IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 17, Issue 2 Ser. III (Mar. – Apr. 2021), PP 01-04
www.iosrjournals.org

Slightly $\tilde{g}\alpha$-Continuous Functions

A.Selvakumar and K.Arunkumar
Rathinam College of Arts and Science, Coimbatore, India

Abstract
The notion of g\tilde\alpha-closed sets in a topological spaces are introduced by R.Devi et. al. [2]. In this paper, we introduced the concept of slightly g\tilde\alpha -continuous functions and study the basic properties and preservation theorems of this function.

Keywords: *clopen set, gα-continuous map, slightly gα-continuous map..*
AMS(2000) Subject classification *: 54A05, 54D05 54D10, 54D45.*

Date of Submission: 20-03-2021 Date of Acceptance: 04-04-2021

I. Introduction

The notion of $\tilde{g}\alpha$-closed sets of a topological space are discussed by R. Devi et. al. [2]. The concept of slightly continuous functions are introduced and investigated by R.C. Jain [4]. The aim of this paper is to introduce the notion of slightly $\tilde{g}\alpha$-continuous functions. Further, the basic properties of slightly $\tilde{g}\alpha$- continuous functions are derived.

Throughout the present paper, X and Y are always topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by $int(A)$ and $cl(A)$ respectively. A subset A of a space X is said to be α-open [5] if $A \subseteq int(cl(int(A)))$. A subset A *of* a space X *is* said to be $\tilde{g}\alpha$-closed [2] if $\alpha cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is $^{\#}gs$-open. The complement of a $g\alpha$-closed set is said to be $g\alpha$-open. The intersection of all $g\alpha$-closed sets of X containing A is called $g\alpha$-closure of A and is denoted by $g\alpha cl(A)$. The union of all $g\alpha$-open sets of X contained in A is called $\tilde{g}\alpha$-interior of A and is denoted by $\tilde{g}\alpha int(A)$. The family of all α-open (resp. $\tilde{g}\alpha$-open, $\tilde{g}\alpha$-closed, clopen, $\tilde{g}\alpha$-clopen) set of X is denoted by $\alpha O(X)$ (resp. $\tilde{g}\alpha O(X)$, $\tilde{g}\alpha C(X)$, $CO(X)$, $\tilde{g}\alpha CO(X)$).

Definition1.1. [2] A function $f: X \to Y$ is $g\alpha$-continuous if $f^{-1}(V)$ is $g\alpha$-open set in X for each open set V of Y.

Definition 1.2. [4] A function $f : X \to Y$ is slightly- continuous if $f^{-1}(V)$ is open set in X for each clopen set V of Y.

II. Slightly $\tilde{g}\alpha$-Continuous Functions

Definition 2.1. A function $f : X \to Y$ is said to be slightly $g\alpha$-continuous if for each $x \in X$ and for each $v \in CO(Y, f(x))$, there exists $U \in G\alpha O(X, x)$ such that $f(U) \subseteq V$.

Definition 2.2. Let $(D, \leq)$ be a directed set. A net $\{x_\lambda : \lambda \in D\}$ in X is said to be $\tilde{g}\alpha$-convergent to a point $x \in X$ if $\{x_\lambda\}_{\lambda \in D}$ is eventually in each $V \in \tilde{G}\alpha O(X, x)$.

Theorem 2.3. For a function $f : X \to Y$, the following are equivalent:

(a) f is slightly $g\alpha$-continuous.

(b) $f^{-1}(v) \in g\alpha O(X)$ for each $V \in CO(Y)$.

(c) $f^{-1}(v)$ is $g\alpha$-clopen for each $V \in CO(Y)$.

(d) for each $x \in X$ and for each set $x_\lambda\}_{\lambda \in D}$ in X.

Proof. $(a) \Rightarrow (b)$. Let $V \in CO(Y)$ and let $x \in f^{-1}$. Then $f(X) \in V$. Since f is slightly $g\alpha$-continuous, there is a $U \in g\alpha O(X, x)$ such that $f(U) \subseteq V$. Thus $f^{-1}(U) = \cup_X\{U : x \in f^{-1}(V)\}$, that is $f^{-1}(U)$ is a union of $\tilde{g}\alpha$-open sets. Hence

$f^{-1}(U) \in \tilde{g}\alpha O(X)$.

(b) $\Rightarrow (c)$. Let $V \in CO(Y)$. Then $(Y - V) \in CO(X)$. By hypothesis $f^{-1}(Y - V) = X - f^{-1}(V) \in \tilde{G}\alpha O(X)$. Thus $f^{-1}(V)$ is $\tilde{g}\alpha$-closed.

(c) $\Rightarrow (d)$. Let $\{x_\lambda\}_{\lambda \in D}$ be a set in X $\tilde{g}\alpha$- converging to x and let $V \in CO(Y, f(x))$. There is thus a $U \in \tilde{g}\alpha O(X, x)$ such that $f(U) \subseteq V$. There is

thus a $\lambda_0 \in D$ such that $\lambda_0 \leq \lambda$ implies $x_\lambda \in U$ since $\{x_\lambda\}_{\lambda \in D}$ is $g\alpha$- convergent to x. Thus $f(x_\lambda) \in f(U) \subseteq V$ for all λ. Thus $\{f(x_\lambda)\}_{\lambda \in D}$ is $g\alpha$- convergent to $f(x)$.

(d) $\Rightarrow (a)$. Suppose that f is not slightly $g\alpha$-continuous at a point $x \in X$, then there exists a $V \in CO(Y, f(x))$ such that $f(U)$ does not contained in V for each

~ $U \in g\alpha O(X, x)$. So $f(U) \cap (Y - V) f= \varphi$ and thus $U \cap f^{-1}(Y - V) f= \varphi$ for

each $U \in \tilde{g}\alpha O(X, x)$, since $g\alpha\tilde{O}(X, x)$ is directed by set inclusion C, there exists a selection function x_U from $\tilde{g}\alpha O(X, x)$ into X for each $U \in \tilde{g}\alpha O(X, x)$. Thus $\{x_U\}_U \in \tilde{g}\alpha O(X, x)$ is a net in X $g\tilde{\alpha}$-converging to x. Since $X_U \in U \cap f^{-1}(Y - V) = U - f^{-1}(V)$ and so $f(x_U) \notin V$, for each U, $\{f(x_U)\}_U \in \tilde{g}\alpha O(X, x)$ is not eventually in $V \in CO(Y, f(x))$, which is a contradiction. Hence (a) holds.

Theorem 2.4. If $f : X \longrightarrow Y$ is slightly $\tilde{g}\alpha$-continuous and $g : Y \longrightarrow Z$ is slightly continuous, then their composition $g \circ f$ is slightly $g\alpha$-continuous.

Proof. Let $V \in CO(Z)$, then $g^{-1}(V) \in CO(Y)$ [6]. Since f is slightly $\tilde{g}\alpha$-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \in g\alpha O(X)$. Thus $g \circ f$ is slightly $g\alpha$-continuous.

Theorem 2.5. The following are equivalent for a function $f : X \to Y$

(a) f is slightly $g\alpha$-continuous,

(b) for each $x \in X$ and for each $V \in CO(Y, f(x))$, there exists $g\alpha$-clopen set U

such that $f(U) \subseteq U$,

(c) for each closed set F of Y, $f^{-1}(F)$ is $g\alpha$-closed,

(d) $f(cl(A)) \subseteq \tilde{f}\alpha cl(f(A))$ for each $A \subseteq X$ and

(e) $cl(f^{-1}(B)) \subseteq f^{-1}(\tilde{g}\alpha cl(B))$ for each $B \subseteq Y$.

Proof. (a) $\Rightarrow$ (b) Let $x \in X$ and $V \in CO(Y, f(x))$ by Theorem 2.3. $f^{-1}(V)$ is clopen. Put $U = f^{-1}(V)$, then $x \in U$ and $f(U) \subseteq V$.

(b) $\Rightarrow$ (c) It is obvious.

(c) $\Rightarrow$ (d) Since $g\alpha cl(f(A))$ is the smallest $g\alpha$-closed set containing $f(A)$, hence by (c), we have (d).

(d) $\Rightarrow$ (e) For each $B \subseteq Y$, $f(cl(f^{-1}(B))) \subseteq g\alpha cl(f(f^{-1}(B))) \subseteq g\alpha cl(B)$. Hence $f(cl(f^{-1}(B))) \subseteq g\alpha cl(B) \Rightarrow cl(f^{-1}(B)) \subseteq f^{-1}(g\alpha cl(B))$.

(e) $\Rightarrow$ (a) Let $V \in CO(Y)$. Then $(Y - V) \in CO(X)$, by (e), we have $cl(f^{-1}(Y - V)) \subseteq f^{-1}(g\alpha cl(Y - V)) = f^{-1}(Y - V)$, since every closed set is $g\alpha$-closed, thus $f^{-1}(Y - V) = X - f^{-1}(V)$ is closed and thus $g\alpha$-closed, thus $f^{-1}(V) \in g\alpha O(X)$ and f is slightly $\tilde{g}\alpha$-continuous.

Theorem 2.6. If $f : X \to Y$ is a slightly $\tilde{g}\alpha$-continuous injection and Y is clopen T_1, then X is $\tilde{g}\alpha$-T_1.

Proof. Suppose that Y is clopen T_1. For any distinct points x and y in X, there exist V, $W \in CO(Y)$ such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is slightly $g\alpha$-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $g\alpha$-open subsets of X

such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that X is $\tilde{g}\alpha$-T_1.

Theorem 2.7. If $f : X \to Y$ is a slightly $\tilde{g}\alpha$-continuous surjection and Y is clopen T_2, then X is $\tilde{g}\alpha$-T_2.

Proof. For any pair of distinct points x and y in X, there exist disjoint clopen sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is slightly $g\alpha$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $g\alpha$-open in X containing x and y respectively. Therefore $f^{-1}(U) \cap f^{-1}(V) = \varphi$ because $U \cap V = \varphi$. This shows that X is $\tilde{g}\alpha$ -T_2.

Definition 2.8. A space is called $\tilde{g}\alpha$-regular if for each $\tilde{g}\alpha$-closed set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subseteq U$ and $x \in V$.

Definition 2.9. A space is said to be $\tilde{g}\alpha$-normal if for every pair of disjoint $g\alpha$-closed subsets F_1 and F_2 of X, there exist disjoint open sets U and V such that $F_1 \subseteq U$ and $F_2 \subseteq V$.

Theorem 2.10. If f is slightly $\tilde{g}\alpha$-continuous injective open function from a $g\alpha$-regular space X onto a space Y, then Y is clopen regular.

Proof. Let F be clopen set in Y and be $y \notin F$. Take $y = f(x)$. Since f is slightly $g\alpha$-continuous, $f^{-1}(F)$ is a $g\alpha$-closed set. Take $G = f^{-1}(F)$, we have $x \notin G$.

Since X is $g\alpha$-regular, there exist disjoint open sets U and V such that $G \subseteq U$ and $x \in V$. We obtain that $F = f(G) \subseteq f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ and $f(V)$ are disjoint open sets. This shows that Y is clopen regular.

Theorem 2.11. If f is slightly $\tilde{g}\alpha$-continuous injective open function from a $g\alpha$-normal space X onto a space Y, then Y is clopen normal.

Proof. Let F_1 and F_2 be disjoint clopen subsets of Y. Since f is slightly $\tilde{g}\alpha$-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are $g\alpha$-closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \varphi$. Since X is $g\alpha$-regular, there exist disjoint open sets A and B such that $U \subseteq A$ and $V \subseteq B$. We obtain that $F_1 = f(U) \subseteq f(A)$ and $F_2 = f(V) \subseteq f(B)$ such that $f(A)$ and $f(B)$ are disjoint open sets. Thus, Y is clopen normal.

References

[1]. M. Caldas and S. Jafari, On g-US spaces, *Universitatea Din Bacau Stud II SI Cercetari Stiintifice (Mathematica)*, No. 14 (2004), 13-20.

[2]. R. Devi, A. Selvakumar and S.Jafari, On $g\alpha$-closed sets in Topological spaces, Asia Mathematika, Vol. 3, No. 3 (2019), 16-22.

[3]. Erdal Ekici and Miguel Caldas, Slightly γ-continuous Functions, *Bol. Soc. Oaran. Mat.*, (35) Vol. 22, No. 2 (2004), 63-74.

[4]. R.C. Jain, The role of regularly open sets in general topology, *Ph.D Thesis, Meerut University, Institute of Advanced Studies, Meerut, India* (1980).

[5]. O. Njastad, On some classes of nearly open sets, *Pacif. J. Math.*, 15 (1965), 961-970.

[6]. T.Noiri, On semi-continuous mappings, *Atti Accad NAz. Lincei Rend. Cl. Sci. Fis. Mat. Natur*, (8) 54 (1973), 210-214.