the exponential distribution or not, the following hypothesis should be formulated:
H_o: The failure time of the machine (Mill troup) follows exponential distribution
H_1: The failure time of th machine (Mill troup) not following exponential distribution

Table 3
Kolmogorov-Smirnov test for failure time distribution of machine (Mill troup)

Sample Size Statistic P-Value Rank λ	12 0.2956 0.2003 54 0.2268				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.2958	0.3382	0.3754	0.4192	0.4491
Reject?	No	No	No	Yes	Yes

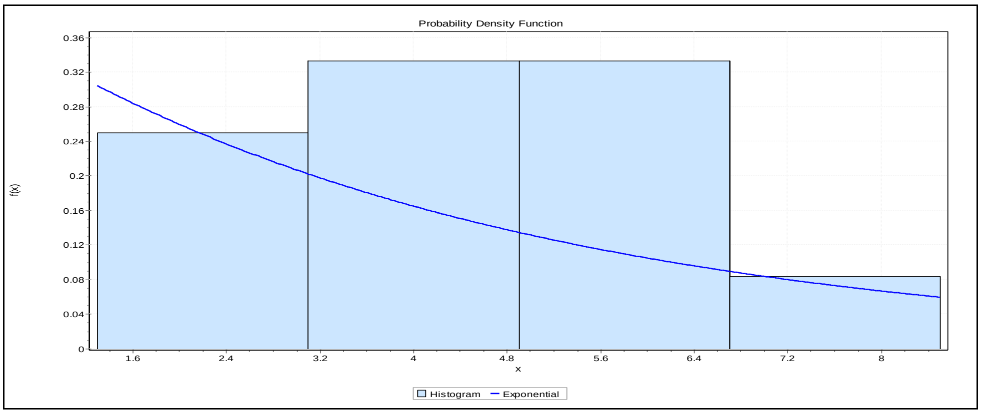

Figure 3: Shows failure times' distribution of machine (Mill troup)

Table 3 shows Kolmogorov–Smirnov test p-value (0.2003 > 0.05), that means the failure time distribution pattern is Exponential distribution with rate ($\lambda = 0.2268$), and repair rate ($\mu = 0.8721$).

4.2.2 Estimate Continuous Time Markov Chain for Machine (Mill troup)

Table 4
Estimate the continuous time Markov chain for machine (Mill troup) :

Failure time 1/2019–12/2019	**Machine (Mill troup)**
Generator matrix	$Q = \begin{bmatrix} -0.8721 & 0.8721 \\ 0.2268 & -0.2268 \end{bmatrix}$
Transition matrix	$p = \begin{bmatrix} 0.4708 & 0.5292 \\ 0.1376 & 0.8624 \end{bmatrix}$
Stationary vector	$V^T = \begin{bmatrix} 0.2064 & 0.7936 \end{bmatrix}$
Mean time of state 1 (hour)	4.4091 ≈ 4 hours
Mean time of state 0 (hour)	1.1467 ≈ 1 hour
Availability	0.7935 ≈ 0.79 %

The results in table 4 show, the transition probability of machine in state 0 is (0.4708) means 47% of the operating time for a machine in a failed state, the transition probability from state 0 to state1 is (0.5292) means 53% operating time of a machine under repaired, the transition probability form state1 to state 0 is (0.1376) that means 14% the operating time a machine fails state , transition probability of machine in state 1(0.8624) which indicates a machine is in operating state. It becomes clear from the results of the estimate stationary probabilities that in the long time approximately 21% of the available operating time of a machine in state 0 (fail) and 79% of the time in state 1 (operating state), On Mean, a machine stays in state 1(operating state) 4 hours, while the machine stays in state 0 (fail state) one hour. The availability percent of the machine is 80%.

4.3 Continuous time Markov chain Model of machine (Mill troup):
4.3.1 Test of Failure Time Distribution:
H_o: The failure time of machine (Boiler) follows exponential distribution
H_1: The failure time of machine (Boiler) not following exponential distribution

Table 4
Kolmogorov-Smirnov test for failure time of machine (Boiler)

Sample Size Statistic P-Value Rank λ	12 0.2743 0.2735 49 0.2459				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.2958	0.3382	0.3754	0.4192	0.4491
Reject?	No	No	No	Yes	Yes

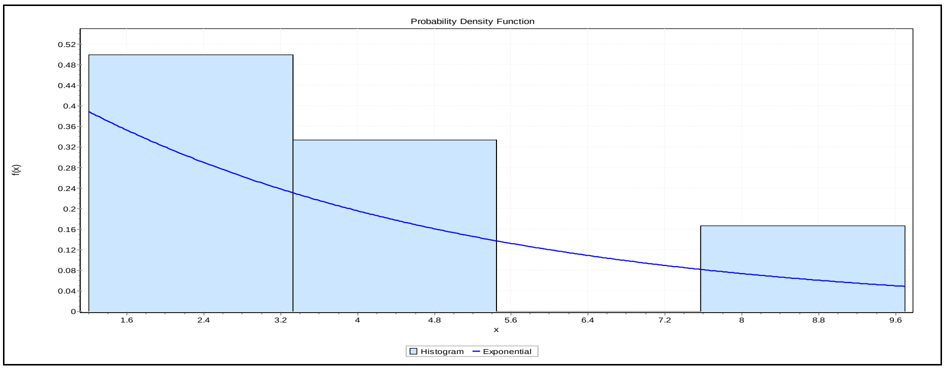

Figure 4: Shows the failure time distribution of machine (Boiler)

Table 4 shows Kolmogorov–Smirnov test p-value (0.2735> 0.05), that means the failure time distribution pattern is Exponential distribution with rate ($\lambda = 0.2459$), and repair rate ($\mu = 0.9687$).

4.3.2 Estimate Continuous time Markov chain for machine (Boiler) :

Table 5
Estimate Continuous time Markov chain for machine (Boiler) :

Failure time 1/2019–12/2019	Machine (Boiler)
Q-Matrices	$Q = \begin{bmatrix} -0.9687 & 0.9687 \\ 0.2459 & -0.2459 \end{bmatrix}$
Transition matrix	$p = \begin{bmatrix} 0.4392 & 0.5608 \\ 0.1424 & 0.8576 \end{bmatrix}$
Stationary vector	$V = \begin{bmatrix} 0.2025 & 0.7975 \end{bmatrix}$
Mean time of state 1 (hour)	$4.0667 \approx 4$ hours
Mean time of state 0 (hour)	$1.0323 \approx 1$ hour
Availability	$0.7975 \approx 0.80$ %

The results in table 5 show , the transition probability of the machine in state (0) is (0.4392) means 43% of the time a machine in a failed state, the transition probability from state 0 to state1 is (0.5608) means 56% of the time a machine under repaired, the transition probability form state(1) to state (0) is (0.1424) that means 14% of time a machine is fails, transition probability of machine in state (1) (0.7975) which indicates 80% of time a machine in operating state. It becomes clear from the results of the estimate stationary probabilities that in the long time approximately 20% of the available operating time of the machine in state (0) (fail state) and 80% of the time in state (1) (operating state), On Mean, a machine stays in state 1(operating state) 4 hours, while the machine stays in state (0) one hour. The availability percent of the machine is 80%.

4.4 Continuous time Markov chain Model of both Machines

4.4.1 Test of Failure Time Distribution:

H_o: The failure time of both machines follow exponential distribution

H_1: The failure time of both machines not follow exponential distribution

Table 6

Kolmogorov-Smirnov test for failure time of both machines

Sample Size Statistic P-Value Rank λ	24 0.2628 0.0596 54 0.2360				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.2958	0.3382	0.3754	0.4192	0.4491
Reject?	No	No	No	Yes	Yes

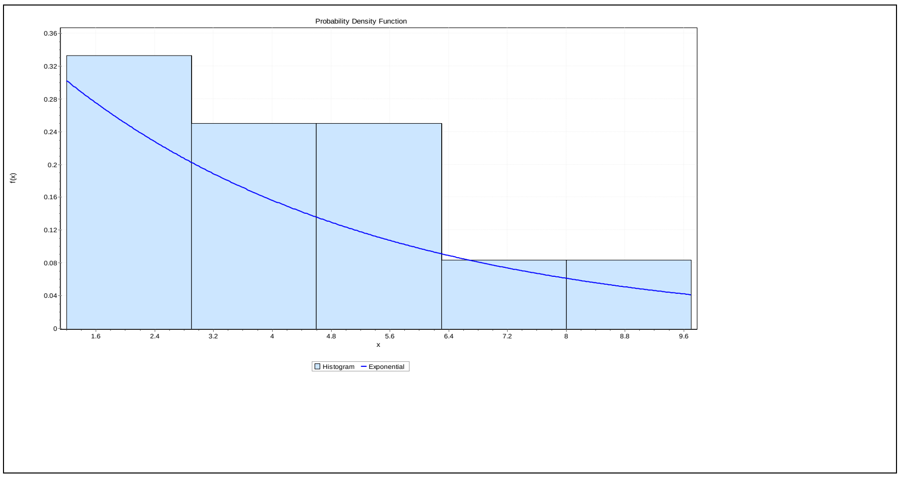

Figure 4: Shows failure time distribution of both machines

Table 6 shows Kolmogorov–Smirnov test p-value (0.0596> 0.05), that means the failure time distribution pattern is Exponential distribution with rate (λ = 0.2360), and repair rate (μ = 0.9417).

Table 7

Estimate Continuous time Markov chain for both machines :

Failure time 1/2019–12/2019	Both machines
Q-Matrices	$Q = \begin{bmatrix} -0.9147 & 0.9417 \\ 0.2360 & -0.2360 \end{bmatrix}$
Transition matrix	$p = \begin{bmatrix} 0.4467 & 0.5533 \\ 0.1387 & 0.8613 \end{bmatrix}$
Stationary vector	$V^T = \begin{bmatrix} 0.2025 & 0.7975 \end{bmatrix}$
Mean time of state 1 (hour)	4.2373 $\approx$ 4 hours
Mean time of state 0 (hour)	1.0619 $\approx$ 1 hour
Availability	0.7996 $\approx$ 0.80 %

The results in Table 7 show , the transition probability of both machines in state (0) is (0.4467) means 45% of the time at machines are failed state, the transition probability from state (0) to state (1) is (0.5533) means 55% of time both machines under repaired, transition probability form state (1) to state (0) is (0.1387) that means 14% of time a both machines are fails, transition probability of both machines in state (1) (0.7975) which indicates 80% of time a machine in operating state. It becomes clear from the results of the estimate stationary probabilities that in the long time approximately 20% of the available operating time of the machine in state (0) (fail state) and 80% of the time in state (1) (operating state), On Mean, both machines stay in state 1 (operating state) 4 hours, while the both machines stay in state (0) one hour. The availability percent of the both machines is 80%.

V. Conclusion

From the through application of the CTMC model on the failure time of two machines in Asalaya Sugar Factory has contributed to assessment state machines through the transition from operating state to fail state and versa. The failure time of machine follows exponential distribution, and this main conditional distribution of the applied CTMC. It become clear from the results of transition matrix, the main diameter elements of the matrix are greater than the off- diameter ones, and sum of each row equal one that indicates the transition matrix represents the CTMC. The transition probability from state (1) to state (0) and transition probability from state (0) to state (1) for machine (Mill troup) ,machine (Boiler), and both machines were very close . It became clear from the mean time of machine stay in state 1 (operating state) an estimated 4 hours for each machine, while the both machine stay in state (0) one hour. The probability of available time to repair machines when it faults is approximately (0.80). The probability of available time to repair machines when it faults is approximately (0.80), and this indicates that the machine have high availability. The study recommends: improving the operational efficiency of the machines through total maintenance based on the results obtained by the CTMC model, it provides a more accurate measure of the condition of machines.

Conflicts of interest

The authors declare no conflict of interest.

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References

[1]. Hussein Nassreldeen Eltahir, Khalid Rahamtalla Khedir, Mohammedelameen Eissa Qurashi (2020). "Markov chains model to estimate the transition operational status probabilities for machines in Asalaya Sugar Company." IOSR Journal of Mathematics (IOSR-JM), 16(6), pp. 06-13. https://doi.org: 10.9790/5728-1606040613

[2]. Rabta, B., van den Boom, B. and Molini , V. (2016), A Continuous-time Markov Chain Approach for Modeling of Poverty Dynamics: Application to Mozambique, African Development Review, 28: 482495.

[3]. https://doi.org/: 10.1111/1467-8268.12225

[4]. Chen, Y. and J. Chen (2011), 'On the Imbedding Problem for Three-State Time Homogeneous Markov Chains with Coinciding Negative Eigenvalues', Journal of Theoretical Probability, Vol. 24, No. 4, pp. 928–38. https://doi.org/:10.1007/s10959-010-0316-5

[5]. Lotfi, T.(2011) Replacement Models with Minimal Repair, London: Springer-Verlag.

[6]. https://doi.org/10.1007/978-0-85729-215-5

[7]. Van Der, J.A., Weide, S. & Van Noortwijk, J.M.(2008), Renewal Theory with Exponential and Hyperbolic Discounting. Probability in the Engineering and Informational Sciences, Vol. 22., pp. 53–74 https://doi.org/10.1017/S0269964808000041

[8]. Guangbin, Y.(2007) Life Cycle Reliability Engineering, NewYork : John Wiley & Sons. https: //doi.org/10.1002/9780470117880

[9]. Hoang Pham,(2006),System Software Reliability, Publisher,Springer-Verlag London https://doi.org/10.1007/1-84628-295-0

[10]. Zhou, X., Xi, L. and Lee, J. (2006), "A dynamic opportunistic maintenance policy for continuously monitored systems", Journal of Quality in Maintenance Engineering, Vol. 12 No. 3, pp. 294-305. https://doi.org/10.1108/13552510610685129

[11]. Adamantios Mellas & Wenbiao Zhao(2005), Modeling and Analysis of Repairable System with Gerenral, Publisher: IEEE. https://doi.org/10.1109/RAMS.2005.1408358

[12]. Dhillon, B.S(2002), Engineering Maintenance: A Modern Approach. United States, CRC Press,.

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