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Application of Homotopy Perturbation Method for Solving Burgers Equations

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Abstract.
Mohand transforms are very useful integral transforms for solving many advanced problems of engineering and sciences like heat conduction problems, vibrating beams problems, population growth and decay problems, electric circuit problems etc. In this article,we present a reliable combination of homotopy perturbation method and Mohandtransform (MHPM) to solve Burger's equations ,the nonlinear terms in the equations can be handled by using of homotopy perturbation method(HPM). The results show that the new method is more effective and convenient to use and high accuracy of it is evident..
Keywords :*Mohand Transform, Homotopy Perturbation method, Burgers equation, Nonlinear Partial differential equation...*

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I. Introduction

In modern time, integral transforms (Laplace transform [10], Z-transform , sawi transform [8], Mahgoub transform [5], Kamal transform [6], Elzaki transform [7], Mohand transform [4], Sumudu transform , etc.) have very useful role in mathematics, physics, chemistry, social science, biology, radio physics, astronomy, nuclear science, electrical and mechanical engineering for solving the advanced problems of these fields

Burger's equation serves as useful model for many interesting problems inapplied mathematics. It models effectively certain problems of a fluid flow nature, in which either shocks or viscous dissipation is significant factor. The first steady-state solutions of Burger's equation were given by Bateman[2] in 1915, and the equation got its name from the extensive research of Burger[3]beginning in 1939. Burger focused on modeling turbulence, but the equationis useful for modeling such diverse physical phenomena as shock flows, trafficflow, acoustic transmission in fog, etc

In the resent years, many researchers mainly had paid attention to studying the solution of nonlinear partial differential equations [9] by using various methods. Among these are the adomian decomposition method[10,13,15], homotopyperturbation method[1], variational iteration method [20], differential transform method[21, 22], and projected differential transform method[23].

Mohand transform is a useful technique for solving linear differential equations but this transform is totally incapable of handling nonlinear equations [1, 14] because of the difficulties that are caused by the nonlinear terms. This paper is using homotopy perturbation method [11,12,14,16-20]to decompose the nonlinear term, so that the solution can be obtained by iteration procedure. This means that we can use both Mohand transform and homotopy perturbation methods to solve Burger's equations.

The main aim of this paper is to consider the effectiveness of the Mohand transform homotopy perturbation method in solving Burger's equation.

II. Definition Of Mohand Transforms:

In year 2017, MohandMahgoub [4] defined "Mohand transform"" of the function $F(t)$for$t \geq 0$ as
$M[F(t)] = R(v) = v^2 \int_0^\infty F(t)e^{-vt}dt$. , $k_1 \leq v \leq k_2$(2.1)
where the operator $M$is called the Mohand transform operator

Linearity property of Mohand transforms:
If Mohand transform of functions $F_1(t)$and $F_2(t)$are $R_1(v)$and $R_2(v)$respectively then Mohand transform of $[aF_1(t) + bF_2(t)]$is given by $[aR_1(v) + bR_2(v)]$wherea, b are arbitrary constants

Mohand transforms of the derivatives of the function
Theorem 2.1:
Let $R[v]$is the SMohand transform of $[M[f(t)] = R[v]]$ then:
(i) $M[f'(t)] = v \quad R(v) - v^2 f(0)$
(ii) $\mathrm{M}[\mathrm{f}''(\mathrm{t})] = v^2 R(v) - v^3 f(0) - v^2 f'(0)$
(iii) $M\left[f^{(n)}(t)\right] = v^{(n)} R(v) - \sum_{k=0}^{n-1} v^{n-k+1} f^{(k)}(0)$

III. MOHAND HOMOTOPY PERTURBATION METHOD

The basic idea of this method can be illustrated by considering the general nonlinear homogeneous partial differential equation with initial conditions of the form [10]
$\mathrm{D\,U(x,t) + RU(x,t) + NU(x,t) = g(x,t)}$(3.1)
$\mathrm{(x,0) = h(x), U_t = (x,t) = f(x)}$(3.2)
Where Dthe second order linear differential operator is $\mathrm{D} = \frac{\partial^2}{\partial t^2}$, $R$is the linear differential operator of less order thanD, Nrepresent the general nonlinear differential operator and$\mathrm{g(x,t)}$is the source term
Applying Mohand Transform on both sides of equation(3.1),
$M[\mathrm{D\,U(x,t) + RU(x,t) + NU(x,t)] = M[g(x,t)]}$(3.3)
Applying the linearity property of the Mohand Transform on equation (3.3), we have
$A[\mathrm{D\,U(x,t)] + [RU(x,t)] + [NU(x,t)] = g(x,t)}$(3.4)
$\mathrm{v^2U(x,v) - U(x,0)} - \frac{1}{v}\mathrm{U}_t\mathrm{(x,0) + M[RU(x,t)] + M[NU(x,t)] = M[g(x,t)]}$(3.5)
Substituting the initial conditions in equation (3.2) into equation (3.5), we obtain
$\mathrm{v^2U(x,v) - h(x)} - \frac{1}{v}\mathrm{U}_t\mathrm{(x,0) = M[g(x,t)] - M[RU(x,t)] - M[NU(x,t)]}$ (3.6)
$\mathrm{U(x,v)} = \frac{1}{\mathrm{v}^2}\mathrm{h(x)} + \frac{1}{v^3}\mathrm{f(x)} + \frac{1}{\mathrm{v}^2}\mathrm{M[g(x,t)]} - \frac{1}{\mathrm{v}^2}\mathrm{M[RU(x,t)]} - \frac{1}{\mathrm{v}^2}\mathrm{M[NU(x,t)]}$(3.7)
Taking Mohand inverse on both sides of equation (3.7), we obtain
$\mathrm{U(x,t) = M^{-1}}\left[\frac{1}{\mathrm{v}^2}\mathrm{h(x)} + \frac{1}{v^3}\mathrm{f(x)} + \frac{1}{\mathrm{v}^2}\mathrm{M[g(x,t)]}\right] - \mathrm{M^{-1}}\frac{M}{\mathrm{v}^2}\left[\mathrm{[RU(x,t)] - [NU(x,t)]}\right]$(3.8)

$$\mathrm{G(x,t) = M^{-1}}\left[\frac{1}{\mathrm{v}^2}\mathrm{h(x)} + \frac{1}{v^3}\mathrm{f(x)} + \frac{1}{\mathrm{v}^2}\mathrm{M[g(x,t)]}\right]$$

$\mathrm{U(x,t) = G(x,t) - M^{-1}}\frac{M}{\mathrm{v}^2}\left[\mathrm{[RU(x,t)] - [NU(x,t)]}\right]$(3.9)
Where $\mathrm{G(x,t)}$represent the terms arising from the source term and the prescribed initial conditions
Assuming the solution of equation (3.1) is of the form
$\mathrm{U = U_0 + p^1U_1 + p^2U_2 + p^3U_3 + p^4U_4} + \cdots$(3.10)
To consider the nonlinear operator, we apply the homotopy perturbation method.
$\mathrm{U(x,t)} = \sum_{n=0}^{\infty} \mathrm{p}^n \mathrm{U}_n\mathrm{(x,t)}$(3.11)
By substituting equation (3.11) into (3.9), we have
$\sum_{n=0}^{\infty} \mathrm{p}^n \mathrm{U}_n\mathrm{(x,t) = G(x,t) - pM^{-1}}\left[\frac{M}{\mathrm{v}^2}\left[\mathrm{[R}\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{H}_n\mathrm{(x,t)] - [N}\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{H}_n\mathrm{(x,t)]}\right]\right]$(3.12)
This is the coupling of the Mohand Transform and the homotopy perturbation method. Comparing the coefficients of like powers of p, we have
$\mathrm{p}^0 \quad : : \mathrm{U_0(x,t) = G(x,t)}$
$\mathrm{p}^1 \quad : : \mathrm{U_1(x,t) = -M^{-1}}\left[\frac{M}{\mathrm{v}^2}\left[\mathrm{[RU_0(x,t)] - [NU_0(x,t)]}\right]\right]$
$\mathrm{p}^2 \quad : : \mathrm{U_2(x,t) = -M^{-1}}\left[\frac{M}{\mathrm{v}^2}\left[\mathrm{[RU_1(x,t)] - [NU_1(x,t)]}\right]\right]$
$\mathrm{p}^3 \quad : : \mathrm{U_3(x,t) = -M^{-1}}\left[\frac{M}{\mathrm{v}^2}\left[\mathrm{[RU_2(x,t)] - [NU_2(x,t)]}\right]\right]$
$\mathrm{p}^4 \quad : : \mathrm{U_4(x,t) = -M^{-1}}\left[\frac{M}{\mathrm{v}^2}\left[\mathrm{[RU_3(x,t)] - [NU_3(x,t)]}\right]\right]$
In general, the recursive relation is given by:
$\mathrm{p}^m \quad : : \mathrm{U}_m\mathrm{(x,t) = -M^{-1}}\left[\frac{M}{\mathrm{v}^2}\left[\mathrm{[RU}_m\mathrm{(x,t)] - [NU}_m\mathrm{(x,t)]}\right]\right]$
Then, the solution can be expressed as
$\mathrm{U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t)} + \cdots$(3.13)

IV. Application

In this section, the MohandHomotopy perturbation method is implemented for solving Burgers equation with initial conditions. We demonstrate the applicability and the effectiveness of this method with three (3) numerical examples. The results obtained by this proposed method are compared with other known results.

Example1

Consider the following one dimensional Burgers equation as [10]

$\mathrm{U}_t = \mathrm{U}_{xx} - \mathrm{UU}_x$(4.1)

With initial conditions

$U(\mathrm{x},0) = 1 - \frac{2}{x}$(4.2)

Applying Mohand Transform to equation (4.1)

$M[\mathrm{U}_t] = \mathrm{M}[\mathrm{U}_{xx} - \mathrm{UU}_x]$(4.3)

Applying the linearity properties of Mohand transform

$$M[\mathrm{U}_t] = \mathrm{M}[\mathrm{U}_{xx}] - \mathrm{M}[\mathrm{UU}_x]$$

$v\mathrm{U}(\mathrm{x},\mathrm{v}) - v^2\mathrm{U}(\mathrm{x},0) = \mathrm{M}[\mathrm{U}_{xx}] - \mathrm{M}[\mathrm{UU}_x]$(4.4)

Substituting the initial conditions(4.2) into equation (4.4) we have

$$v\mathrm{U}(\mathrm{x},\mathrm{v}) - v^2\left(1 - \frac{2}{x}\right) = \mathrm{M}[\mathrm{U}_{xx}] - \mathrm{M}[\mathrm{UU}_x]$$

$\mathrm{U}(\mathrm{x},\mathrm{v}) = v\left(1 - \frac{2}{x}\right) + \frac{\mathrm{M}}{v}[\mathrm{U}_{xx}] - \frac{\mathrm{M}}{v}[\mathrm{UU}_x]$(4.5)

Now taking the Mohand inverse on both sides of equation (4.5), we obtain

$$\mathrm{U}(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left\{v\left(1 - \frac{2}{x}\right) + \frac{\mathrm{M}}{v}[\mathrm{U}_{xx}] - \frac{\mathrm{M}}{v}[\mathrm{UU}_x]\right\}$$

$\mathrm{U}(\mathrm{x},\mathrm{t}) = \left(1 - \frac{2}{x}\right) + \mathrm{M}^{-1}\left\{\frac{\mathrm{M}}{v}[\mathrm{U}_{xx}] - \frac{\mathrm{M}}{v}[\mathrm{UU}_x]\right\}$(4.6)

Now, applying homotopy perturbation method,

$\mathrm{U}(\mathrm{x},\mathrm{t}) = \sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n(\mathrm{x},\mathrm{t})$(4.7)

Substituting equation (4.7) into (4.6)

$\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n(\mathrm{x},\mathrm{t}) = \left(1 - \frac{2}{x}\right) + \mathrm{pM}^{-1}\left[\frac{M}{\mathrm{v}}\left\{\left(\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n(\mathrm{x},\mathrm{t})\right)_{xx}\right\} - \frac{M}{\mathrm{v}}\left\{\left(\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n\,\mathrm{U}_n(\mathrm{x},\mathrm{t})\right)_x\right\}\right]$(4.8)

Comparing the coefficients of the corresponding powers of $p$in equation (4.8), we have

$\mathrm{p}^0 : \mathrm{U}_0(\mathrm{x},\mathrm{t}) = 1 - \frac{2}{x}$

$\mathrm{p}^1 : \mathrm{U}_1(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{0\,xx}(\mathrm{x},\mathrm{t})]\} - \frac{M}{\mathrm{v}}\{\mathrm{U}_0(\mathrm{x},\mathrm{t})\mathrm{U}_{0\,x}(\mathrm{x},\mathrm{t})\}\right]$

$$= \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\left(\frac{-4}{x^3}\right) - \frac{M}{\mathrm{v}}\left\{\left(1 - \frac{2}{x}\right)\left(\frac{2}{x^2}\right)\right\}\right] = \mathrm{M}^{-1}\left[\frac{-4}{x^3} - \frac{2}{x^2} + \frac{4}{x^3}\right]$$

$$\mathrm{U}_1(\mathrm{x},\mathrm{t}) = -\frac{2}{x^2}t$$

$\mathrm{p}^2 \quad : \mathrm{U}_2(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{1\,xx}(\mathrm{x},\mathrm{t})]\} - \frac{M}{\mathrm{v}}\{\mathrm{U}_1\mathrm{U}_{0\,x} + \mathrm{U}_0\mathrm{U}_{1\,x}\}\right]$

$$\mathrm{U}_2(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\left(\frac{-12}{x^4}t\right) - \frac{M}{\mathrm{v}}\left\{\left(\frac{-2}{x^2}t\right)\left(\frac{2}{x^2}\right) + \left(1 - \frac{2}{x}\right)\left(\frac{4}{x^3}t\right)\right\}\right]$$

$$\mathrm{U}_2(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\left\{-\frac{12}{x^4}t + \frac{12}{x^4}t - \frac{4}{x^3}t\right\}\right]\mathrm{U}_2(\mathrm{x},\mathrm{t}) = -\frac{2}{x^3}t^2$$

$\mathrm{p}^3 : \mathrm{U}_3(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{2\,xx}(\mathrm{x},\mathrm{t})]\} - \frac{M}{\mathrm{v}}\{\mathrm{U}_2\mathrm{U}_{0\,x} + \mathrm{U}_1\mathrm{U}_{1\,x} + \mathrm{U}_0\mathrm{U}_{2\,x}\}\right]$

$$\mathrm{U}_3(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\left(\frac{-24}{x^5}t^2\right) - \frac{M}{\mathrm{v}}\left\{-\frac{12}{x^5}t^2 + \frac{6}{x^4}t^2 - \frac{12}{x^5}t^2\right\}\right]$$

$$\mathrm{U}_3(\mathrm{x},\mathrm{t}) = -\frac{2}{x^4}t^3$$

$\mathrm{p}^4 \ : \mathrm{U}_4(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{3\,xx}(\mathrm{x},\mathrm{t})]\} - \frac{M}{\mathrm{v}}\{\{\mathrm{U}_3\mathrm{U}_{0\,x}\} + \mathrm{U}_2\mathrm{U}_{1\,x} + \mathrm{U}_1\mathrm{U}_{2\,x} + \mathrm{U}_0\mathrm{U}_{3\,x}\}\right]$

$$\mathrm{U}_4(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\left(\frac{-40}{x^6}t^3\right) - \frac{M}{\mathrm{v}}\left\{-\frac{24}{x^6}t^3 + \frac{8}{x^5}t^3 - \frac{16}{x^6}t^3\right\}\right]$$

$$\mathrm{U}_4(\mathrm{x},\mathrm{t}) = \mathrm{M}^{-1}\left[\frac{-8}{x^5}t^3\right] = -\frac{2}{x^5}t^4$$

Then, the solution $(x,\, t)$ is expressed as

$$\mathrm{U}(\mathrm{x},\mathrm{t}) = \mathrm{U}_0(\mathrm{x},\mathrm{t}) + \mathrm{U}_1(\mathrm{x},\mathrm{t}) + \mathrm{U}_2(\mathrm{x},\mathrm{t}) + \cdots$$

$$\mathrm{U}(\mathrm{x},\mathrm{t}) = \left(1 - \frac{2}{x}\right) + \left(-\frac{2}{x^2}t\right) + \left(\frac{-2}{x^3}t^2\right) + \left(-\frac{2}{x^4}t^3\right) + \left(-\frac{2}{x^5}t^4\right)$$

$\mathrm{U(x,t)} = 1 - \frac{2}{x} - \frac{2}{x^2}t - \frac{2}{x^3}t^2 - \frac{2}{x^4}t^3 - \frac{2}{x^5}t^4$(4.9)

Thus, the solution can be written in the closed form as:

$\mathrm{U(x,t)} = 1 - \frac{2}{x-t}$(4.10)

Equation (4.10) is the exact solution for equation (4.1) which is the same as the solution in [10]

Example 2

Consider one-dimensional Burger's equation of the form [10]

$\mathrm{U}_t = \mathrm{U}_{xx} - \mathrm{UU}_x$(4.11)

Subject to initial condition

$U(\mathrm{x},0) = \mathrm{x}$(4.12)

Applying Mohand Transform to equation (4.11)

$M[\mathrm{U}_t] = \mathrm{M}[\mathrm{U}_{xx} - \mathrm{UU}_x]$(4.13)

Applying the linearity properties of Mohand transform

$AM[\mathrm{U}_t] = \mathrm{M}[\mathrm{U}_{xx}] - \mathrm{M}[\mathrm{UU}_x]$

$v\mathrm{U(x,v)} - v^2\mathrm{U(x,0)} = \mathrm{M}[\mathrm{U}_{xx}] - \mathrm{M}[\mathrm{UU}_x]$(4.14)

Substituting the initial conditions(4.12) into equation (4.14) we have

$$v\mathrm{U(x,v)} - v^2 x = \mathrm{M}[\mathrm{U}_{xx}] - \mathrm{M}[\mathrm{UU}_x]$$

$\mathrm{U(x,v)} = vx + \frac{\mathrm{M}}{v}[\mathrm{U}_{xx}] - \frac{\mathrm{M}}{v}[\mathrm{UU}_x]$(4.15)

Now taking the Mohand inverse on both sides of equation (4.15), we obtain

$$\mathrm{U(x,t)} = \mathrm{M}^{-1}\left\{vx + \frac{\mathrm{M}}{v}[\mathrm{U}_{xx}] - \frac{\mathrm{M}}{v}[\mathrm{UU}_x]\right\}$$

$\mathrm{U(x,t)} = x + \mathrm{M}^{-1}\left\{\frac{\mathrm{M}}{v}[\mathrm{U}_{xx}] - \frac{\mathrm{M}}{v}[\mathrm{UU}_x]\right\}$(4.16)

Now, applying Homotopy perturbation method

$\mathrm{U(x,t)} = \sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n\,\mathrm{(x,t)}$(4.17)

Substituting equation (4.17) into (4.16)

$\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n\,\mathrm{(x,t)} = x + \mathrm{pM}^{-1}\left[\frac{M}{\mathrm{v}}\left\{\left(\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n\,\mathrm{(x,t)}\right)_{xx}\right\} - \frac{M}{\mathrm{v}}\left\{\left(\sum_{n=0}^{\infty}\mathrm{p}^n\mathrm{U}_n\,\mathrm{U}_n\mathrm{(x,t)}\right)_x\right\}\right]$ (4.18)

Comparing the coefficients of the corresponding powers of p in equation (4.18), we obtain

$\mathrm{p}^0 : \mathrm{U}_0\mathrm{(x,t)} = \mathrm{x}$

$\mathrm{p}^1 : \mathrm{U}_1\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{0\,xx}\mathrm{(x,t)}]\} - \frac{M}{\mathrm{v}}\{\mathrm{U}_0\mathrm{(x,t)}\mathrm{U}_{0\,x}\mathrm{(x,t)}\}\right]$

$$= \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}(0) - \frac{M}{\mathrm{v}}\{\mathrm{x}\}\right]$$

$$\mathrm{U}_1\mathrm{(x,t)} = -xt$$

$\mathrm{p}^2 \quad : \quad \mathrm{U}_2\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{1\,xx}\mathrm{(x,t)}]\} - \frac{M}{\mathrm{v}}\{\mathrm{U}_1\mathrm{U}_{0\,x} + \mathrm{U}_0\mathrm{U}_{1\,x}\}\right]$

$$\mathrm{U}_2\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}(0) - \frac{M}{\mathrm{v}}\{(-\mathrm{x}t)(1) + (\mathrm{x})(-t)\}\right]$$

$$\mathrm{U}_2\mathrm{(x,t)} = -\mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{-2\mathrm{x}t\}\right]\mathrm{U}_2\mathrm{(x,t)} = xt^2$$

$\mathrm{p}^3 : \mathrm{U}_3\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{2\,xx}\mathrm{(x,t)}]\} - \frac{M}{\mathrm{v}}\{\mathrm{U}_2\mathrm{U}_{0\,x} + \mathrm{U}_1\mathrm{U}_{1\,x} + \mathrm{U}_0\mathrm{U}_{2\,x}\}\right]$

$$\mathrm{U}_3\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}(0) - \frac{M}{\mathrm{v}}\{xt^2 + \mathrm{x}t^2 + \mathrm{x}t^2\}\right]$$

$$\mathrm{U}_3\mathrm{(x,t)} = \mathrm{M}^{-1}[3\mathrm{x}t^2]$$

$$\mathrm{U}_3\mathrm{(x,t)} = -xt^3$$

$\mathrm{p}^4 \; : \mathrm{U}_4\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{[\mathrm{U}_{3\,xx}\mathrm{(x,t)}]\} - \frac{M}{\mathrm{v}}\{\{\mathrm{U}_3\mathrm{U}_{0\,x}\} + \mathrm{U}_2\mathrm{U}_{1\,x} + \mathrm{U}_1\mathrm{U}_{2\,x} + \mathrm{U}_0\mathrm{U}_{3\,x}\}\right]$

$$\mathrm{U}_4\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}(0) - \frac{M}{\mathrm{v}}\{-xt^3 - \mathrm{x}t^3 - xt^3 - \mathrm{x}t^3\}\right]$$

$$\mathrm{U}_4\mathrm{(x,t)} = \mathrm{M}^{-1}\left[\frac{M}{\mathrm{v}}\{4\mathrm{x}t^3\}\right] = xt^4$$

Then, the solution (x, t) is expressed as

$$\mathrm{U(x,t)} = \mathrm{U}_0\mathrm{(x,t)} + \mathrm{U}_1\mathrm{(x,t)} + \mathrm{U}_2\mathrm{(x,t)} + \cdots$$

$$\mathrm{U(x,t)} = x - \mathrm{xt} + xt^2 - xt^3 + xt^4 - \cdots$$

$\mathrm{U(x,t)} = \mathrm{x}(1 - \mathrm{t} + t^2 - t^3 + t^4 - \cdots)$(4.19)

Thus, the solution can be written in the closed form as:

$\mathrm{U(x,t)} = \frac{x}{1+t}$(4.20)

Example 3
Consider the(2+1)-dimensional Burger's equation
$U_t + \left(u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}\right) - \varepsilon(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) = 0$(4.21)
Subject to initial condition
$U(x, y, 0) = x + y$(4.22)
Applying Mohand Transform to equation (4.21)
$U(x,t) = v \quad (x + y) - \frac{1}{v} M\left\{\left(u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}\right) - \varepsilon(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})\right\}$(4.23)
Now, taking the Mohand inverse on both sides of equation (4.23), we obtain
$U(x,t) = \quad (x + y) - M^{-1}\left\{\frac{1}{v} M\left(u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y}\right) - \varepsilon(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})\right\}$(4.24)
Now, applying Homotopy perturbation method to get
$U(x,t) = \sum_{n=0}^{\infty} p^n U_n(x,t) = (x + y) - p\left\{M^{-1}\left\{\frac{1}{v} M\{\sum_{n=0}^{\infty} p^n H_n(u)\}\right\}\right\}$(4.25)
That is
$p(uu_x + uu_y) - \varepsilon p(u_{xx} + u_{yy}) = 0$,

$$u = u_0 + pu_1 + p^2 u_2 + \cdots$$

the equation (4.25) can be written as
$p[u_0 + pu_1 + p^2u_2 + \cdots][u_{0x} + pu_{1x} + p^2u_{2x} + \cdots] + p[u_0 + pu_1 + p^2u_2 + \cdots][u_{0y} + pu_{1y} + p^2u_{2y} +$ $\ldots - \varepsilon p u 0xx + pu 1xx + p2 u 2xx + \ldots - \varepsilon p u 0yy + pu 1yy + p2 u 2yy + \ldots = 0$(4.26)
The first few components of He's polynomials, are given by

$$H_0(u) = u_0u_{0x} + u_0u_{0y} - \varepsilon u_{0xx} - \varepsilon u_{0yy}$$

$H_1(u) = u_0u_{1x} + u_1u_{0x} + u_0u_{1y} + u_1u_{0y} - \varepsilon u_{1xx} - \varepsilon u_{1yy}$ ….
Comparing the coefficients of the corresponding powers of $p$in equation (4.26), we obtain
p^0 : $U_0(x, y, t) = x + y$, $H_0(u) = 2(x + y)$
p^1 : $U_1(x, y, t) = -M^{-1}\left\{\frac{M}{v}\{\sum_{n=0}^{\infty} H_0(u)\}\right\} = -2t(x + y)$

$$H_1(u) = -8t(x + y)$$

p^2 : $U_2(x,t) = -M^{-1}\left[\frac{M}{v}\{\sum_{n=0}^{\infty} H_1(u)\}\right] = 4t^2(x + y)$
p^3 : $U_3(x, y, t) = -8t^3(x + y)$
p^4 : $U_4(x, y, t) = 16t^4(x + y)$
Then, the solution (x, t) is expressed as
$U(x, y, t) = U_0(x, y, t) + U_1(x, y, t) + U_2(x, y, t) + \cdots = \frac{x+y}{1+2t}$(4.27)
Equation (4.27) is the same as the result obtained in [25]

V. Conclusion

In this study, Mohandhomotopy perturbation method has been successfully employed to obtain solution of Burgers equations .This method has been successfully employed to obtain the approximate solutions of Burger's equation. Also the obtained result by this method is almost accurate with the exact solution

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