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Unified Theory of Mathematical Operations

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I. Introduction:

Problems in both physical and social sciences are generally described by mathematical models in multi-dimensional variable spaces. Then their solutions require skills and a good understanding of mathematical operations. When you ask most professionals and students to define ZERO and ONE or UNITY, they are puzzled and can't really answer it. They were taught in the elementary school that multiplication of numbers is like a recurrent addition, but when you ask them to explain why the product of two negative numbers is positive, they are again at a loss. So, in this paper we first define a set of unified elementary mathematical operations consisting of eight steps, which include definitions of ZERO and ONE/UNITY for all number systems used in formulating and solving problems. A dimensional space is introduced for expressing all types of measurements from REAL variables, as single dimensional on a line from $-\infty$ to $+\infty$ passing through ZERO in the middle, to multi-dimensional vector spaces. The unified mathematical operations are applied to all number systems from scalars, logarithms, complex variables, multi-dimensional space vectors, computer arithmetic, matrices, and determinants. These unified elementary operations show us why DIVISION can be carried out for complex variables in the two dimensional space but the same is not true for vectors in the multi-dimensional space. It is shown that why multiplication of scalars and complex variables is COMMUTATIVE but not for multi-dimensional vectors and matrices. Contrary to general perception, it is shown that $[i + j + k]$ is not a UNITY vector in three-dimensional space, and that's why there is no DIVISION of three-dimensional vectors.

Elementary Mathematical Operations

Mathematics is a language that we use to express relationships amongst various data, so as to understand the system behavior, in order to solve everyday problems, we encounter in our lives. These data comprise quantitative measurements of variables for which we need a 'Number System'. For example, real, complex, binary, hexadecimal etc. are typical number systems used in solving real life problems. Each of these number systems consist of a set of elements, such as 0,1, 2, 3……………to infinity in REAL number system.

Common to all number systems, we first define a set of " *Elementary Mathematical Operations* ". While these operations may appear to be different for different systems, they follow a unified pattern consisting of the steps described below:

1. Define rule for **ADDITION** of two elements A and B such that

$$A + B = C \text{ (a third element)}$$

2. Search for existence of a **ZERO** (**O**) element such that when it is added to any element A of the system using the rule defined in step (1) it does not alter its value. That is,

$$A + O = A?$$

3. If a ZERO element exists, then search for an element A' such that when it is added to element A the resultant yields the ZERO element. That is,

$$A + A' = O?$$

If such an element exists then define the element A' as negative (complement) of the element A.

$$(A' = - A)$$

4. Define **SUBTRACTION** as an operation of ADDITION of one element to the complement of another element. That is,

$$A - B = A + B'$$

5. Define rule for **MULTIPLICATION** of two elements A and B such that

$$(A) \times (B) = C \text{ (a third element)}$$

6. Search for the existence of a **UNITY** `U', as an element such that when multiplied to another element A, using the rule of multiplication in step (5), it does not alter the value of the element A. That is,

$$(U) \times (A) = (A)?$$

7. If the unity element exists, then search for an element A^{-1} such that when it is multiplied by the element A the resultant yields the unity element `U'. That is,

$$(A) \times (A^{-1}) = U?$$

If such an element exists, then define the element A^{-1} as inverse (reciprocal) of the element A. That is,

$$A^{-1} = \text{Inverse of } A$$

8. Define **DIVISION** of an element A by another element B as MULTIPLICATION of the element A with that of the inverse of B. That is,

$$A \div B = (A) \times (B^{-1})$$

These elementary mathematical operations are further governed by three properties with respect to the order in which the elements are selected for these elementary operations.

1. *COMMUTATIVE*, if $A + B = B + A$
   $(A) \times (B) = (B) \times (A)$
   $(A) \times (B^{-1}) = (B^{-1}) \times (A)$ etc.

2. *ASSOCIATIVE*, if $A + (B + C) = (A + B) + C$
   $(A) \times [(B)\times(C)] = [(A)\times(B)] \times (C)$

3. *DISTRIBUTIVE*, if $(A) \times (B+C) = (A) \times (B) + (A) \times (C)$

When variables in systems have more than one quantitative attribute, they are often expressed by multi-dimensional quantities called **VECTORS**. For example,

Complex numbers, sinusoidal waveforms, force and velocity of bodies in motion.

Scalars with positive and negative values can be treated as vectors of single dimension, expressed by magnitude and an angle of zero radians for the positive numbers and an angle of π radians for the negative numbers with respect to the reference axis, along the REAL LINE. On the other hand, so called COMPLEX numbers are expressed as vectors in a 2-dimensional plane defined by their components along the HORIZONTAL (*REAL*) and the VERTICAL (*IMAGINERY*) axes.

When the values of a number of variables are needed to describe the condition (STATE) of a given system, these variables can be treated as components of a single variable, and expressed as a multi-dimensional vector variable. For Example, air gap flux, armature current and armature voltage for a DC motor can be treated as three components of a vector variable which describes the `state' of the motor at the specified instant of time.

All the elementary operations are applicable to the single and two-dimensional vectors. However, the same is not true for the three and higher dimensional vectors. Step (6) for search of a UNITY element for the set of three-dimensional vectors results in the conclusion that no such vector exists. No inverse of three-dimensional vectors exist and hence no division of two vectors is possible in the three-dimensional space. The vector notations and the elementary operations for the multi-dimensional vectors are defined to facilitate expression of relationships and physical phenomena observed experimentally. For Example, Power in AC circuits can be expressed as a DOT product of voltage and current because it is a scalar quantity. Similarly, electro-magnetic force can be expressed as a CROSS product of current and flux density because it is vector quantity with direction of motion perpendicular to the plane of the current and the flux vectors.

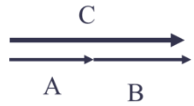
Scalars

We define scalars as 'single-dimensional' vectors along the real-line and treat them as a sub-set of the so called "Complex Numbers". That is, we define all positive scalars as vectors at zero radian angle with respect to the real-axis and with the length representing their value.

1. Addition: Add scalars A & B by placing the start of scalar B to the end of scalar A on the REAL line

$$A + B = C$$

The total length represents the resultant C.

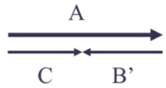


2. Zero: Scalar of length zero.

3. Complement (negative): Scalars along the real axis pointing in the opposite direction and of length equal to their respective positive scalars. That is, vectors along the real-line with an angle of π radians as opposed to the vectors with the radian angle of zero.

4. Subtraction: Subtract scalar B from scalar A by adding the complement of B to A.

$$A - B = A + (B') = C \text{ if } A > B \text{ and } C' \text{ if } A < B$$



5. Multiplication: Multiply scalars A and B by multiplying their magnitudes and by adding their directional angles of '0' (for positive scalars) and 'π' (for negative scalars).

$$(A) \times (B) = |A| \times |B| \angle (0 + 0) = AB \angle 0 = AB$$
$$(A) \times (-B) = (A) \times (B') = |A| \times |B| \angle (0 + \pi) = AB \angle \pi = - AB$$
$$(-A) \times (-B) = (A') \times (B') = |A| \times |B| \angle (\pi + \pi) = AB \angle 2\pi = AB$$

6. Unity: Scalar of one-unit length pointing in the direction of positive scalars. [$U = 1 \angle 0$]

$$(A) \times (U) = |A| .1. \angle(0 + 0) = A$$

7. Inverse: Inverse of scalar A is a scalar with the magnitude such that its product with the magnitude of A equals the unity scalar and its angle is same as the angle of A. NOTE: The Zero scalar has no inverse.

$$(A) \times (A^{-1}) = |A| . [1/|A|] \angle (0 + 0) = 1 \angle 0 = \text{'U'}$$
$$(-A) \times (-A^{-1}) = |A| . [1/|A|] \angle (\pi + \pi) = 1 \angle 2\pi = \text{'U'}$$

8. Division: Divide scalar A by scalar B, by multiplying the scalar A by inverse of scalar B.

$$A \div B = (A) \times (B^{-1}) = |A| . [1/|B|] \angle (0 + 0) = A/B$$

When a scalar is multiplied by itself, like (A) x (A), it is raised to powers by use of exponents. Certain basic rules apply when adding and multiplying such scalars. For example: Consider positive scalars (A) and (B) and positive or negative exponents (x) and (y).

$(B)^{-x} = 1/(B)^x$; $(B)^x. (B)^y = (B)^{(x+y)}$; $(A.B)^x = (A)^x.(B)^x$; $(B)^{x.y} = [(B)^x]^y$; $(B)^x+(B)^x= 2(B)^x$

Logarithms

Logarithm answers the simple question of how many times can a given number be multiplied to get another number. For scalars, *b, y* and *x*, if $b^y = x$ for positive *b* and *x*, then $\log_b x = y$, where *b* is defined as 'Base' of the logarithm. Two base values are commonly used.

b = 10 for common or ***Briggsian*** logarithms
e = 2.718 for ***Natural, Naperian or Hyperbolic*** logarithms [*logs*]

Rules in use of logarithms are:

$$\log ab = \log a + \log b$$
$$\log a/b = \log a - \log b$$
$$\log (a)^n = n \log a$$
$$\log \sqrt[n]{a} = \frac{1}{n} \log a$$

Antilogarithms or inverse logarithms are also defined so that transformed operations can be performed in the logarithmic domain and final solutions can be obtained in the original number domain.

Since most numbers are irrational powers of 10, a common logarithm, in general, consists of an integer called the *CHARACTERISTIC* and an endless decimal called the *MANTISSA*.

Example: log 256 = 2.**40824** → Mantissa (from the Log table)
↓
Characteristic (1 less than the number of digits of the number left of the decimal point)

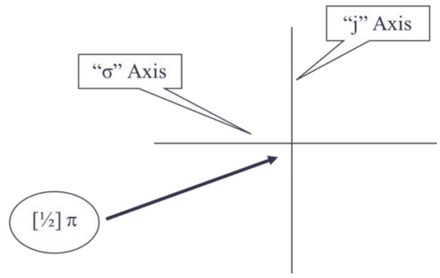
$\log 256{,}000{,}000 = \log (256 \times 10^6) = \log 256 + \log 10^6 = \log 256 + 6 \log 10 = 2.40824+6 = 8.40824$

$\log 0.00000256 = \log (2.56 \times 10^{-6}) = \log 2.56 + \log 10^{-6} = \log 2.56 - 6\log 10 = 0.40824-6 = -5.59176$

Since the hand calculators can instantly give us results for multiplication problems, the use of logarithms for simple multiplications is no longer needed. However, numerical data over a large range can be compressed for presentation on a "semi-log" or "log-log" graph papers. Also, the technique is very valuable in frequency domain analysis of control systems.

Complex Numbers

Complex numbers are not really complex, and $\sqrt{-1}$ does exist. It doesn't exist on the REAL line, but on the line perpendicular to it, referred to in most books as the ***j*** axis. These are two-dimensional numbers that can be represented in four different forms.



* Rectangular Coordinate Representation $Z = \alpha + j\beta$
* Polar Coordinate Representation $Z = |Z| \angle\theta$
* Trigonometric Representation $Z = |Z| (\cos\theta + j \sin\theta)$
* Exponential Representation $Z = |Z| e^{j\theta}$ where, $j = \sqrt{-1}$

1. Addition: Add real components for the real part of the sum and *j* components for the *j* part of the sum

$$Z_1 = a_1 + j b_1 \quad ; \quad Z_2 = a_2 + j b_2$$

$$Z_1 + Z_2 = (a_1 + a_2) + j(b_1 + b_2)$$

2. Zero: Both components α and β equal to zero; $|Z| = 0$

$$Z_1 + 0 = (a_1 + 0) + j(b_1 + 0) = Z_1$$

3. Complement (Negative) : Both components *a* and *b* in opposite direction. Add 180 degrees to the polar form and others.

$$Z_1 = a_1 + j b_1 \quad ; \quad Z_{1'} = -a_1 - j b_1$$

$$Z_1 + Z_{1'} = (a_1 - a_1) + j(b_1 - b_1) = 0$$

4. Subtraction: Subtract Z_2 from Z_1 by adding complement of Z_2 $[Z_{2'}]$ to Z_1.

$$Z_1 - Z_2 = Z_1 + Z_{2'} = (a_1 - a_2) + j(b_1 - b_2)$$

5. Multiplication: Multiply Z_1 and Z_2 by multiplying their magnitudes $|Z_1|$ and $|Z_2|$ and adding their

angles θ_1 and θ_2.

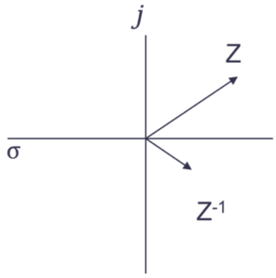
$(Z_1) \times (Z_2) = |Z_1| . |Z_2| ; \angle (\theta_1 + \theta_2)$

6. Unity: Complex number of unit length along the real axis. Also Z with a = 1 and b = 0.

$(Z_1) \times (U) = |Z_1| . 1 ; \angle(\theta_1+0) = Z_1$

7. Inverse: Inverse of a complex number Z is the complex number with magnitude [1/|Z|] and angle negative of the angle of Z .

$(Z_1) \times (Z_1)^{-1} = |Z_1| . [1/|Z_1|] = 1; \angle (\theta_1 + (- \theta_1) = 0;$ That is 'U'



8. Division: To divide complex number Z_1 by Z_2, multiply Z_1 by the inverse of Z_2.

$(Z_1) \div (Z_2) = (Z_1) \times (Z_2)^{-1} = |Z_1| . 1/|Z_2|; \angle (\theta_1 - \theta_2)$

Example 1. If $f(z) = z^2 + 6z + \frac{1}{z}$, find $f(3j)$

Solution:

$$f(3j) = (3j)^2 + 6(3j) + \frac{1}{3j} = 9j^2 + 18j + \frac{(j)1}{(3j)j} = -9 + 18j + \frac{j}{3j^2} = -9 + 18j - \frac{1}{3}j$$

Simplifying the above, we get $f(3j) = -9 + \frac{53}{3}j$

Example 2. Capacitive and inductive elements with impedances (in ohms) 7-12j and 3+26j, respectively, are connected in parallel. Find the circuit impedance in rectangular and polar forms.

Solution:

Let $z_1 = 7 - 12\,j$ and $z_2 = 3 + 26\,j$

The total impedance z for z_1 and z_2 in parallel, $Z = \frac{z_1 z_2}{z_1 + z_2} = \frac{(7-12j)(3+26j)}{(7-12j)+(3+26j)} = \frac{21+182j-36j+312}{10+14j}$

$$Z = \frac{333+146j}{10+14j} = \left[\frac{333+146j}{10+14j}\right]\frac{(10-14j)}{(10-14j)} = \frac{3330-4662j+1460J-2044\,j^2}{100+196} = 18.16 - 10.82j\ \Omega$$

Example 3. The total impedance in a circuit is given by $(Z = Z_1 + Z_2)$

Where, $z_1 = 4\angle 0^0$ and $z_2 = 8\angle 45^0$, find the total impedance in polar form.

Solution:

$$Z = Z_1 + Z_2 = 4\angle 0^0 + 8\angle 45^0 = 4(Cos\ 0 + j\ Sin\ 0) + 8(Cos\ 45 + j\ Sin\ 45)$$

$$Z = (1 + j\ 0) + 8\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = \left(4 + \frac{8}{\sqrt{2}}\right) + j\frac{1}{\sqrt{2}} = 9.657 + j\ 0.707$$

$$Z = \sqrt{(9.657^2 + 0.707^2)}\ \angle \tan^{-1}\left(\frac{0.707}{9.567}\right) = 9.683\ \angle 4.19\ \Omega$$

Multi-dimensional Space Vectors

Force and velocity measurements in mechanical systems require a vector representation in a three-dimensional space. They are represented with three components along the three space axes X, Y, and Z with symbols *i*, *j*, and *k*. For example,

$A = a_1\,i + a_2\,j + a_3\,k$; the linear length of the vector $|A| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

1. Addition: Add respective components along the three axes of the two elements to be added.

$A = a_1 i + a_2 j + a_3 k$; and $B = b_1 i + b_2 j + b_3 k$; then

$A + B = (a_1+b_1) i + (a_2+b_2) j + (a_3+b_3) k = C = c_1 i + c_2 j + c_3 k$

2. Zero: All three components are zero length. Z = 0

$A + Z = (a_1+0) i + (a_2+0) j + (a_3+ 0) k = A$

3. Complement (Negative) : All three components along axes X, Y and Z in opposite direction.

$A + A' = (a_1-a_1) i + (a_2-a_2) j + (a_3-a_3) k = Z = 0$

4. Subtraction: Subtract B from A by adding complement of B to A.

$A - B = A + B' = (a_1-b_1) i + (a_2-b_2) j + (a_3-b_3) k = C = c_1 i + c_2 j + c_3 k$

5. Multiplication: There two types of multiplication rules for three-dimensional vectors.

CROSS Product of A and B: The resultant is also a vector (C), which is perpendicular to the plane of vectors A and B.

$$A \text{ x } B = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

Some properties of this multiplication rule are:

1) A x B = - B x A

2) A x (β B + γ C) = β (A x B) + γ (A x C); where β and γ are scalars.

3) A x A = - (A x A) = 0

DOT Product of A and B: The resultant is a scalar

$A . B = |A| . |B| \, Cos\, \theta$; Where |A| and |B| are the lengths of the vectors and θ is the angle between them. Note that while the Cross Product of a vector A by itself is zero, the Dot product is $|A|^2$. These multiplication rules are actually mathematical representations of phenomena in physical systems. For example, interaction of current I and flux ϕ in an electromagnetic field produces a force F proportional to their product but perpendicular to the plane of the current and flux vectors. On the other hand, the work done (W) by an object moving at a velocity V under an applied force F is a scalar quantity proportional to the product of the velocity and component of the force along the direction of the velocity vector.

6. Unity: No unity vector exists in multi-dimensional vector space since no vector, using either Cross or Dot product, can produce the vector it multiplies with.

CAUTION: Vector [$i + j + k$] with unit lengths along the X, Y, and Z axes is NOT a Unity vector.

7. Inverse: Since no Unity vector exists, there can be no pair of vectors whose product would result in a unity vector.

8. Division: Since no inverse exists, division of two vectors is not defined.

Example 1: V_1: V_2 ; V_3 ; and V_4 are three dimensional vectors as follows:

$V_1 = 2 i + 2 j + 2 k$; $V_2 = 4 i + 3 j + k$; $V_3 = 4 i + 3 j$; $V_4 = 2 j + 3 k$

Compute: $(V_1 + V_2)$; $(V_1 – V_4)$; $(V_1 \text{ x } V_3)$; $(V_3 \text{ x } V_1)$; $(V_2 * V_4)$; $(V_4 * V_2)$;

$(V_1 \text{ x } V_2) \text{ x } V_4$; $V_1 \text{ x } (V_2 \text{ x } V_4)$

Solution:

$(V_1 + V_2) = (2+4)\, i + (2+3)\, j + (2+1)\, k = 6\, i + 5\, j + 3\, k$

$(V_1 - V_4) = (2-0)\, i + (2-2)\, j + (2-3)\, k = 2\, i - k$

$(V_1 \text{ x } V_3) = \begin{bmatrix} i & j & k \\ 2 & 2 & 2 \\ 4 & 3 & 0 \end{bmatrix} = -6\, i + 8\, j - 2\, k\, ;\ (V_3 \text{ x } V_1) = \begin{bmatrix} i & j & k \\ 4 & 3 & 0 \\ 2 & 2 & 2 \end{bmatrix} = 6\, i - 8\, j + 2\, k$

Note: $(V_1 \text{ x } V_3) = -(V_3 \text{ x } V_1)$

$(V_2 * V_4) = (4\text{x}0) + (3\text{x}2) + (1\text{x}3) = 9 = (V_4 * V_2)$

$(V_1 \text{ x } V_2) \text{ x } V_4$: First compute $(V_1 \text{ x } V_2)$ then multiply it with V_4

$(V_1 \text{ x } V_2) = \begin{bmatrix} i & j & k \\ 2 & 2 & 2 \\ 4 & 3 & 1 \end{bmatrix} = -4i+6j-2k;\ (V_1 \text{ x } V_2) \text{ x } V_4 = \begin{bmatrix} i & j & k \\ -4 & 6 & -2 \\ 0 & 2 & 3 \end{bmatrix} = 22i+12j-8k$

$V_1 \text{ x } (V_2 \text{ x } V_4)$: First compute $(V_2 \text{ x } V_4)$ then pre-multiply with V_1

$(V_2 \text{ x } V_4) = \begin{bmatrix} i & j & k \\ 4 & 3 & 1 \\ 0 & 2 & 3 \end{bmatrix} = 7i-12j+8k;\ V_1 \text{ x } (V_2 \text{ x } V_4) = \begin{bmatrix} i & j & k \\ 2 & 2 & 2 \\ 7 & -12 & 8 \end{bmatrix} = 40i-2j-38k$

Note: $(V_1 \text{ x } V_2) \text{ x } V_4 \neq V_1 \text{ x } (V_2 \text{ x } V_4)$

Example 2: For a three-dimensional space with *i* , *j* , and *k* as unit vectors along the X, Y and Z axes respectively,

a. Determine the value of α such that $A = -2i + \alpha j - 2k$ and $B = 3i + 4j + 19k$ are perpendicular

b. Prove that $|A\text{x}B|^2 + |A.B|^2 = |A|^2\, |B|^2$

Solution:

a. The dot product of *A* and *B*; $A*B = |A|\, |B| \text{ Cos } \varphi$.

For A and B to be perpendicular, Cos (90) = 0

The dot product of $A = -2i + \alpha j - 2k$ and $B = 3i + 4j + 19k = (-2\text{x}3) + 4\alpha - 19\text{x}2 = 0$; $\alpha = 11$

b. $|A*B| = |A||B| \text{ Cos } \varphi$ and $|A\text{x}B| = |A||B| \text{ Sin } \varphi$

$|A\text{x}B|^2 + |A.B|^2 = |A|^2|B|^2 \text{ Cos}^2 \varphi + |A|^2|B|^2 \text{ Sin}^2 \varphi = |A|^2|B|^2 (\text{Cos}^2 \varphi + \text{Sin}^2 \varphi) = |A|^2|B|^2$

Example 3: The Hay Bridge is an AC circuit used in electrical measurements. It is governed by the following equation. Express R_x and L_x in terms of the other circuit constants.

$$\left(R_1 - j\frac{1}{\omega\, C_1}\right)(R_x + j\omega\, L_x) = R_2 R_3$$

Solution:

$$\left(R_1 - j\frac{1}{\omega\, C_1}\right)(R_x + j\omega\, L_x) = R_2 R_3 \qquad (1)$$

$$R_1\, R_x + \frac{L_x}{C_1} + j\left(\omega L_x R_1 - \frac{R_x}{\omega C_1}\right) = R_2 R_3 \qquad (2)$$

Equating the real and imaginary parts:

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \qquad (3)$$

And, $$\omega L_x R_1 - \frac{R_x}{\omega C_1} = 0;\ L_x = \frac{R_x}{\omega^2 C_1 R_1} \qquad (4)$$

Substituting the value of L_x in from (4) into (3)

$$R_1 R_x + \frac{R_x}{\omega^2 C_1^2 R_1} = R_2 R_3; \;\; R_x \left(R_1 + \frac{1}{\omega^2 C_1^2 R_1}\right) = R_2 R_3$$

$$R_x = \frac{(\omega C_1)^2 R_1 R_2 R_3}{(\omega C_1 R_1)^2 + 1}; \text{ and } \; L_x = \frac{C_1 R_2 R_3}{(\omega C_1 R_1)^2 + 1}$$

Computer Arithmetic

Binary number system is a 'Base 2' system consisting of only two base integers of the scalar system; the **Zero** and the **Unity**. Numbers larger than 2 are represented by digits to the left in a manner similar to the decimal number system. Though numbers up to the infinity can be represented in the binary system, the number of digits (called **BITS** in the computer language) available in a given computing system limit the maximum decimal number that can be handled.

Elementary operations in a four-bit system:

There are 16 elements in this binary system ranging from 0000 to 1111. The “Hexadecimal” system is “Base 16” system, represented by a single digit. After the numbers 0- 9, A, B, C, D, E and F are used to represent numbers 10, 11, 12, 13, 14, and 15, as shown in the circle diagram below.

1. Addition:

Rule 1: 1 + 1 = 0 with one (1) carry to the left bit.

Rule 2: 1 + 0 = 0 + 1 = 1

Rule 3: 0 + 0 = 0

Example: 1110
+ 1010
1 1000

Any carry beyond the bit size of the system is ignored by the computer.

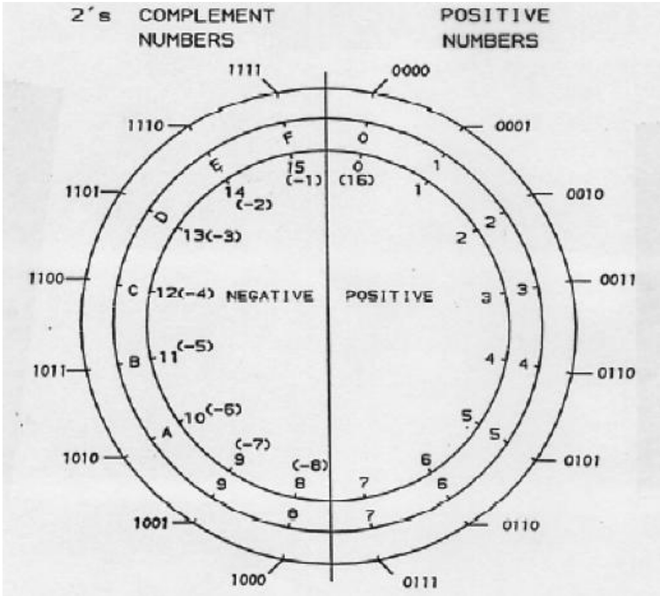
2. Zero: All four bits are zero. That is Zero = 0000 since it will not alter the value of any of the sixteen numbers, if added to it by the foregoing rules.

3. Complement (Negative): Since there are only sixteen numbers in this four-bit system, they can be divided up into two groups of eight numbers each requiring only three bits for their representation. Refer to the circle diagram. *Note: Just like the general scalar system, the zero is treated as a positive number.* The complement of number 0101 (decimal 5) is 1011 (decimal 11) because the sum of these two numbers is a five-bit binary number 10000 (decimal 16) which wrap arounds to 0000.

4. Subtraction: Take the two's complement of the subtrahend and add it to the minuend.

Decimal: 7 - 3 = 4

Binary: 0111 - 0011 = 0111 + 1101 = [1]0100



5. Multiplication: Repetitive addition can be used to perform the multiplication. However, the number of bits available in a system restrict the operands to those for which the product remains within the available number of bits unless two sets of bits are used for storing the resultant.

The multiplicand is repetitively added (multiplier-1) times to itself. Or, multiplier is repetitively added (multiplicand - 1) times to itself, whichever yields lower number of operations.

Decimal: 3 x 2 = 2 x 3 = 6

Binary: (0011) x (0010) = (0010)x(0011) = 0011 + 0011 = 0110
= 0010 + 0010 + 0010 = 0110

6. Unity: The unity (0001) exists since any number added zero (unity - 1) times to itself would not alter the original multiplicand.

Decimal: 5 x 1 = 1 x 5 = 5

Binary: (0101) x (0001) = (0001) x (0101) = 0101

7. Inverse: Since binary numbers are integers, their inverses would have to be less than one so that the product of number and its inverse would yield a resultant equal to the unity. The inverses, therefore, do not exist.

8. Division: Since inverses do not exist, division of two binary numbers is carried out by repetitive subtraction. The divisor is subtracted from the dividend until there is no remainder. It can also be carried out in the same way as division of decimal numbers. The four general rules are:

$0 \div 0$ and $1 \div 0$ have no meaning; $0 \div 1 = 0$ and $1 \div 1 = 1$; Decimal: $\frac{4}{2} = 2$

Binary: (0100) ÷ (0010) = 0010

Matrix Algebra

Matrix is an array of elements (quantities or operators) arranged in rows and columns.

Examples:

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -2 \\ 3 & 5 & 1 \end{bmatrix} \quad ; \quad \begin{bmatrix} s & \frac{s}{1+s} \\ 2s^2 & \frac{2}{s} \end{bmatrix} \quad ; \quad \begin{bmatrix} (3 + j6) & (5 + j2) \end{bmatrix}$$

The number of rows and columns determine the dimensions of a given matrix. That is, a matrix A of

$$A = [a_{ij}]_{mn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdot & \cdot & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{m}1 & a_{m}2 & a_{m}3 & \cdot & \cdot & a_{mn} \end{bmatrix}$$

dimension (***m x n***) is said to have ***m*** number of rows and ***n*** number of columns.

Here, a_{ij} represents the element in the *ith.* row and the *jth.* column.

Elementary mathematical operations

1. Addition**:**

$A = [a_{ij}]_{mn}$; $B = [b_{ij}]_{pq}$ then $A + B = C$ is defined only if $m = p \wedge n = q$

$C = [c_{ij}]_{mn}$; where $c_{ij} = a_{ij} + b_{ij}$

Addition is commutative and associative

$A + B = B + A \quad \wedge \quad A + (B + C) = (A + B) + C$

2. Zero or Null matrix**:**

$O = [\alpha_{ij}]_{mn}$ where each element $\alpha_{ij} = 0$

$A + O = A$ because each element of the sum is $a_{ij} + 0 = a_{ij}$

3. Negative (Complement)

$A' = [-A]_{mn}$ that is, each element of A' is negative of the corresponding element of A

4. Subtraction:

$A - B = A + B' = C$; that is, element $c_{ij} = a_{ij} + (b_{ij})' = a_{ij} - b_{ij}$

5. Multiplication:

If $A = [a_{ij}]_{mn}$ and $B = [b_{ij}]_{pq}$ then,

(A) (B) = C_{mq} is defined only for matrices with $(n = p)$ and

(B) (A) = D_{pn} is defined only for matrices with $(q = m)$

where,

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

6. Unity (Identity) :

[U] or [I] of order (*mxn*) exists only if $m = n$.

[U] = [I] = {u_{ij}} where, $u_{ij} = 1$ for $i = j$ and $u_{ij} = 0$ *for* $i \neq j$

That is, (*A*) (*U*) = (*U*) (*A*) = A

7. Inverse (Reciprocal):

A^{-1} for A exists only if $m = n$ and, det [A] ≠ 0. Then [A] $[A]^{-1} = [A]^{-1}$[A] = [U]

Calculation of the inverse of matrix A requires following steps.

i. Calculate the determinant of [A] to make sure det [A] ≠ 0

ii. Replace each element of A by its *Cofactor* to obtain the *Adjugate of* A

iii. Transpose the resulting matrix to obtain *Adjoint of* A [*adj* (A)]

$$A^{-1} = \frac{1}{\det A}\,(adj\ A)$$

iv. Divide each element of *adj* (A) by the det A

For given *i, j*, the *Cofactor* of the element a_{ij} of A is obtained by calculating the determinant of [A] after striking out the i^{th}. row and j^{th}. column, multiplied by $(-1)^{i+j}$.

Another method of finding inverse of a matrix is the Gauss-Jordan reduction technique. An 'Identity' matrix [I] is appended to the matrix to be inverted and then linear operations are performed on the rows of the appended matrix until the two interchange their positions within the appended matrix. See example below.

Example: Find inverse of

$$A = \begin{bmatrix} 3 & -2 & -1 \\ -1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[\,A\_I] = \left[\begin{array}{ccc|ccc} 3 & -2 & -1 & 1 & 0 & 0 \\ -1 & 3 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Subtract 2 times the last row from the first and add third row to the second.

$$\begin{bmatrix} 1 & -4 & -3 & | & 1 & 0 & -2 \\ 0 & 4 & 3 & | & 0 & 1 & 1 \\ 1 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Add second row to the first.

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & 1 & -1 \\ 0 & 4 & 3 & | & 0 & 1 & 1 \\ 1 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

Subtract first row from the last

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & 1 & -1 \\ 0 & 4 & 3 & | & 0 & 1 & 1 \\ 0 & 1 & 1 & | & -1 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & 1 & -1 \\ 0 & 1 & 0 & | & 3 & 4 & -5 \\ 0 & 1 & 1 & | & -1 & -1 & 2 \end{bmatrix}$$

Subtract 3 times the last row from the second Subtract 3 times the last row from the second.
Subtract the second row from the last.

The inverse of A is:

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -5 \\ -4 & -5 & 7 \end{bmatrix}$$

As a check, we compute $[A]\,[A]^{-1}$ to confirm that it equals the identity matrix I .

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & 1 & -1 \\ 0 & 1 & 0 & | & 3 & 4 & -5 \\ 0 & 0 & 1 & | & -4 & -5 & 7 \end{bmatrix} = [\ I \mid A^{-1}\ ]$$

8. Division:

Division of matrix A by matrix B is obtained by either pre-multiplying or post-multiplying the matrix A by the inverse of matrix B if it exists. Following conditions for B be must be met.

i. B must be a square matrix with non-zero determinant.

ii. Dimensions of B must equal the number of columns in A for $A.\,B^{-1}$ product.

and dimensions of B must equal the number of rows in A for $B^{-1}.A$ product.

Terms and definitions associated with matrices

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 5 & 11 & 1 \\ 2 & 9 & -3 \end{bmatrix}$$

1. Singular matrix: A matrix whose determinant does not exist or is zero.

det A = 3(-33-9) - 2(-15-2) + 4(45-22) = 0

2. Non-singular matrix: A matrix whose determinant is non-zero.

3. Rank of a matrix: The dimension of the largest non-singular submatrix of A_{mn}

$$Rank \quad of \quad A = \left[\begin{array}{cc|c} 3 & 2 & 4 \\ 5 & 11 & 1 \\ \hline 2 & 9 & 3 \end{array}\right] = 2$$

$$Rank \quad of \quad B = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 9 & -3 \\ 5 & 0 & -10 \end{bmatrix} = 3$$

because the *det* [B] ≠ 0

4. Diagonal matrix: A matrix with all its off-diagonal elements equal to zero.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (-2+3j) & 0 \\ 0 & 0 & (-2-3j) \end{bmatrix}$$

$[A]_{mn} = (a_{ij})_{mn}$ is diagonal if $a_{ij} = 0$ for all $i \neq j$

5. Diagonalizable matrix: A matrix that can be represented by a non-singular matrix P and a diagonalizable matrix Λ such that $A P = P \Lambda$; that is $A = P \Lambda P^{-1}$

Example:

$$A = \begin{bmatrix} -3 & -2 & 4 \\ 7 & 7 & 13 \\ 5 & 6 & -10 \end{bmatrix} ; \quad P = \begin{bmatrix} 3 & -2 & -1 \\ -1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix} ; \quad \Lambda = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

6. Similar matrices: Matrices **A** and **B** are said to be similar if there is a non-singular matrix **P** such that $\mathbf{A} = \mathbf{P}^{-1} \mathbf{B} \mathbf{P}$.

$$P^* = \begin{bmatrix} 3 & -1 & 1 \\ -2 & 3 & 1 \\ -1 & 2 & 1 \end{bmatrix} \quad \text{is transp ose of} \quad P = \begin{bmatrix} 3 & -2 & -1 \\ -1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

7. Transposed matrix: A matrix obtained by interchanging rows and columns of another matrix

8. Symmetric matrix: A square matrix which is equal to its transposed matrix. That is, $A^* = A$. For example,

$$A = \begin{bmatrix} (s+1) & 1 & s \\ 1 & (s+2) & 2 \\ s & 2 & (s+3) \end{bmatrix}$$

9. Characteristic polynomial of a matrix: The polynomial in λ obtained from [det $(A-\lambda I)$] for a given

$$A = \begin{bmatrix} -3 & 1 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} ; \quad [A - \lambda I] = \begin{bmatrix} -(3+\lambda) & 1 & 0 \\ -2 & -\lambda & 1 \\ 0 & 0 & -\lambda \end{bmatrix}$$

square matrix A. For example,

Characteristic polynomial is $\det[A - \lambda I] = -(3+\lambda)\lambda^2 - 2\lambda = -[\lambda^3 + 3\lambda^2 + 2\lambda]$

10. Eigenvalues of a matrix: The roots of the characteristic equation [det $(A-\lambda I) = 0$] are called the eigenvalues of matrix A. For example,

$\lambda^3 + 3\lambda^2 + 2\lambda = 0 \;;\; \lambda(\lambda+1)(\lambda+2) = 0$

$\lambda_1 = 0 \;;\; \lambda_2 = -1 \;;\; \lambda_3 = -2$ are eigenvalues of the matrix A .

11. Eigenvectors of a matrix: The eigenvectors ξ_i of an n-dimensional square matrix A are vectors such that $[\lambda_i \mathrm{I} - \mathrm{A}]\xi_i = 0$; i = 1,2,.... where, λ_i are the eigenvalues of A .

For example, for $\lambda_1 = 0$, and the A matrix of (9) and (10) above,

$$\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_{11} \\ \xi_{12} \\ \xi_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \;; \quad \xi_1 = \begin{bmatrix} \xi_{11} \\ \xi_{12} \\ \xi_{13} \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$[\lambda_1 \mathrm{I} - \mathrm{A}]\,\xi_1 = -\mathrm{A}\,\xi_1 = 0$$

ξ_1 is the eigenvector for $\lambda_1 = 0$. **NOTE:** Since the last equation results in 0 = 0, the first two are solved for ξ_{12} and ξ_{13} by setting $\xi_{11} = \alpha_1$.

12. Adjoint of a square matrix [A]: A matrix obtained by taking the transpose of matrix of cofactors of the original matrix A .

$$\mathrm{A} = \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{Cofactors of A} = \begin{bmatrix} -2 & -6 & 3 \\ 3 & 2 & -1 \\ -1 & -3 & 5 \end{bmatrix} \quad \mathrm{Adj(A)} = \begin{bmatrix} -2 & 3 & -1 \\ -6 & 2 & -3 \\ 3 & -1 & 5 \end{bmatrix}$$

Example:

13. Unitary matrix: If the product of a matrix and its adjoint is equal to a Unity (Identity) matrix, then the matrix is called a unitary matrix. That is, if the determinant of a matrix is equal to 1 then it is unitary.

14. Bordering of a matrix: It is the process of building a matrix of dimension *n+1* from an *nxn* matrix A by adding a vector U, transpose of a vector V and a scalar α.

$$\tilde{\mathrm{A}}_{n+1} = \left[\begin{array}{c|c} \mathrm{A}_n & \mathrm{U}_n \\ \hline \mathrm{V}_n^{\mathrm{T}} & \alpha \end{array}\right]$$

Example: For the matrices A and B find A^2, BA^{-1}, and (A - B).

$$A = \begin{bmatrix} -3 & 2 & 1 \\ 2 & -3 & -2 \\ -1 & -2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -1 & -2 \\ 2 & 3 & -3 \\ -3 & -4 & 2 \end{bmatrix}$$

Solution:

$$A^2 = \begin{bmatrix} -3 & 2 & 1 \\ 2 & -3 & -2 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 & 1 \\ 2 & -3 & -2 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -14 & -7 \\ -10 & 17 & 8 \\ -1 & 4 & 3 \end{bmatrix}$$

To calculate $[A]^{-1}$: *det A = 9*;

$$Adjugate\ of\ A = \begin{bmatrix} -4 & 2 & -7 \\ -2 & 1 & -8 \\ -1 & -4 & 5 \end{bmatrix}; Adjoint\ of\ A = \begin{bmatrix} -4 & -2 & -1 \\ 2 & 1 & -4 \\ -7 & -8 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} Adjoint\ (A) = \begin{bmatrix} -0.444 & -0.222 & -0.111 \\ 0.222 & 0.111 & -0.444 \\ -0.777 & -0.888 & 0.555 \end{bmatrix}$$

$$B^{*}A^{-1} = \begin{bmatrix} -1 & -1 & -2 \\ 2 & 3 & -3 \\ -3 & -4 & 2 \end{bmatrix}\begin{bmatrix} -0.444 & -0.222 & -0.111 \\ 0.222 & 0.111 & -0.444 \\ -0.777 & -0.888 & 0.555 \end{bmatrix} = \begin{bmatrix} 1.778 & 1.889 & -0.556 \\ 2.111 & 2.556 & -3.222 \\ -1.111 & -1.556 & -3.222 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -2 & 3 & 3 \\ 0 & -6 & 1 \\ 2 & 2 & -2 \end{bmatrix}$$

Determinants

While matrices are an array of numbers arranged in rows and columns, a determinant is a number. It is a convenient way to express the number [ad-bc] in the form $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$. Number of rows and columns are always equal in the array. It is denoted by the symbol Δ or letter D. This 2x2 array is called a 'Second Order' determinant. A 'Third Order' determinant would be an array of 3x3. [*a*,*b*,*c*,*d*,] are called elements, [*ad*-*bc*] is the expansion or the value of the determinant, [a and b] make the first column while [a and c] make the first row. [a] is the leading element, [*a*, *d*] lie on the 'Leading' or 'Principal' diagonal. Δ' is transpose of the determinant with rows and columns switched. Some properties of determinants are:

1) Δ and Δ' have identical expansions.

2) A common factor of the elements of a row (column) is a factor of Δ, for example;

$\begin{vmatrix} abx & az \\ by & t \end{vmatrix} = abxt - abzy = ab(xt - zy)$, so you can remove common factors before expanding

3) Interchange of two rows (or columns) changes Δ in to -Δ

$\begin{vmatrix} a & c \\ b & d \end{vmatrix} = (ad - bc)$ but $\begin{vmatrix} c & a \\ d & b \end{vmatrix} = (bc - ad)$

4) If two rows or columns are identical then Δ = 0. $\begin{vmatrix} a & b \\ a & b \end{vmatrix} = (ab - ab) = 0.$

5) If two rows or columns are proportional then Δ = 0

6) The product of any row (column) and any arbitrary constant may be added to any other row (column) without changing the value of the determinant.

$\Delta = \begin{vmatrix} a & c \\ b & d \end{vmatrix} = (ad - bc)$and, $\Delta = \begin{vmatrix} (a+kc) & c \\ (b+kd) & d \end{vmatrix} = [(a+kc)d - (b+kd)c] = ad - bc$

7) The value of a triangular determinant is equal to the product of all elements on its principal diagonal. Triangular determinant has all elements on one side of the diagonal equal to zero.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11}.a_{22}.a_{33}$$

Evaluation of Determinants

1. a_{jk} is an element in the j^{th} row and k^{th} column.

2. a_{jk} Minor written M_{jk} is a determinant with j^{th} row and k^{th} column eliminated.

3. a_{jk} Cofactor = $[(-1)^{j+k} . M_{jk}]$ is also written as Δ_{jk}

4. A determinant of any order can be expanded about any row or column by summing the products of the elements of rows(columns) and their respective Cofactor.

5. An alternate method is to reduce the given determinant in to a triangular determinant and then multiply the elements on the principal diagonal for the answer.

Probability & Statistics

Engineering and social sciences problems involving random processes, or variables, are defined as **stochastic** (non-deterministic). Instead of deterministic values, the measurements are expressed by the "Mean" of the distribution ($\bar{X}$) and its variance σ^2. Smaller the variance, closer the random values of the signal/variable to the 'Mean', and closer it is to a deterministic system behavior. Mathematical operations of deterministic variable can. be applied using 'Mean' values of the variables involved.

Probability of an event taking place under specified conditions is expressed in per-unit ranging from 0 (zero) for occurrence of an impossible outcome to 1 (one) for occurrence of an outcome with certainty (full assurance).

There are 6 general rules when we are dealing with probabilities of single and multiple events.

Rule 1.

Probability of an impossible outcome of an event ......P{0} = 0

Rule 2.

If A_i [i = 1, 2, ... n] are n possible outcomes of a process, then

$P\{A_1$ or A_2 or ....$A_n\} = P\{A_1\} + P\{A_2\} + ...+P\{A_n\}$

Rule 3.

If A_i, B_i, ... Z_i are independent processes with multiple outcomes having probabilities of $P\{A_i\}$, $P\{B_i\}$, ... $P\{Z_i\}$, then $P\{A_i$ and B_i and ..... $Z_i\} = P\{A_i\}\ p\{B_i\}..........P\{Z_i\}$

Rule 4

Probability of an outcome not occurring is complement of the probability of its occurrence.

$P\{\text{not } A_1\} = 1- P\{A_1\}$

Rule 5

$P\{A_i \text{ or } B_i\} = P\{A_i\} + P\{B_i\} - P\{A_i\}\ P\{B_i\}$

Rule 6

Probability that A will occur given that B has already occurred, where the two event are dependent.

$P\{A\ given\ B\} = \frac{P\{A\ and\ B\}}{P\{B\}}$

Probability density functions are mathematical functions describing probabilities of numerical events. Numerical events are outcomes which have numerical values expressed by real numbers. There are four such types of density functions.

1. Binomial

n = the number of trials

x = the number of successes desired

P = the probability of a success in a single trial

q = (1-P) the probability of failure

The binomial coefficient is given by

$$\frac{n!}{(n-x)!}\ P^x\ q^{(n-x)}$$

The probability of obtaining x successes in n trials is given by

$$P(x) = \begin{bmatrix} n \\ x \end{bmatrix} P^x q^{(n-x)}$$

The mean of the binomial distribution is nP and the variance of the distribution is nPq

2. Poisson

If an event occurs, on the average, λ times per period, the probability that it will occur x times per period is given by $f\ [\ x\ ] = \frac{e^{-\lambda}\ \lambda^{x}}{x!}$

3. *Exponential*

If the mean of the exponential distribution is 1/u, then

$$f[x] = u(e^{-ux})$$

The variance is $(\ 1/\ u\ )^2$ and the probability F[x] of x or less occurring is $(1 - e^{-ux})$.

4. Normal (Gaussian)

Although f[x] may be expressed mathematically for the normal distribution, tables are used to evaluate F[x] since f[x] cannot be easily integrated. Since the x-axis of the normal distribution will seldom correspond to actual sample variables, the sample values need to be normalized for use of the standard tables. Given the mean (u), and the standard deviation (σ), the normalized variable is

$$z = \frac{(sample\ value - u)}{\sigma}$$

Then, the probability of a sample exceeding the given sample value is equal to the area in the tail past point z.

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