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A Statistical Model for Water-Supply Strategies and Drought Mitigation Plans in Agriculture

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Abstract
The agricultural sector of Fars province plays a substantial role in ensuring food security, production, and employment in Iran. Unfortunately, in recent decade wide areas of the province have been affected by drought phenomenon. Due to interdependence drought characteristics and the existence of ungagged areas, in this research, a regional bivariate analysis is proposed for meteorological drought analysis. The cluster analysis and the L-moments method is used to identify homogenous regions of Fars. For meteorological drought analysis of the monthly rainfall series, standardized precipitation index method is used and the crucial drought characteristics, namely drought duration and severity, are determined. Marginal probability distributions of these characteristics are identified by fitting Gamma and Exponential distributions. Three types of bivariate copulas (i.e., Frank, Clayton, and Gumbel–Hougaard) are evaluated for modeling and the best-fit copula for each homogenous region is then employed to estimate conditional probability of the drought characteristics. The conditional drought probability is further described to explain the drought properties comprehensively. The probabilities of drought occurrences under certain circumstances with a specific duration can be determined in order to verify the possibility of drought episodes.

Key words*: copula functions, l- moment method, Regional bivariate frequency analysis, Water-Supply*

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I. Introduction

At present, drought is the most complicated climatic phenomenon affecting society and the environment and is considered as one of the most harmful natural disasters (Wilhite 2000).

Among the several proposed indices for meteorological drought analysis, the standardized precipitation index (SPI) is the commonly used type which was originally developed by McKee et al. (1993). The commencement and ending time, duration, severity, occurrence frequency and area width, which have been evaluated by many researchers, are the main characteristics of each drought(Bazrafshan, Hejabi, and Rahimi 2014b).

The interdependence of drought characteristics makes the univariate frequency analysis inefficient (Mirakbari, Ganji, and Fallah 2012). Furthermore, in univariate severity analysis, the duration variable is usually ignored or considered as fictitious, thus it cannot be assured that the calculated quantiles are a real reflection of the natural events.

In this regard, copula functions, which were introduced in 1959 by Sklar(Sklar, 1959), have been used by researchers in recent years,(Chebana and Ouarda 2009). Copula functions can describe joint distributions of the associated variables with different marginal distributions (Sklar, 1959). Insufficient data at the stations and the existence of ungagged areas necessitate regional analysis. Regional frequency analysis, on the one hand, provides the possibility of analysis for ungagged regions, and, on the other hand, provides better and more complete information for meteorological stations using a combination of points and regional data. The index variable based on linear moments is one of the most advanced methods used in drought frequency analysis (Núñez EZ et al. 2011). The main objective of this research are employed copulas to facilitate the identification and construction of dependence structure of droughts, and to further our understanding into the statistical nature of droughts and our ability to characterize them in Fars province, Iran.

II. Materials And Methods

Study area

Fars province is located in the south of Iran at 50º 42′ to 55 º 36′ east longitudes and 27 º 2′ to 31º 42′ north latitudes (Figure 1.a). The province area is over 122,000 km2 and makes up 7.5% of Iran's total area (Figure 1.a). In Fars province, 42 rain gauge stations with different recorded periods (1974-2014) are available

(Figure 1.b). These 42 stations have period length records of more than 30 years up to a maximum 47 years. Besides, data quality tests such as run test, trend analysis, and rainfall time series stationary were investigated in 42 stations.

Drought Characteristic Analysis

For drought monitoring, McKee et al. (1993) introduced the standardized precipitation index (SPI) based on long-term time series of precipitation in various month scales (3, 6, 9 and 12). Details of SPI calculation are presented in the studies of, (N. B. Guttman, 1993). The positive and negative values of SPI indicate dry and wet conditions, respectively (Tosunoglu & Can, 2016). According to the studies of McKee et al. (1993), drought event is a period when SPI value is constantly negative. The sum of the negative values of SPI is defined as the severity of the drought, which is equal to:

$$S = -\sum_{i=1}^{D} SPI_i \tag{1}$$

In this equation (Eq.1), S is drought severity and D is the duration of drought (month). In this study, the 6-month scale was used to calculate the SPI.

Regional analysis using the index- drought method and application of linear moments

For a homogeneous region according to Eq. (2), we will have:

$$x_i(F) = \mu_i q(F) \tag{2}$$

Where μ_i is the scale parameter of station i or index drought. $q(F)$ is known as the regional frequency curve and is called a non-dimensional variable in the references(Grimaldi et al., 2011). In this research, for the nondimensionalization, severity and duration data of each station were divided by the station mean. A few main steps characterize any regionalization procedure. If the index-variable is selected as the regionalization scheme and one is interested in estimating the variable quantile at a given site, these steps can be summarized as follows :(1) estimation of the index-variable, μ_i; (2) estimation of the regional quantile, $q(F)$, that is, (2a) identification of homogeneous pooling group of sites, (2b) choice of a frequency distribution, and (2c) estimation of the regional frequency distribution; and (3) validation of the regional model(J.R.M. Hosking & J.R.Wallis, 1997).

Identification of candidate homogenous

The formation of regions was based on the cluster analysis of five 'site characteristics': longitudes, latitude, elevation, mean annual precipitation and mean ratio of summer half-year to winter half-year precipitation. These site characteristics are considered to be important in defining a sites precipitation climate; they include indicators of precipitation amounts, distributions of the amounts through the year, and geographic location. Ward's minimum variance hierarchical clustering algorithm (Ward, 1963), which is reported to be useful for the identification of a homogenous region in a regionalization process (Hosking and Wallis, 1997), was applied to cluster analysis. The appropriate number of groups (clusters) was determined based on spatially continuous and physically reasonable.

Testing for homogeneity of the regions

The first step in regional frequency analysis is determining the heterogeneity of the region. In this regard, the study area is considered as a homogeneous meteorological region and then the opposite hypothesis is tested based on two discordancy (D) and heterogeneity (H) statistics. Discordancy test is done in order to specify stations with different linear moment ratios compared with other stations. If the value $D_i \geq 3$, the station is discordant compared to other stations. The second criterion used for determining the heterogeneity of the region was presented by J.R.M. Hosking and J.R.Wallis (1997). If H is used as the level of significance, the criterion used for rejecting the heterogeneity assumption at a significance level of 10% (assuming normal distribution for V) would be $H = 1$ (Mirakbari et al., 2012). Therefore, the criterion $H \leq 1$ can be used for accepting the homogeneity of the area. In this study, both discordancy and heterogeneity criteria were used for drought characteristics of
severity and duration. J.R.M. Hosking and J.R.Wallis (1997) proposed a goodness-of-fit (GOF) test Z^{DIST} used as a measure of fitness of the applied distributions to a region's data:

$$Z^{DIST} = (t_4^R - \tau_4^{DIST}) / \sigma_4 \tag{3}$$

In Eq. 3, t_4^R is an average L-kurtosis value computed from the data of the region, τ_4^{DIST} is a theoretical L-kurtosis value computed from the simulation for a fitted distribution, and σ_4 is the standard deviation of L-kurtosis values obtained from simulated data. The fit of distribution is considered satisfactory if $\left|Z^{DIST}\right| \leq 1.64$. Once the appropriate frequency distribution for each homogenous region is identified quintiles are estimated for several probability levels or return periods using the index-drought method. All statistical computations and graphical displays were made using R statistical software version 3.0-1. The R package lmomRFA developed by J.R.M. Hosking and J.R.Wallis (1997) was used for L-moment analysis.

Copula functions

Copulas functions connect multivariate distributions functions ($F_{X,Y}$) to their marginal distributions ($C[F_X(X), F_Y(Y)]$)(Sklar, 1959). There is a unit copula function called C:

$$F_{X,Y} = C[F_X(X), F_Y(Y)] \tag{4}$$

Assuming that marginal functions are continuous and have density functions of $f_x(x)$ and $f_y(y)$, their copula probability function will be equal to:

$$f_{X,Y} = c[F_X(X), F_Y(Y)]\, f_X(x).\, f_Y(y)] \tag{5}$$

Where c is the density function of C , which is defined as follows:

$$C(u,v) = \frac{\partial^2 c(u,v)}{\partial u \partial v} \tag{6}$$

Where u and v are $F_X(X)$ and $F_Y(Y)$, respectively. Further details related to copula functions are available in the study of (Schweizer & Sklar, 1983). In the current study, for the development of bivariate probability distribution, Archimedean copulas functions were used due to the simplicity of functions structure and the allowable range of dependence (Zhang et al. 2012). Three families of Archimedean copula functions used include Gumbel, Clayton and Frank, (J. T. Shiau, 2006).In order to fit the aforementioned functions to the desired data, the parameter was first estimated. There are several methods for estimating the parameter of copula functions. In this research, a semi-parametric method was used to estimate the parameter of copula functions. In the semi-parametric estimation, the relationship between the generating function of each copula and Kendall correlation coefficient is used (Genest & Rivest, 1993). The most important step in modeling the copula functions is to select the superior copula function using goodness of fit method (Aissia et al. 2015) . Different methods have been presented for goodness of fit of copula functions so far (Genest, Rémillard, and Beaudoin 2009a) performed comprehensive studies on various goodness of fit tests, and finally proposed the parametric bootstrapping procedure, also known as Cramer-von Mises statistic (S_n), that is illustrated as the following equation:

$$S_n = \int n\left\{\left\{C_n(u,v) - C_{\theta_n}(u,v)\right\}^2 dC_n(u,v)\right. \tag{7}$$

Where C_n is the empirical copula (Deheuvels, 1979) calculated using n observation data and C_{θ_n} is an estimation of C obtained assuming $C \in C_0$. The estimation of C_{θ_n} is based on the estimator θ_n of θ .

Multivariate index-drought model

According to the studies of Zhang et al. (2015), the index drought - method can be developed for bivariate models. In this study, since the variables of drought severity and duration jointly describe the characteristics of the drought regime, the equations (2) should be developed for bivariate modes. Assume those random variables and marginal distribution of X and Y are F_X and F_Y , respectively, then their frequency quantiles are $x(F_X)$ and $y(F_Y)$, respectively, and copula function in order to describe the joint distribution of the random variables X and Y is F_{XY} function. In this case, frequency quantiles at station i are equal to:

$$\begin{cases} x_i(F_X) = \mu_{i,X}\, q_X(F_X) \\ y_i(F_Y) = \mu_{i,Y}\, q_Y(F_Y) \\ F_{XY}{}^i(x,y) = C(F_X{}^i(x), F_Y{}^i(y)) \end{cases} \tag{8}$$

In Eq. (8), the symbol C is copula function, $q(X)$ and $q(Y)$ are the regional marginal frequency curves for random variables of X and Y . It should be noted here Eq. (25) a used to fit a regional copula function. In the present study, R programming language (version 2.15.3) and its related software were used to perform bivariate analysis. In this way, CDVINE, COPULA and COP BASIC packages, which have been developed for use in R software, were also used.

III. Results and discussion

SPI calculation

Based on monthly precipitation data at the 42 rain gauges (Fig. 1), series of the SPI at 6-month time scales were obtained. In the study area the average maximum severity and duration in Lar stations are respectively 4.74 and 6.56 month according to run Theory.

Cluster analysis

hierarchical cluster analysis with Ward's method was first applied to identify initial homogeneous regions. The result of Ward's clustering with three clusters is depicted in the dendrogram drawn in Figure 2a. The spatial distribution of the rainfall groups is illustrated in Figure 2b.

Regional Homogeneity Tests and Regional Frequency Distributions

Discordancy and heterogeneity test

The result of the discordancy test indicated that three stations, namely Fasa, Sarvestan, and Kazeron (in region 1) were discordant with the other stations in their groups. These stations were deleted after it proved impossible to reassign them to other regions. After modification of initial region all of which were acceptably homogeneous ($H \leq 1$) with no discordant stations as indicated in Table 1. It shows the results of range of values discordancy test for severity and duration among 39 stations in the study area. As can be seen in the study area, the value of heterogeneity index for variables of duration and severity of drought for cluster 1,2 and 3 were -0.69, -2.4, -0.39 and-1.26, -0.7, -1.5 respectively. Therefore, all 39 stations were non-discordant and the study area was homogeneous. The second stage in the regional frequency analysis is the determination of univariate marginal distributions of severity and duration. Linear moment method was used to estimate the marginal distribution parameters. In this method, the distributions of generalized extreme value, generalized logistic, generalized Pareto, normal, three-parameter log-normal, and Pearson three type were fitted to non-dimensional data, and Z statistic was calculated for goodness of fit for three-parameter distributions based on Hosking and Wallis's. The results of goodness of fit are shown in Table (2). If statistic $|Z| \leq 1.64$, the distribution is suitable for regional analysis. According to Table 2, only Pearson type three distribution is acceptable for non-dimensional drought severity data in group (2) and none is appropriate for fitting the drought duration variable. Zelenhasić and Salvai (1987), Mathier et al. (1992) and Shiau, Feng, and Nadarajah (2007) indicated that the marginal distributions for duration and severity variables are exponential and 2-parameter gamma, respectively. According to the studies of Hosking and Wallis, Z statistic test is suitable only for trivariate distributions. Therefore, the Kolmogorov-Smirnov (K-S) test was used for goodness of fit of exponential and gamma distributions and the results are presented in Table (3). Based on Table 4, gamma and exponential distributions were confirmed at 95% confidence level for duration and severity in all groups. Therefore, gamma and exponential distributions are selected as regional marginal distributions of severity and duration variables and their parameters are presented in Table 4.

Copula estimation for homogenous region

The copula functions parameter can be estimated using the semi-parametric method. In order to select the best copula function, the function presented by Cramer-von Mises and the Eq. (7) presented in the studies of Genest and Rémillard were used (Genest et al., 2009). It can be observed from Table 5 that the goodness-of-fit of the Clayton copula functions was not acceptable for drought series of any subdivision. However, the goodness-of-fit of the Gumbel and Frank copulas was acceptable with p values ranging between 0.42 and 0.44 for the Gumbel copula and 0.20–0.35 for the Frank copula.

Finally, in order to select the best model, different criteria Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), were also used. The function that has the maximum absolute values of AIC, BIC, and log-likelihood function is chosen as the most suitable function for multivariate modeling of non-dimensional drought severity and duration. According to the results presented in Table (6), Gumbel copula function with the maximum values of AIC, BIC and log-likelihood function was selected as the most suitable copula function in group 1,2 and 3 , which is in agreement with the findings of (Zhang et al., 2015). To construct the empirical copula, first, the variables of non-dimensional duration and severity should be ranked based on their observed values. Then, empirical CDFs are created based on the ranks. Thereafter, the empirical CDFs are used to evaluate the empirical copula. Briefly, in the empirical copula, theoretical CDFs of drought duration and severity are replaced by their empirical CDFs. The values of root mean square error (RMSE) compares the closeness between empirical copula and theoretical copula (Table 6). According to Table 6, it is obvious that the Gumbel–Hougaard copula has the minimum AIC value the all regions., confirming the results obtained from the selection of the copula model based on BIC, and log-likelihood function. univariate regional growth curves of duration and severity are estimated directly using regional parameters of marginal distributions. Fig. 3 shows the univariate and bivariate estimated growth curves corresponding to nonexceedance probabilities p= 0.9,0.95 and 0.98 as well as the quantile curve in the unit square and the marginal distributions for (D, S). Univariate regional growth curves of duration and severity are also presented in Table 7. Univariate and bivariate quantiles can be assessed by multiplying growth curves by the corresponding index drought. Base on table 7 for example corresponding to p= 0.9 dimensionless duration and severity (D, S) for groups 1 to 3 is equal to (2.19,2.72), (2.21,2.71) and (2.16,2.63). Regionalization in this study clarified the changing properties or nature of droughts.

IV. Conclusion

Drought characteristics are dependent variables, with the magnitude of one variable affecting the magnitude of other variables. To develop effective water-supply strategies and drought mitigation plans, sufficient information is required on the probability of exceeding a certain magnitude for each drought characteristic. In the present paper, practical aspects of copula model are presented and investigated jointly for the drought characteristics of the considered dataset. The results show that the appropriate fitted marginal distributions are gamma and exponential distributions for drought duration and severity, as well as the Gumbel copula for their dependence structure for the whole region. The selected distributions of drought characteristics are in agreement with the findings of (Zhang et al., 2015). Then, the derived Gumbel–Hougaard copulas were used to estimate the conditional probabilities of the drought characteristics, for each homogenous region. Furthermore, droughts in the regions without meteorological data can also be estimated in terms of joint probability using the index-drought method proposed in this study. In conclusion, the method can provide helpful information in assessing joint return periods, drought risk, optimizing the water resources and planning drought strategies to reduce the effects of future droughts in Fars province.

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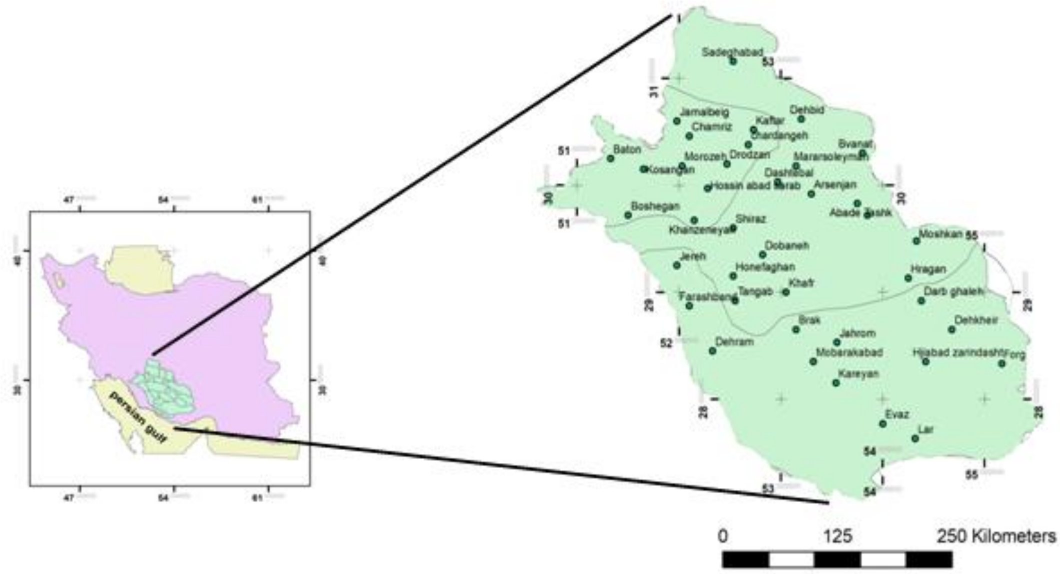

Figure 1. Location of Fars province in Iran (a), distribution of stations in Fars province (b)

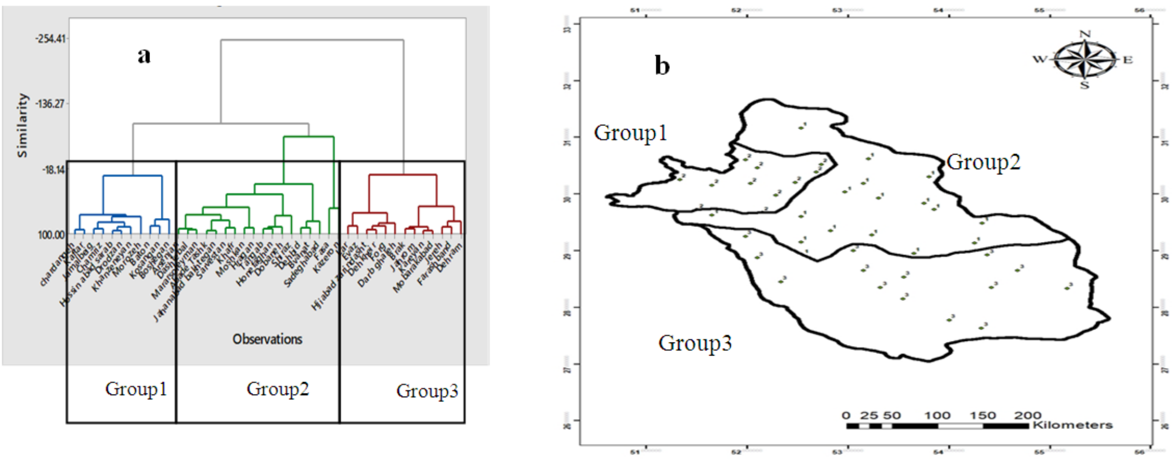

Figure 2a. Dendrogram of clustered stations by the Ward's method.2b. Location of sites in the three relatively homogeneous clusters within the Fars province

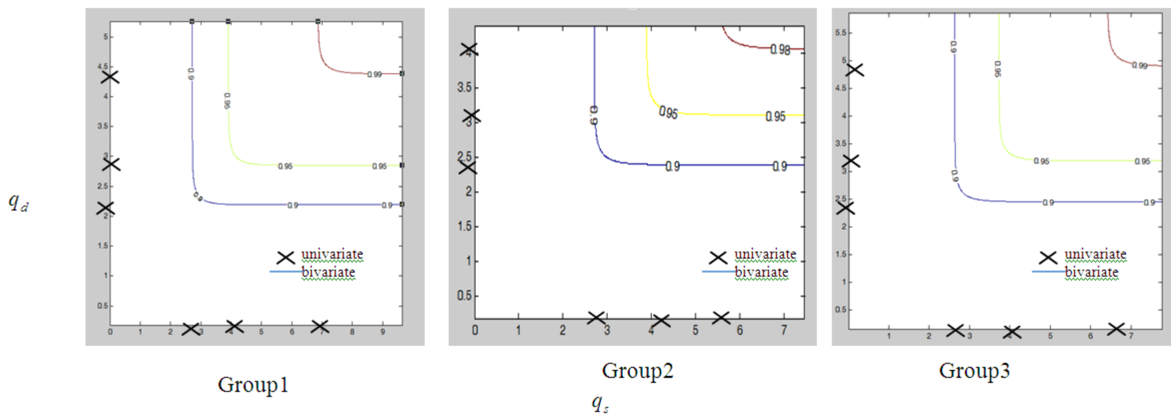

Figure 3. Estimated regional bivariate and univariate growth curves of non-dimensional drought severity and duration q_s and q_d for a given level p(0.9-0.95-0.98) in group 1,2 and 3

Table 1. The range of values discordancy index for data of drought duration and severity

Region	D_d	D_S
Group1	(0.51-1.88)	(0.13-2.01)
Group2	(0.11-2.03)	(0.04-1.92)
Group3	(0.06-2.57)	(0.02-2.54)

Table 2. The results of (Z) statistic test for drought duration and severity data in group 1,2 and 3

distribution	$Z_s(G1)$	$Z_s(G2)$	$Z_s(G3)$	$Z_d(G1)$	$Z_d(G2)$	$Z_d(G3)$
Gen. logistic	7.02	6.74	8.23	13.3	12.83	12.4
Gen. extreme value	6.51	6.23	7.52	11.71	11.51	10.76
Gen. normal	4.59	4.5	5.55	9.97	10.08	9.4
Pearson type III	1.32*	1.56*	2.17	6.94	7.6	7.02
Gen. Pareto	4.19	4.02	4.7	6.99	7.62	6.21
	* accepted distribution					

Table 3. The results of K-S test (p-value) for severity and duration variables of drought

Region	% of goodness of fit*	D	S
Group1	100	0.45	0.86
Group2	100	0.48	0.93
Group3	100	0.48	0.88
			*% of accepted stations

Table 4. Selected regional distribution parameters for severity and duration variables

Region	Duration(Exponential)	severity (Gamma)
Group1	$\xi = 0.048$ $\alpha = 0.95$	$\alpha = 0.46$ $\beta = 2.15$
Group2	$\xi = 0.0381$ $\alpha = 0.9618$	$\alpha = 0.47$ $\beta = 2.11$
Group3	$\xi = 0.06$ $\alpha = 0.9394$	$\alpha = 0.51$ $\beta = 1.92$

Table 5. The results of goodness of fit using Cramer function in group 1,2 and 3

Region	Copula	P-value(Mean)	*Rate	*τ_R	*R
Group1	Clayton	0.024	8	0.786	7.34
	Gumbel	0.2	100	0.786	4.67
	Frank	0.43	100	0.786	16.86
Group2	Clayton	0.1278	0.36	0.7999	7.95
	Gumbel	0.350	100	0.7999	4.97
	Frank	0.441	1	0.7999	18.09
Group3	Clayton	0.1796	0.35	0.788	7.38
	Gumbel	0.3260	100	0.788	4.69
	Frank	0.4233	100	0.788	16.95

*Mean in the table denotes the arithmetic mean of test results; Rate denotes the percentage of stations that passed the test to the total stations under consideration τ_R is the Kendall correlation coefficient for regional drought duration-severity, R is the parameter of the copula function

Table 6. Goodness of fit statistics of copula functions and values of calculated error (RMSE) of theoretical functions

Region	Copula	AIC	BIC	Log-like hood	RMSE
Group1	Clayton	-635.86	-631.33	318.93	0.033
	Gumbel	**-1169.32**	**-1169.32**	**587.92**	**0.022**
	Frank	-1148.33	-1148.33	577.43	0.023
Group2	Clayton	-356.31	-352	179.15	0.036
	Gumbel	**.0356-9**	**.7252-9**	**.51794**	**70.02**
	Frank	.8637-9	.5532-9	.43694	50.02
Group3	Clayton	-751.92	-747.44	376.96	0.024
	Gumbel	**-1201.9**	**-1197.42**	**601.95**	**0.011**
	Frank	-1164.02	-1159.54	583.01	0.013

Table 7. Bivariate regional growth curve values

		p		
Region	Marginal distribution	**0.9**	**0.95**	**0.98**
1	(Duration, Severity)	(2.19,2.72)	(2.85,3.91)	(4.38,6.86)
2	(Duration, Severity)	(2.21,2.71)	(2.88,3.89)	(4.42,6.8)
3	(Duration, Severity)	(2.16,2.63)	(2.81,3.73)	(4.32,6.42)

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