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The Logistic-Weibull distribution with Properties and Applications

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Abstract:
A three-parameter univariate distribution is presented named Logistic Weibull distribution along with its relevant properties like cumulative distribution function, probability density function, survival and hazard rate function. Parameter estimation methods like maximum likelihood estimation (MLE) are discussed and applied for the estimation of purposed parameter distribution. With the help of a real dataset, the distribution's goodness-of-fit has been evaluated in comparison to other existing distributions.
Key Word*: Estimation method, Goodness-of-fit ,Logistic distribution, , MLE ,Weibull distribution*

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I. Introduction

Lifetime distribution finds its use for studying the lifespan of components of a system in fields like life-science, medicine, engineering, biology, econometric and so on. Recently most of the researchers are attracted towards a univariate continuous distribution called logistic distribution for its applicability in modeling life-time data due to its excellent performance.

The CDF and PDF of the logistic distribution have been used in fields like neural networks, logit models and logistic regression. As this distribution has wider tails in comparison to Gaussian distribution, we see more consistency with the outcomes. Let us denote a positive random variable be denoted by *Y* and shape parameter $\zeta > 0$, then CDF of logistic distribution is given by

$$T(y;\zeta) = \frac{1}{1+e^{-\zeta t}}; \quad \zeta > 0, y \in \Re \tag{1.1}$$

Then probability density function is

$$t(y;\zeta) = \frac{\zeta e^{-\zeta t}}{\left(1+e^{-\zeta t}\right)^2}; \quad \zeta > 0, y \in \Re \tag{1.2}$$

Joshi et al. (2020) has introduced logistic exponential power distribution with various shapes of hazard rate. Logistic-modified Weibull distribution has introduced by (Mandouh, 2018) has more flexibility in comparison to modified Weibull distribution for survival analysis. Joshi and Kumar (2020) have created half-logistic NHE distribution.

Chaudhary & Kumar (2020) have presented the logistic inverse exponential distribution using the parent distribution as inverse exponential distribution.

An approach that helps in defining logistic compounded model was given by Lan and Leemis (2008) and they purposed logistic–exponential survival distribution whose survival function is

$$S(t;\alpha,\beta) = \frac{1}{1+\left(e^{\beta t}-1\right)^{\alpha}}; \quad \alpha > 0, \beta > 0, t \ge 0 \tag{1.3}$$

where shape and scale parameters are denoted by α and β for LE distribution. Chaudhary & Kumar (2020) have introduced the logistic modified exponential distribution. Chaudhary & Kumar (2020) have presented the logistic NHE distribution with more flexible hazard rate. Chaudhary & Kumar (2020) studied properties and goodness-of-fit of the Logistic inverse Weibull distribution. Via (Lan & Leemis, 2008)'s approach, we present logistic Weibull (LoW) distribution. In this paper we have used Weibull distribution as a base-line distribution inserting one extra parameter to it aiming for a distribution with more flexibility and better goodness-of-fit.
We have structured the study in the following way. In section 2, Logistic Weibull distribution along with its relevant properties is presented. Parameters estimation are carried out in section 3. In Section 4, a real data set

has been analyzed for exploring the applications of the distribution purposed and examining the goodness of fit in comparison to other models. In this section, we also present the test criterion like AIC, BIC, AICC and HQIC criterion for ML, LSE, and CVME. In Section 5 conclusion has been presented

II. The Logistic Weibull distribution

Taking Weibull distribution as baseline, Logistic Weibull (LoW) distribution has been introduced whose CDF is

$$F(x) = 1 - \frac{1}{1+\left(\exp(\lambda x^{\beta})-1\right)^{\alpha}} \quad ; x \ge 0, \alpha > 0, \beta > 0, \lambda > 0. \tag{2.1}$$

And PDF is

$$f(x) = \frac{\alpha\ \beta\ \lambda x^{\beta-1}\exp(\lambda x^{\beta})\left(\exp(\lambda x^{\beta})-1\right)^{\alpha-1}}{\left\{1+\left(\exp(\lambda x^{\beta})-1\right)^{\alpha}\right\}^{2}} \quad ; x \ge 0, \alpha > 0, \beta > 0, \lambda > 0. \tag{2.2}$$

LoW distribution's survival function is

$$S(x) = \frac{1}{1+\left(\exp(\lambda x^{\beta})-1\right)^{\alpha}} \quad ; x \ge 0, \alpha > 0, \beta > 0, \lambda > 0. \tag{2.3}$$

LoW distribution's hazard function is

$$h(x) = \frac{\alpha\ \beta\ \lambda x^{\beta-1}\exp(\lambda x^{\beta})\left(\exp(\lambda x^{\beta})-1\right)^{\alpha-1}}{1+\left(\exp(\lambda x^{\beta})-1\right)^{\alpha}} \tag{2.4}$$

For varying values of α, β and λ, the PDF and hazard rate function's graphs of LoW distribution is illustrated in Figure 1.

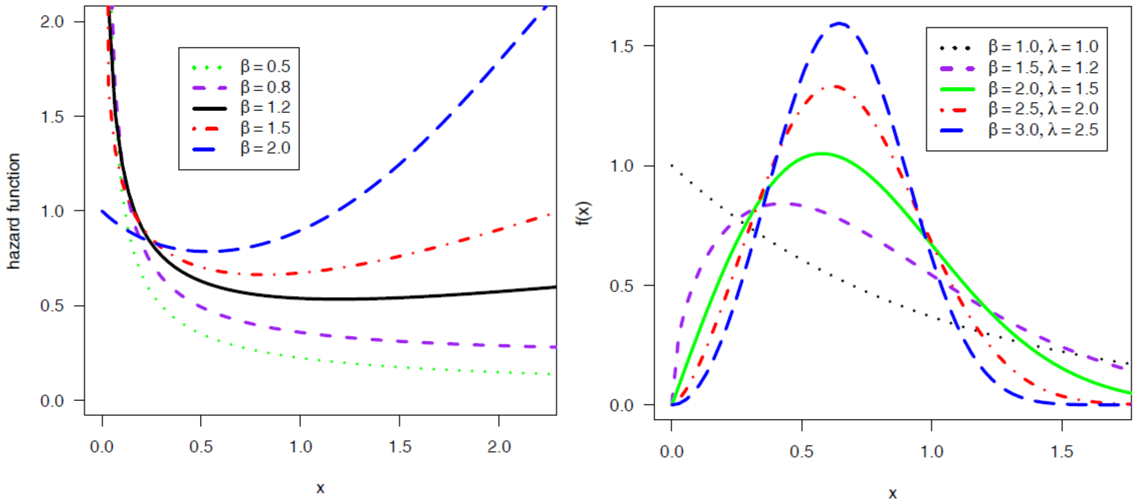

Figure 1. For various values of α, β and λ. graphs of hazard function (left panel) and PDF (right panel)

LoW distribution's quantile function is

$$Q(q) = \left[\frac{1}{\lambda}\log\left\{1+\left(\frac{q}{1-q}\right)^{1/\alpha}\right\}\right]^{1/\beta} \quad ; 0 < q < 1. \tag{2.5}$$

LoW distribution's Random deviate generation:

$$x = \left[\frac{1}{\lambda} \log \left\{ 1 + \left(\frac{1}{u} - 1 \right)^{-(1/\alpha)} \right\} \right]^{1/\beta} \qquad ; 0 < u < 1.$$

(2.6)

Skewness and Kurtosis:

The coefficient of skewness can be expressed as

$$Skewness = \frac{Q(0.75) + Q(0.25) - 2Q(0.5)}{Q(3/4) - Q(1/4)} \text{ and} \tag{2.7}$$

Coefficient of kurtosis defined by (Moors, 1988) is

$$Kurtosis = \frac{Q(0.875) - Q(0.625) + Q(0.375) - Q(0.125)}{Q(3/4) - Q(1/4)} \tag{2.8}$$

III. Methods of estimation

For the parameter estimation of the LoW distribution we have employed the following well-known estimation methods.

a) MLE method
b) LSE method
c) CVM estimation method

3.1. MLE method

Consider a random sample be denoted as $x_1, x_2, \dots, x_n$ from $LoW(\alpha, \beta, \lambda)$ and the likelihood function can be expressed as,

$$L = \alpha \beta \lambda \prod_{i=1}^{n} \frac{x_i^{\beta-1} \exp(\lambda x_i^{\beta}) \left(\exp(\lambda x_i^{\beta}) - 1 \right)^{\alpha-1}}{\left\{ 1 + \left(\exp(\lambda x_i^{\beta}) - 1 \right)^{\alpha} \right\}^2} \quad ; x \geq 0.; \ (\alpha, \beta, \lambda) > 0, \ x > 0$$

Now log-likelihood density is

$$\log L = n \log \alpha + n \log \beta + n \log \lambda + (\beta - 1) \sum_{i=1}^{n} \log x_i + \lambda \sum_{i=1}^{n} x_i^{\beta} + (\alpha - 1) \sum_{i=1}^{n} \log \left(\exp(\lambda x_i^{\beta}) - 1 \right)$$
$$- 2 \sum_{i=1}^{n} \log \left\{ 1 + \left(\exp(\lambda x_i^{\beta}) - 1 \right)^{\alpha} \right\}$$

(3.1.1)

Differentiation of (3.1.1) w.r.t α, β and λ following is obtained

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \log \left(\exp(\lambda x_i^{\beta}) - 1 \right) - 2 \sum_{i=1}^{n} \frac{\left(\exp(\lambda x_i^{\beta}) - 1 \right)^{\alpha} \log \left(\exp(\lambda x_i^{\beta}) - 1 \right)}{\left\{ 1 + \left(\exp(\lambda x_i^{\beta}) - 1 \right)^{\alpha} \right\}}$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \log x_i + \lambda \sum_{i=1}^{n} x_i^{\beta} \log x_i + (\alpha - 1)\lambda \sum_{i=1}^{n} \frac{x_i^{\beta} \exp(\lambda x_i^{\beta}) \log x_i}{\left\{\exp(\lambda x_i^{\beta}) - 1\right\}}$$

$$- 2\alpha\lambda \sum_{i=1}^{n} \frac{\left\{\exp(\lambda x_i^{\beta}) - 1\right\}^{\alpha - 1} x_i^{\beta} \exp(\lambda x_i^{\beta}) \log x_i}{1 + \left\{\exp(\lambda x_i^{\beta}) - 1\right\}^{\alpha}}$$

$$\frac{\partial \log L}{\partial \lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n} x_i^{\beta} + \sum_{i=1}^{n} x_i^{\beta} \log x_i + (\alpha - 1) \sum_{i=1}^{n} \frac{x_i^{\beta} \exp(\lambda x_i^{\beta})}{\left\{\exp(\lambda x_i^{\beta}) - 1\right\}}$$

$$- 2\alpha \sum_{i=1}^{n} \frac{\left\{\exp(\lambda x_i^{\beta}) - 1\right\}^{\alpha - 1} x_i^{\beta} \exp(\lambda x_i^{\beta})}{1 + \left\{\exp(\lambda x_i^{\beta}) - 1\right\}^{\alpha}}$$

Solving these non-linear functions for (α, β, λ) by equating to zero we will obtain the ML estimators $\hat{\alpha},\ \hat{\beta}\ and\ \hat{\lambda}$. The computer softwares like R, Mathmatica, Matlab etc can be used to solve them manually. The observed information matrix for α, β and λ is as follows

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

where

$$Y_{11} = \frac{\partial^2 l}{\partial \alpha^2},\ Y_{12} = \frac{\partial^2 l}{\partial \alpha \partial \beta},\ Y_{13} = \frac{\partial^2 l}{\partial \alpha \lambda}$$

$$Y_{21} = \frac{\partial^2 l}{\partial \beta \partial \alpha},\ Y_{22} = \frac{\partial^2 l}{\partial \beta^2},\ Y_{23} = \frac{\partial^2 l}{\partial \beta \partial \lambda}$$

$$Y_{31} = \frac{\partial^2 l}{\partial \lambda \partial \alpha},\ Y_{32} = \frac{\partial^2 l}{\partial \beta \partial \lambda},\ Y_{33} = \frac{\partial^2 l}{\partial \lambda^2}$$

Let $\underline{\phi} = (\alpha, \beta, \lambda)$ represent the parameter space and the ML estimate of ϕ as $\hat{\underline{\phi}} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, then $\left(\hat{\underline{\phi}} - \underline{\phi}\right) \rightarrow N_3\left[0, \left(U\left(\underline{\phi}\right)\right)^{-1}\right]$ where $U\left(\underline{\phi}\right)$ is the information matrix of Fisher. Applying Via the algorithm of Newton-Raphson, maximization of likelihood gives the observed information matrix and the var-cov matrix is,

$$\left[U\left(\underline{\phi}\right)\right]^{-1} = \begin{pmatrix} \operatorname{var}(\hat{\alpha}) & \operatorname{cov}(\hat{\alpha}, \hat{\beta}) & \operatorname{cov}(\hat{\alpha}, \hat{\lambda}) \\ \operatorname{cov}(\hat{\beta}, \hat{\alpha}) & \operatorname{var}(\hat{\beta}) & \operatorname{cov}(\hat{\beta}, \hat{\lambda}) \\ \operatorname{cov}(\hat{\lambda}, \hat{\alpha}) & \operatorname{cov}(\hat{\lambda}, \hat{\beta}) & \operatorname{var}(\hat{\lambda}) \end{pmatrix}$$

(3.1.2)

Thus for α, β and λ, using MLEs' asymptotic normality, approximate 100(1-α) % confidence intervals is given as

$$\hat{\alpha} \pm Z_{\alpha/2} SE(\hat{\alpha}),\ \hat{\beta} \pm Z_{\alpha/2} SE(\hat{\beta}) \text{ and, } \hat{\lambda} \pm Z_{\alpha/2} SE(\hat{\lambda})$$

Upper percentile of standard normal variate is denoted by $Z_{\alpha/2}$

3.2. LSE Method

The estimation of α, β and λ of LoW distribution employing least-square estimation method is carried out with maximization of

$$B(t;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\left[F(t_i)-\frac{i}{n+1}\right]^2 \quad (3.2.1)$$

w.r.t α, β and λ.

From a distribution function F(.), suppose $F(X_i)$ represents ordered random variables ($X_{(1)} < X_{(2)} < \dots < X_{(n)}$)'s CDF where random sample is denoted as $\{X_1, X_2, \dots, X_n\}$ with "n" size. The LSEs of the unknown parameters is acquired with minimization of

$$B(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{i}{n+1}\right]^2 ;x\geq 0,\alpha>0,\beta>0,\lambda>0. \quad (3.2.2)$$

w.r.t α, β and λ.

Differentiation of (3.2.2) w.r.t α, β and λ we obtain

$$\frac{\partial B}{\partial\alpha}=2\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{i}{n+1}\right]\frac{\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\ln\left[\exp(\lambda x_i^{\beta})-1\right]}{\left\{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\right\}^2}$$

$$\frac{\partial B}{\partial\beta}=2\alpha\lambda\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{i}{n+1}\right]\frac{x_i^{\beta}\ln(x_i)\exp(\lambda x_i^{\beta})\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha-1}}{\left\{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\right\}^2}$$

$$\frac{\partial B}{\partial\lambda}=2\alpha\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{i}{n+1}\right]\frac{x_i^{\beta}\exp(\lambda x_i^{\beta})\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha-1}}{\left\{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\right\}^2}$$

Using the similar computational method as above, the weighted LS estimators is attained with minimization of

$$C(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]$$

w.r.t α, β and λ.

Weights $w_i=\frac{1}{Var(X_{(i)})}=\frac{(n+1)^2(n+2)}{i(n-i+1)}$

Thus, the weighted LS estimators for α, β and λ respectively can be obtained by minimizing,
We maximization of

$$C(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{i}{n+1}\right]^2$$

(3.2.3)

with respect to α, β and λ, weighted LS estimators for unknown parameter is attained

3.3. CVM Method

The CVM estimators is attained with minimization of

$$D(X;\alpha,\beta,\lambda)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F(x_{i:n}\mid\alpha,\beta,\lambda)-\frac{2i-1}{2n}\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{2i-1}{2n}\right]^2$$

(3.3.1)

Differentiation of (3.3.1) w.r.t α, β and λ following is obtained:

$$\frac{\partial D}{\partial\alpha}=2\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{2i-1}{2n}\right]\frac{\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\ln\left[\exp(\lambda x_i^{\beta})-1\right]}{\left\{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\right\}^2}$$

$$\frac{\partial D}{\partial\beta}=2\alpha\lambda\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{2i-1}{2n}\right]\frac{x_i^{\beta}\ln(x_i)\exp(\lambda x_i^{\beta})\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha-1}}{\left\{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\right\}^2}$$

$$\frac{\partial D}{\partial\lambda}=2\alpha\sum_{i=1}^{n}\left[1-\frac{1}{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}}-\frac{2i-1}{2n}\right]\frac{x_i^{\beta}\exp(\lambda x_i^{\beta})\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha-1}}{\left\{1+\left(\exp(\lambda x_i^{\beta})-1\right)^{\alpha}\right\}^2}$$

We will get the CVM estimators after solving non-linear equations $\frac{\partial D}{\partial\alpha}=0,\frac{\partial D}{\partial\beta}=0\ and\ \frac{\partial D}{\partial\lambda}=0$ simultaneously.

IV. Application to real dataset

We have illustrated the applicability of logistic Weibull distribution by considering a real data-set as used by Lee and Wang (2003) which represents the remission times (in months) of a random sample of 128 bladder cancer patients (sorted data)

0.08, 0.20, 0.40, 0.50, 0.51, 0.81, 0.90, 1.05, 1.19, 1.26, 1.35, 1.40, 1.46, 1.76, 2.02, 2.02, 2.07, 2.09, 2.23, 2.26, 2.46, 2.54, 2.62, 2.64, 2.69, 2.69, 2.75, 2.83, 2.87, 3.02, 3.25, 3.31, 3.36, 3.36, 3.48, 3.52, 3.57, 3.64, 3.70, 3.82, 3.88, 4.18, 4.23, 4.26, 4.33, 4.34, 4.40, 4.50, 4.51, 4.87, 4.98, 5.06, 5.09, 5.17, 5.32, 5.32, 5.34, 5.41, 5.41, 5.49, 5.62, 5.71, 5.85, 6.25, 6.54, 6.76, 6.93, 6.94, 6.97, 7.09, 7.26, 7.28, 7.32, 7.39, 7.59, 7.62, 7.63, 7.66, 7.87, 7.93, 8.26, 8.37, 8.53, 8.65, 8.66, 9.02, 9.22, 9.47, 9.74, 10.06, 10.34, 10.66, 10.75, 11.25, 11.64, 11.79, 11.98, 12.02, 12.03, 12.07, 12.63, 13.11, 13.29, 13.80, 14.24, 14.76, 14.77, 14.83, 15.96, 16.62, 17.12, 17.14, 17.36, 18.10, 19.13, 20.28, 21.73, 22.69, 23.63, 25.74, 25.82, 26.31, 32.15, 34.26, 36.66, 43.01, 46.12, 79.05

The MLEs are calculated by applying R software's optim() function (R Core Team, 2020) and (Mailund, 2017) for maximization of (3.1.1). We have obtained Log-Likelihood value is l = -409.4744 and the MLE's with their standard errors (SE) and 95% asymptotic confidence interval are displayed in Table 1.

Table 1
for α, β and λ, MLE along with SE and 95% confidence interval

Parameter	MLE	SE	95% ACI
alpha	2.4165	1.1083	(0.2442, 4.5888)
beta	0.5103	0.2221	(0.0750, 0.9456)
lambda	0.2711	0.1142	(0.0473, 0.4950)

Plots of profile log-likelihood function for the unknown parameters in Figure 2 (Kumar & Ligges, 2011) and found that the MLEs can be uniquely determined.

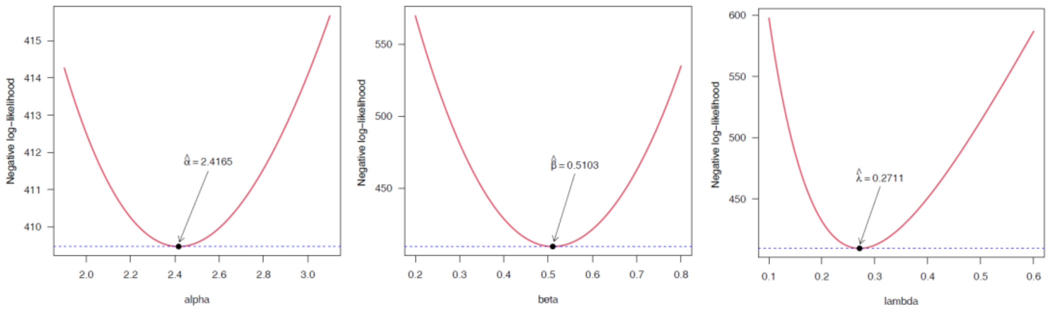

Figure 2. For α, β, and λ, Plot of profile log-likelihood function.

In Figure 3 illustration of plots of Q-Q and P-P plot of LoW distribution.

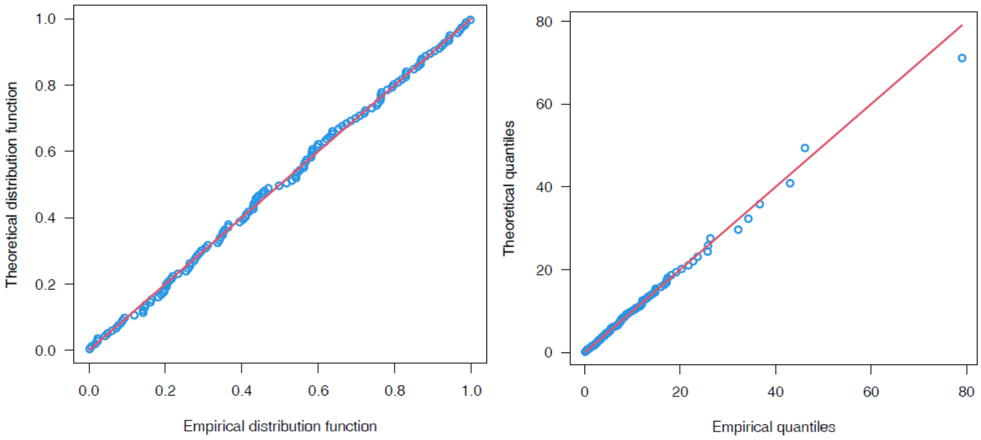

Figure 3. The LoW distribution's plots of P-P (left panel) and Q-Q (right panel)

Via the estimation methods the estimated values of the parameters along with their log-likelihood, and AIC criterion is given in Table 2

Table 2
Estimated parameters, log-likelihood, and AIC

Method of Estimation	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-LL	AIC
MLE	2.4165	0.5103	0.2711	409.4744	824.9487
LSE	3.7217	0.3348	0.3756	409.7356	825.4712
CVE	3.6612	0.3443	0.3691	409.7208	825.4416

In Table 3 we have displayed the KS, W and A^2 statistics with their corresponding p-value of MLE, LSE and CVE estimates.

Table 3
The KS, W and A^2 statistics with a p-value

Method of Estimation	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
MLE	0.0321(0.9994)	0.0149(0.9997)	0.1010(0.9999)
LSE	0.0303(0.9998)	0.0127(0.9999)	0.1038(0.9999)
CVE	0.0307(0.9997)	0.0122(0.9999)	0.1017(0.9999)

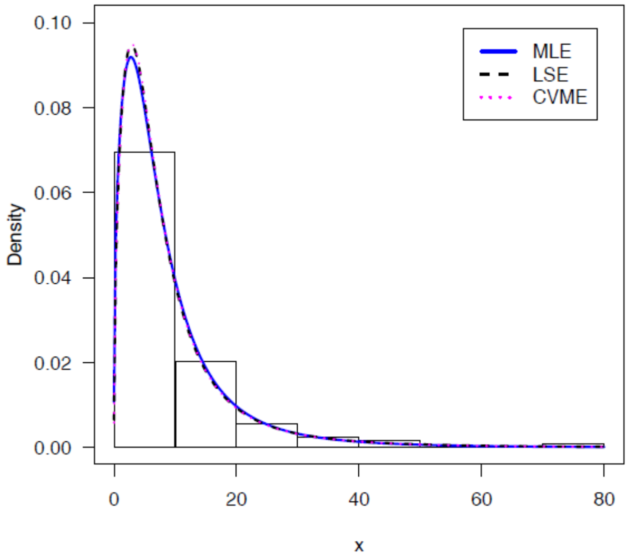

Figure 4. Fitted distributions' PDF and histograms of estimation methods MLE, LSE and CVM.

Following distributions are taken for comparison purpose to evaluate goodness of fit

A. Exponentiated Exponential Poisson (EEP):

The EEP (Ristić & Nadarajah, 2014)'s PDF can be expressed as

$$f(x)=\frac{\alpha\beta\lambda}{\left(1-e^{-\lambda}\right)}e^{-\beta x}\left(1-e^{-\beta x}\right)^{\alpha-1}\exp\left\{-\lambda\left(1-e^{-\beta x}\right)^{\alpha}\right\} \quad ; x>0, \alpha>0, \lambda>0$$

B. Generalized Exponential Extension (GEE) distribution:

GEE's PDF (Lemonte, 2013) is

$$f_{GEE}(x;\alpha,\beta,\lambda)=\alpha\beta\lambda\,(1+\lambda x)^{\alpha-1}\,exp\left\{1-(1+\lambda x)^{\alpha}\right\}$$

$$\left[1-exp\left\{1-(1+\lambda x)^{\alpha}\right\}\right]^{\beta-1} \quad ; \ x\geq 0.$$

C. Logistic-Exponential (LE) distribution:

LE's PDF (Lan & Leemis, 2008) is

$$f_{LE}(x)=\frac{\lambda\ \alpha\ e^{\lambda x}\left(e^{\lambda x}-1\right)^{\alpha-1}}{\left\{1+\left(e^{\lambda x}-1\right)^{\alpha}\right\}^{2}} \quad ; x\geq 0, \alpha>0, \lambda>0.$$

D. Generalized Exponential (GE) distribution:

GE distribution's PDF (Gupta & Kundu, 1999) is.

$$f_{GE}(x;\alpha,\lambda)=\alpha\ \lambda\ e^{-\lambda x}\left\{1-e^{-\lambda x}\right\}^{\alpha-1} \quad ; (\alpha,\lambda)>0, x>0$$

E. Exponential power (EP) distribution:

EP distribution's PDF (Smith & Bain, 1975) is

$$f_{EP}(x) = \alpha\,\lambda^{\alpha}\,x^{\alpha-1}\,e^{(\lambda x)^{\alpha}}\exp\left\{1 - e^{(\lambda x)^{\alpha}}\right\} \quad ;(\alpha,\lambda) > 0, \quad x \geq 0.$$

We have illustrated the Bayesian information criterion (BIC), Akaike information criterion (AIC), Hannan-Quinn information criterion (HQIC) and Corrected Akaike information criterion (CAIC) for the evaluation of the applicability of the purposed distribution in Table 4.

Table 4

AIC, Negative Log-likelihood (LL), CAIC, BIC and HQIC of LoW distribution

Distribution	AIC	-LL	CAIC	BIC	HQIC
LoW	824.9487	409.4744	825.1423	833.5048	828.4251
EEP	825.5056	409.7528	825.6991	834.0617	828.9819
GEE	827.2026	410.6013	827.3961	835.7586	830.6789
LE	829.2507	412.6254	829.3467	834.9548	831.5683
GE	830.1552	413.0776	830.2512	835.8592	832.4728
EP	857.2948	426.6474	857.3893	862.9989	859.6124

Fitted distributions' PDF and Histogram and Empirical and Fitted distribution function of LoW are presented in Figure 5.

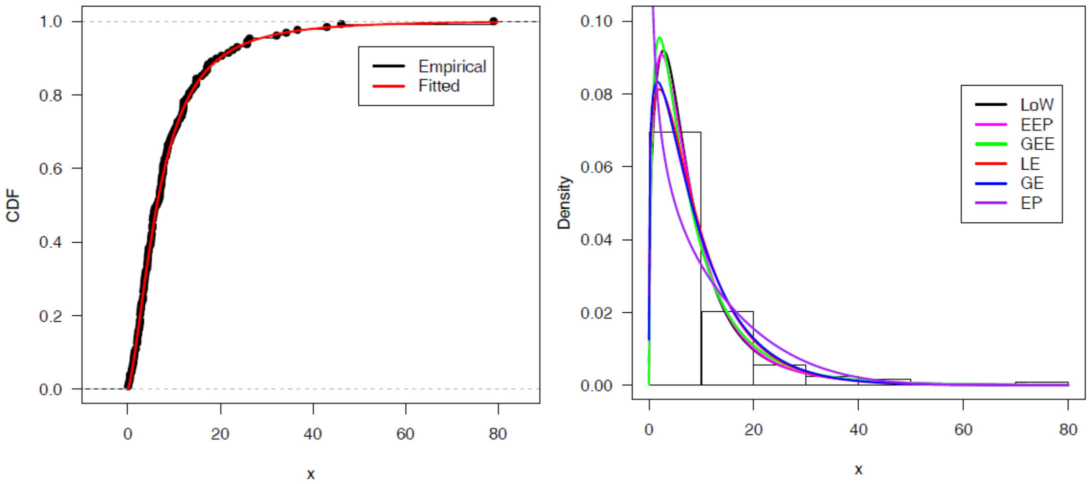

Figure 5. Empirical and Fitted distribution function of LoW distribution (Left panel) and The Histogram and the density function of fitted distributions(Right panel)

To compare the LoW distribution's goodness of fit among different models, different values of goodness of fit statistics which are Anderson-Darling (W), Kolmogorov-Simnorov (KS) and the Cramer-Von Mises (A^2) statistics are presented in Table 5 with corresponding p-value where the test statistics for model purposed was observed to have low value also the p-value was higher. Thus conclusion that LoW distribution shows better fit with more reliability and consistency in results among others taken for comparison

Table 5

The goodness-of-fit statistics and their corresponding p-value (dataset-I)

Distribution	*W(p-value)*	*KS(p-value)*	*A^2(p-value)*
LoW	0.0149(0.9997)	0.0321(0.9994)	0.1010(0.9999)
EEP	0.0220(0.9946)	0.0380(0.9925)	0.1486(0.9987)
GEE	0.0394(0.9367)	0.0442(0.9636)	0.2630(0.9631)
LE	0.1131(0.5252)	0.0691(0.5740)	0.6276(0.6219)
GE	0.1279(0.4652)	0.0725(0.5115)	0.7137(0.5472)
EP	0.5993(0.0223)	0.1199(0.0503)	3.6745(0.0126)

V. Conclusion

Here, a new distribution with three-parameter called Logistic Weibull distribution is introduced with some distributional characteristics like the curve of the probability density, and hazard rate functions and explicit expressions for cumulative distribution, quantile function, survival function, the kurtosis and skewness measures. The model parameters are estimated by using MLE, CVME and LSE method. A real datasets is taken to explore the potentiality of the proposed distribution where we concluded that LoW distribution shows better fit with more reliability and consistency in results among others taken for comparison providing a good alternative to the existing models in survival analysis

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