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Application of Linear Programming to Profit Maximization In Water Production

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Abstract
Optimization has become a common phenomenon in almost all organizations and establishments. In developed, countries managerial decisions are mostly based on the use of optimization techniques the objective of this research work was to apply linear programming for profit maximization of raw material in water production. Viclibo Venturs (VILA), Zango, Lokoja, Kogi State was used as our case study. The decision variables in this research work are the three different sizes of water (Sachet bag of water, 50cl pack of water, and 75cl pack of water) produced by Viclibo Venturs. The researcher focused mainly on three raw materials (production cost, production time and Demand or Sales) used in the production and the amount of raw material required of each variable (water size). The result shows that 600 unit of a bag of sachet water, 150 unit of a pack of 50cl water 100 unit of a pack of 75cl of water respectively which will give a maximum profit of ₦40,000.00
Keywords: Optimization, Water, Production, Linear Programming, Excel Solver
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I. Introduction

Optimization has become a common phenomenon in almost all organizations and establishments. In developed, countries managerial decisions are mostly based on the use of optimization techniques. In a profit seeking organizations, the Board of Directors has many things to tackle which may include: The problem of other competitors in the same business, availability of funds for new capital projects, reduction of operational cost, high level of output and ultimately maximization of profit as explained by Nonso (2005).

In an attempt to address these problems there are two techniques of operation that may be applied. They include: The quantitative technique and the qualitative technique. Quantitative technique which is preferred involves modelling of a 'real form' problem into a mathematical form which can be solved to arrive at a Solution that would aid the decision makers. Linear programming (LP) technique is such a Quantitative technique: It is a widely used Mathematical modelling technique concerned with the efficient allocation of limited resources to known activities with the objective of meeting the desired goal (Taha, 1977).

In 1947, during World War II, George B. Dantzing while working with the US Air force, developed LP model primarily for solving military logistics problems. But now, it is extensively being used in all functional areas of management, airlines, agriculture, military operations, oil refining, education, energy planning, health care system etc.

Frizzone, et al (1997) used linear programming model to optimize the water resource used in irrigation, and obtained an optimal way of water resource usage. Though, considering the numerous constraints, he had to develop a separate mathematical model to achieve his goal. He noted that the mathematical programming quantifies an optimal way of combing scarce resources to satisfy the proposed goals; that is to analyze the cases where the available resources must be combined in a way to maximize the profit or minimize cost.

Nonso (2005) in his work on Application of Linear programming for Managerial Decision found out how an organization can have effective control over materials for input during production. He observed that units produced must be assumed as what is sold in order to achieve the company's goal.

Fagoyinbo, I. S. and Ajibode, I, A. (2010) worked on the Application of Linear programming Techniques in the Effective use of resources for staff training. The method employed gave an integer optimum solution to all the models formulated. The Data used did not yield a feasible solution but when the model reformed gave an optimum solution.

B. I. Ezema and U. Amakom (2012) worked on optimizing profit with the linear programming model: A focus on Golden plastic Industry limited. Enugu. 2012. The result they had showed that only 2 sizes of the total 8 'PVC' pipes should be produced.

I. U.Khan, N. H. Bajuri and I. A. Jadoon (2011) in their work, optimal production levels for the different products manufactured at ICL, a multinational Company in Pakistan had a result that showed that the amount was raised by changing production patterns within the first, second, third and fourth digit respectively. The intention of this paper is therefore to determine the optimum production capacity of Usmer water company, Uyo, Nigeria.

II. Methodology

Linear Programming is a mathematical technique for generating and selecting the optimal or the best solution for a given objective function. It may be defined as a method of optimizing (i.e maximizing or minimizing) a linear function for several constraints stated in the form of linear equations.
The general linear programming problem (or model) with *n* decision variables and *m* constraints can be stated in the following form.
Optimize $z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n$
Subject to Linear Constraints of the form

$$\begin{array}{c} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_{1n}x_n(\le, =, \ge)b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_{2n}x_n(\le, =, \ge)b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_{3n}x_n(\le, =, \ge)b_3 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_{mn}x_n(\le, =, \ge)b_m \end{array}$$

The above formulation can also be expressed in a compact form bellow.
Optimize (Max or Min) $z = \sum_{j=1}^{n} c_jx_i \ldots\ldots (objective\ function)$
Subject to Linear Constraints

$$\sum_{j=1}^{n} a_{ij}x_j(\le, =, \ge)b_i, i = 1,2, \ldots\ldots m\ and$$
$$x_j \ge 0, j = 1,2, \ldots\ldots\ldots n$$

Where $c_1, c_2, c_3 \ldots\ldots\ldots\ldots\ldots c_n$ represent the per unit profit (or cost) of decision variables $x_1, x_2, \ldots\ldots\ldots\ldots\ldots x_n$ to the value of the objective function. And $a_{11}, a_{12}, a_{13} \ldots\ldots\ldots, a_{m1}, a_{m2}, a_{m3} \ldots\ldots\ldots$, represent the amount of resource consumed per unit of the decision variables. The b_1 represents the total availability of the i^{th} resource. Z represent the measure -of – performance which can be either profit, or cost.

Specifically the linear programming model for this paper is given by
$maximize\ z = c_1x_1 + c_2x_2 + c_3x_3$ - objective function
$subject\ to$
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \le b_1$ (cost constraint)
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \le b_2$ (Production time constraint)
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \le b_3$ (Demand/supply constraint)
$x_1, x_2, x_3 \ge 0.$

Model Assumptions

- It is assumed that the raw materials required for production of water are limited (scarce)
- It is assumed that an effective allocation of the variables used (sachet bag of water, 50cl pack of water,75cl pack of water) will aid optimal production and at the same time maximizing the profit of the Viclibo water venture.
- It is assumed that the qualities of variables used in water production are standard (not inferior).

III. Data Presentation

The data for this study are in the table 1 below which were extracted from the sales record of Viclibo Venturs (VILA), Zango, along Kabba-Okene Express Way, Lokoja, Kogi State, Nigeria in July, 2019. The data consist of total amount of raw materials (cost, production time and Demand/Supply) available for daily production of three different sizes of water (Sachet bag of water, 50cl pack of water, and 75cl pack of water) and profit contribution per each unit size of water produced. The data analysis was carried out with Excel Solver software. The content of each raw material per each unit product of water produced is shown in the table below.

Table 1

S/n	Product Package	Production Time(seconds)	Production Cost (N)	Selling Price (N)	Highest Purchase (Daily)	Lowest Purchase (Daily)
1	Sachet Bag	36.4	60	90	600	200 bags
2	50cl (pack)	0.144	368	440	150	30 packs
3	75cl (pack)	0.174	368	480	100	10 packs

Note: i. Working Hours 8am – 4Pm daily, that is 480 mins, i.e 28,800 seconds (8 hrs)

ii. The maximum cost production per day is 87,840 naira

iii. Profit Maximization is production cost – selling price i.e

- $Sachet\ bag\ of\ water\ is\ 90 - 60 = 30$
- $50cl\ pack\ of\ water\ is\ 440 - 368 = 72$
- $75cl\ pack\ of\ water\ is\ 480 - 368 = 112$

The Formulated Model is given as

$Max. z = 30x_1 + 72x_2 + 112x_3$

$s.t$

$36.4x_1 + 0.144x_2 + 0.174x_3 \leq 28{,}800$

$60x_1 + 368x_2 + 368x_3 \leq 187{,}840$

$x_1 \leq 600$

$x_2 \leq 150$

$x_3 \leq 100$

$x_1 \geq 200$

$x_2 \geq 30$

$x_3 \geq 10$

$x_1, x_2, x_3 \geq 0$

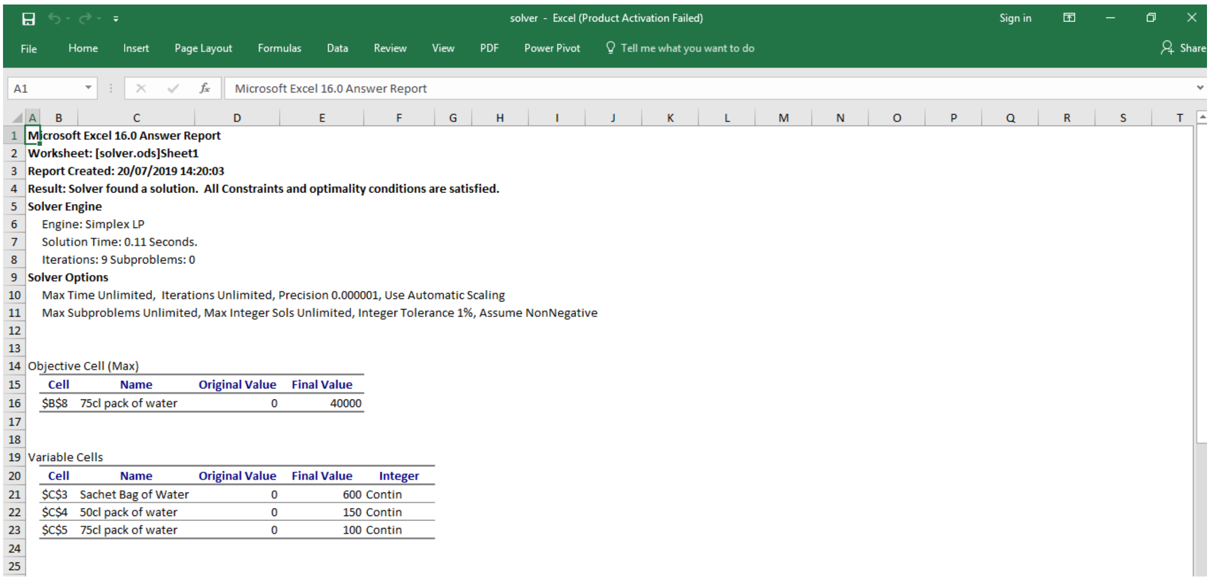
IV. Data Analysis and Results

The data were analyzed using EXCEL SOLVER and the solution is given thus

$x_1 = 600$, $x_2 = 150$, $x_3 = 100$.

Therefore, the final solution Optimum value is 40000

REEN DISPLAYED OF THE ANALYSIS USING EXCEL SOLVER


Microsoft Excel 16.0 Answer Report
Worksheet: [solver.ods]Sheet1
Report Created: 20/07/2019 14:20:03
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.11 Seconds.
Iterations: 9 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$B$8	75cl pack of water	0	40000

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$3	Sachet Bag of Water	0	600	Contin
$C$4	50cl pack of water	0	150	Contin
$C$5	75cl pack of water	0	100	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$11	75cl pack of water	21879	$B$11<=$D$11	Not Binding	6921
$B$12	75cl pack of water	128000	$B$12<=$D$12	Not Binding	59840
$B$13	75cl pack of water	600	$B$13<=$D$13	Binding	0
$B$14	75cl pack of water	150	$B$14<=$D$14	Binding	0
$B$15	75cl pack of water	100	$B$15<=$D$15	Binding	0
$B$16	75cl pack of water	600	$B$16>=$D$16	Not Binding	400
$B$17	75cl pack of water	150	$B$17>=$D$17	Not Binding	120
$B$18	75cl pack of water	100	$B$18>=$D$18	Not Binding	90
$B$19	75cl pack of water	600	$B$19>=$D$19	Not Binding	600
$B$20	75cl pack of water	150	$B$20>=$D$20	Not Binding	150
$B$21	75cl pack of water	100	$B$21>=$D$21	Not Binding	100

Answer Report 1 | Sensitivity Report 1 | Limits Report 1 | Sheet1

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [solver.ods]Sheet1
Report Created: 20/07/2019 14:20:03

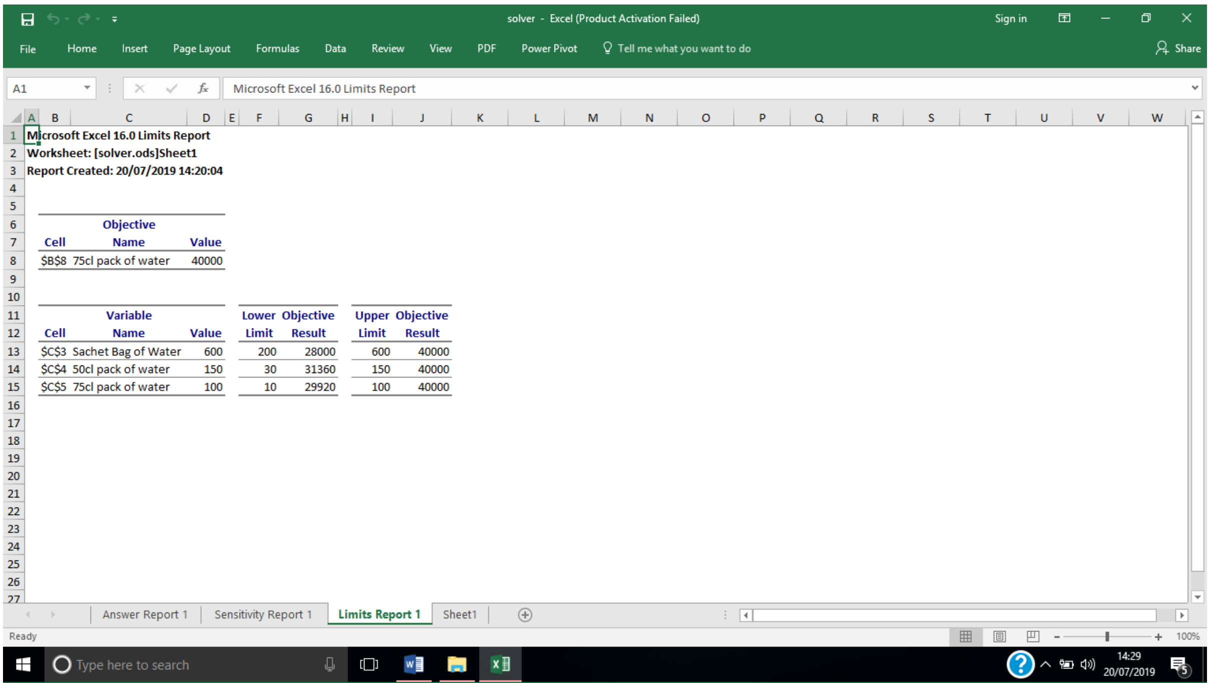
Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$3	Sachet Bag of Water	600	0	30	1E+30	30
$C$4	50cl pack of water	150	0	72	1E+30	72
$C$5	75cl pack of water	100	0	112	1E+30	112

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$11	75cl pack of water	21879	0	28800	1E+30	6921
$B$12	75cl pack of water	128000	0	187840	1E+30	59840
$B$13	75cl pack of water	600	30	600	190.1373626	400
$B$14	75cl pack of water	150	72	150	162.6086957	120
$B$15	75cl pack of water	100	112	100	162.6086957	90
$B$16	75cl pack of water	600	0	200	400	1E+30
$B$17	75cl pack of water	150	0	30	120	1E+30
$B$18	75cl pack of water	100	0	10	90	1E+30
$B$19	75cl pack of water	600	0	0	600	1E+30
$B$20	75cl pack of water	150	0	0	150	1E+30
$B$21	75cl pack of water	100	0	0	100	1E+30

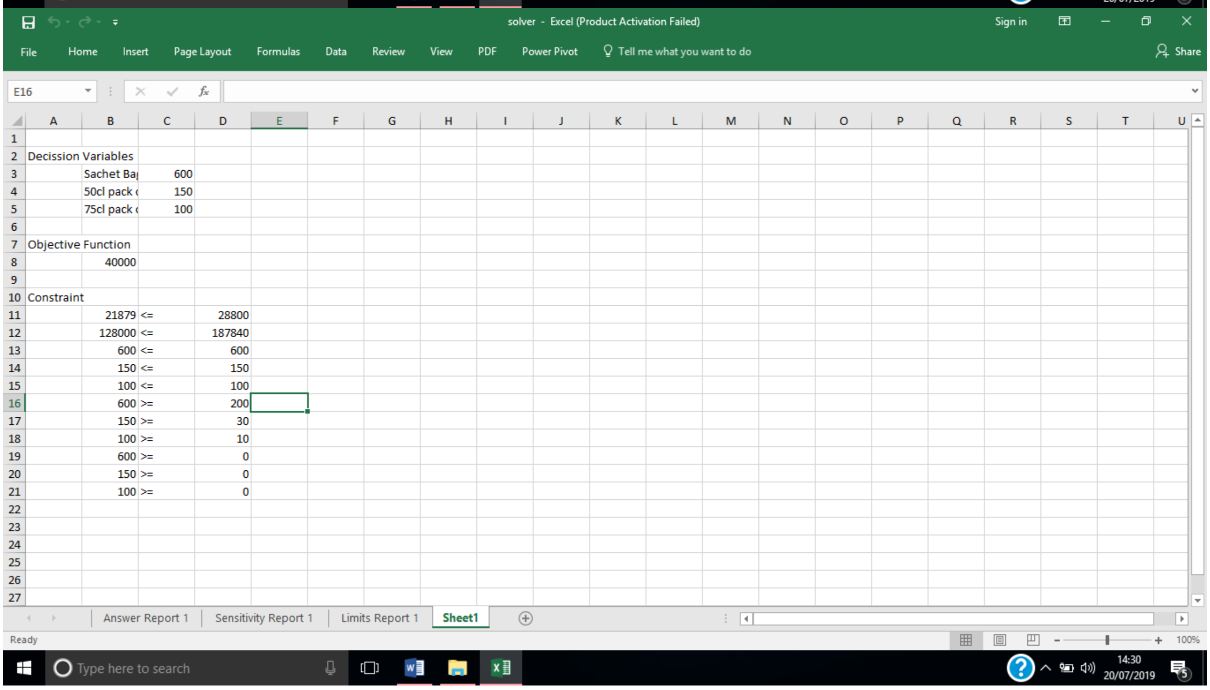
Answer Report 1 | Sensitivity Report 1 | Limits Report 1 | Sheet1



Microsoft Excel 16.0 Limits Report
Worksheet: [solver.ods]Sheet1
Report Created: 20/07/2019 14:20:04

	Objective	
Cell	Name	Value
$B$8	75cl pack of water	40000

	Variable		Lower	Objective	Upper	Objective
Cell	Name	Value	Limit	Result	Limit	Result
$C$3	Sachet Bag of Water	600	200	28000	600	40000
$C$4	50cl pack of water	150	30	31360	150	40000
$C$5	75cl pack of water	100	10	29920	100	40000



Decission Variables			
	Sachet Ba	600	
	50cl pack	150	
	75cl pack	100	
Objective Function			
	40000		
Constraint			
	21879	<=	28800
	128000	<=	187840
	600	<=	600
	150	<=	150
	100	<=	100
	600	>=	200
	150	>=	30
	100	>=	10
	600	>=	0
	150	>=	0
	100	>=	0

The summary of the result is given in Table 2 below

Table 2

	Objective	
Cell	**Name**	**Value**
$B$8	75cl pack of water	40000

	Variable		**Lower**	**Objective**	**Upper**	**Objective**
Cell	**Name**	**Value**	**Limit**	**Result**	**Limit**	**Result**
$C$3	Sachet Bag of Water	600	200	28000	600	40000

$C$4	50cl pack of water	150	30	31360	150	40000
$C$5	75cl pack of water	100	10	29920	100	40000

The results or solutions in table 2 require that the company should produce daily,

- 600 Bags of Sachet water
- 150 packs of 50cl water
- 100 packs of 75cl water

Note that a bag contains 20 Sachets while a pack contains 12 bottles. Also it is noticed that if the company embarks on this plan the following results would follow|:

a. ₦ 128000 of ₦ 187840 (Maximum Production Cost) shall be unused cost.
b. Total working time of 28800 seconds shall be fully exhausted /utilized.
c. A deficit production of 200 bags of sachet water may occur if demand increased to 800 bags and would require

V. Sensitivity Analysis

This was performed still using Excel Solver software and the result is as follows in Table 3. From the analysis, it was discovered that constraints 3, 4, and 5 are **Binding**while other constraints are **not Binding**. Also, an increase or decrease in any of the constraint will only bring change to the **final value** of that particular constraint and the **optimum value** while other **final values** of other constraints remain unchanged provided the change is within the allowable increase or decrease as seen in table3.

Table 3

Microsoft Excel 16.0 Sensitivity Report

Worksheet: [solver.ods]Sheet1

Report Created: 20/07/2019 14:20:03

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$3	Sachet Bag of Water	600	0	30	1E+30	30
$C$4	50cl pack of water	150	0	72	1E+30	72
$C$5	75cl pack of water	100	0	112	1E+30	112

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$11	75cl pack of water	21879	0	28800	1E+30	6921
$B$12	75cl pack of water	128000	0	187840	1E+30	59840
$B$13	75cl pack of water	600	30	600	190.1373626	400
$B$14	75cl pack of water	150	72	150	162.6086957	120
$B$15	75cl pack of water	100	112	100	162.6086957	90
$B$16	75cl pack of water	600	0	200	400	1E+30
$B$17	75cl pack of water	150	0	30	120	1E+30
$B$18	75cl pack of water	100	0	10	90	1E+30
$B$19	75cl pack of water	600	0	0	600	1E+30
$B$20	75cl pack of water	150	0	0	150	1E+30
$B$21	75cl pack of water	100	0	0	100	1E+30

VI. Conclusion

Based on the analysis carried out in this research work and the result shown, Viclibo Venturs (VILA) should produce the three sizes of water (sachet bag of water, 50cl pack of water, and 75cl pack of water) in order to satisfy her customers. Also, more of sachet bag of water should be produce in order to attain maximum profit, because it contribute mostly to the profit earned by the company. Also linear programming method has proven, given the raw data the only way the company production problem can be addressed. The sensitivity report in table 3 shows that constraints 3,4, and 5 are **Binding** while other constraints are **not Binding**.

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