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Markov chains model to estimate the transition operational status probabilities for machines in Asalaya Sugar Company

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Abstract:
In this paper, a Markov chain model was applied for monthly failure timefor season (2019) of two machines (Mill troup -Boiler), important machines in the sugar industry inAsalaya Sugar Company.Through estimating the failure rate and repair rate andthe transition probabilitiesfrom operational state to another , it was found thatfailure rate of the machine(Mill troup) is greater than the failure rate of the machine (Boiler),the probability of both machines in working condition is high ,probability of machines (Mill troup) suffers more failure, which requiresmore effort in maintenance than machine (Boiler).The probability of the overall failure rate of the machines is negligible probability for the machines,which isa good indicator as it is unlikely that both machines will fail at the same time. That means maintenance work should take place immediately for the machine that suffers a malfunction.

Keywords: Failure rate, Markov chain , Repair rate, MTTF,MDT

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I. Introduction:

The concept of maintenance refers to a group of activities aimed at increasing the actual use of machines.The productivity of a machine is defined by how it is operated and maintained throughout its life cycle,proper preparation and installation as well as regular maintenance, inspection and replacement of spare parts contribute to increasing working time and improving performance.the machines of the industrial plant are subject to many faults with the passage of time and a failure is defined as the loss of the machine's ability to perform an operation or set of operations that are necessary for the machine to provide a specific service (Lotfi, 2011,p5).The faults are also known as repairing, preventing and avoiding damage resulting from use (xioaun &Lifeng, p.294).Successful maintenance operations require the manufacturing facility to create plans and effective measures that support implementation processes, one of which is the use of mathematical methods that can provide an indicationcharacterized by a high degree of accuracy about the operational condition of the machines.One of these methods is the use of a Markov chainto measure the total failure rate of production line machines and the transitionprobabilities form state to another. The research aims to apply one of the models of stochastic operations, which is the Markov chains model which can be used to calculate the failure rates and repair rates of machines andthe transitionprobabilities the machinesfrom one operational state to another.In order to ascertain the operating condition of the machines.

II. Study problem:

The maintenance department faces a set of difficulties that are represented in how to use mathematical and statistical models in the process of machine maintenance, among them those that measure the probability of failure rate and the transitionprobabilities the machine from one operating state to another and this provides an understanding of the dimensions of the machine.The problem is consuming the Sudanese Sugar Company.Asalaya Sugar Company was chosen, which belongs to Sudanese Sugar Company, and it was found that the maintenance department did not use mathematical and statistical models to know the actual operational reality of production machines,this research comes as a practical and scientific addition to the use of Markov chains in the maintenance process.

2.1 Proposed solution:

This type of study relies on identifying the problem mainly, then identifying the causes of the problem and making appropriate recommendations that include solutions. In order to identify the main cause of the study problem, the following hypotheses must first be tested:

- Markov chain can be applied in Asalaya Sugar Company to calculate probabilities transmission of machines from one operational state to another.
- Over all failure rate for machinesis negligible probability.

III. Methodology:

3.1Failure Rate:

A fault is defined as the loss of the machine's ability to perform an operation or set of operations that are necessary for the machine to provide a specific service (Lotfi, 2011,p.5).faulty behavior can be described by a number of mathematical functions and quantitative methods, which differ in complexity according to the nature of the machines system (Dhillo, 2002, p.191),The commonly used mathematical function to calculate fault rate in an exponential distribution is:

$$\lambda = \frac{f(t)}{R(t)} \quad \text{……………………..} \quad (1)$$

Where f(t) probability density function (pdf) of fault and R(t) Reliability function

3.2. Mean time to failure (MTTF)

One of basic measures of reliability is mean time to failure (MTTF) .This statistical value is defined as the average time expected until the first failure of machine.MTTF can calculated by the failurerate inverse, $(1/\lambda)$. Assuming failure rate

3.3. Mean time to repair (MTTR)

Mean time to repair (MTTR) can described as the total time that spent to perform all corrective or preventative maintenance repairs divided by the total of repair numbers.

3.4. Mean down time (MDT):

mean down time (MDT) is defined as the mean time that a system is not usable. This includes all time such as repair corrective and preventive maintenance, Failure Analysis and Preventionself-imposed downtime. MDT can calculated by the repairrate inverse $(1/\mu)$ (Walter A.shewhart& Samuel S.Wilks p.314) .

3.1. Application of the Markov Model in Maintenance:

There are many mathematical and statistical models that can be used in maintenance and repair applications,And thismodels are based on models of stochastic processes and the fundamentals of probability theory in their calculations.It is a system evaluation of the operational status of machines. One such model is the Markov Chain Model iscommonly used models for analyzing repairable/maintenance.Markov chains are a model of stochastic processes. The stochastic process is the process that specifies that its future state does not depend on its past states. Knowing its condition in the future depends on knowing its condition in the present.This random process is called a Markov chain denoted by:

$$P_r(X_n = k, X_{n-1} = j, X_{n-2} = i, X_{n-3} = h,..., X_1 = b, X_0 = a)$$

$$P_r(X_n = k / X_{n-1} = j) = p_{ik} \quad \text{…………………………...} (2)$$

The value of the future random variable is (X_{n-1})it depends on the value (X_n) in this time.It is not affected by the variable values in the past $(X_1, X_2,..., X_{n-1})$,The Markov chain in maintenance applications consists of a number of possible states of the operational state of production line machines. And the transitionprobabilities from one operational situation to another (Guangbin 2007, p.381).the transmission probability means the probability of the machine transitionfrom the state in which the machine is valid to work, to the state in which the machine is idle and vice versa.Figure (1) presents the representationdiagram of a Markov chain and the probability of transition from one state to another.

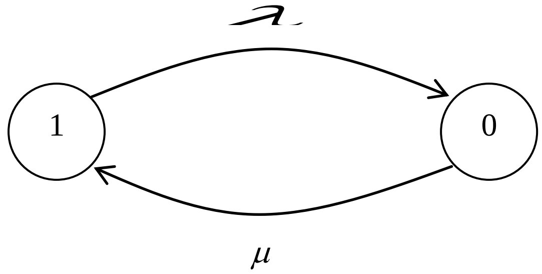

Figure 1*:* Markov Chain Model diagram for a machine.

From above figure, (λ) represent failure rate and (μ) represent repair rate. Transition from state 1 to state 0 means that the machine failsand transition from state 0 to state 1 means that the machine is repaired. The equation for transition from one state to another for any given state will be (Dhillon, 2002, p.49):

$$\frac{dp_k}{dt} = \sum \lambda_{i},_{k} p_i + \sum \mu_{n},_{k} p_n - \left(\sum \mu_{k},_{j} + \sum \lambda_{k},_{n} p_n\right) \text{……………………….(3)}$$

According to the probabilities of transition for any given state (equation no. 2). The probabilities of states system ($P_0, P_1, P_2, P_3)$ .it will be:

$$(\lambda_1 + \lambda_2)P_0 = \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 \text{……………(4)}$$

$$(\lambda_2 + \mu_1)P_1 = \lambda_1 P_0 \text{………………………..….(5)}$$

$$(\lambda_1 + \mu_2)P_2 = \lambda_2 P_0 \text{………………………(6)}$$

$$(\mu_3 P_3) = \lambda_2 P_1 + \lambda_1 P_2 \text{………………………(7)}$$

$$P_0 + P_1 + P_2 + P_3 = 1 \text{……………………..…(8)}$$

Noting that equation (8) represents the initial condition for system probabilities ($\sum_{j=0}^{3} Pj = 1$) .Substituting equation (7) into equation (4), we get:

$$(\lambda_1 + \lambda_2)P_0 = (\lambda_2 + \mu_1)P_1 + (\lambda_1 + \mu_2)P_2 \text{………………(9)}$$

Form equation no.(5) and no.(6), we get:

$$P_1 = \frac{\lambda_1}{\lambda_2 + \mu_1} P_0 \text{……………………(10)}$$

Probability of machine no.(1)non-working -machine no.(2) working

$$P_2 = \frac{\lambda_2}{\lambda_1 + \mu_2} P_0 \text{……………………(11)}$$

Probability of machine no.(2)non-working -machine no.(1) working

And from equation (7) the initial condition (where $P_3 = 0$),So the initial condition equation will become:

$$P_0 + P_1 + P_2 = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Substituting P_1, P_2 in equation no.(10) and no.(11) in equation no.(12),we get:

$$P_0 = \frac{1}{1 + \frac{\lambda_1}{\lambda_2 + \mu_1} + \frac{\lambda_2}{\lambda_1 + \mu_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Probability of all machines areunworkable.

Either the possibility of the total failure rate is represented by the system transition to state (3), when both machines are non-working.According to the "probabilities of entry and exit into state (3), the total failure rate of the system will be:

$$\lambda sys = \lambda_2 P_1 + \lambda_1 P_2 \lambda_2 P_1 + \lambda_1 P_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Substituting values of P_1, P_2 in the probability of total system failure rate will be:

$$\lambda sys = \lambda_2 P_1 + \lambda_1 P_2 = \frac{\lambda_2 \frac{\lambda_1}{\lambda_2 + \mu_1} + \lambda_1 \frac{\lambda_2}{\lambda_1 + \mu_2}}{1 + \frac{\lambda_1}{\lambda_2 + \mu_1} + \frac{\lambda_2}{\lambda_1 + \mu_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(15).$$

IV. Application:

The system under study consists of two machines (Mill troup) related to mill suger cane andmachine (Boiler)related to boil cane juice.The system performs the required function if both machines are in operating condition or that one of them is validfor work because one of them has a fault that does not affect the function of the second machine, it is possible to continue operating until the repair of the faulty machine is completed.The probabilities of the system will be one of the following four states:
State (0): Both machines working.
State (1): Machine (Mill troup) working -machine (Boiler) non-working. .
State (2): Machine (Boiler) working -machines (Mill troup) non-working.
State (3): Both machines non – working.

4.1Estimating failure rate and repairs.

For the purpose of applying the Markov chain model in maintenance, data for number of failures and time repair during 12 consecutive months for the year (2019) for machines depended on mechanical faults. The tabular method was used to calculate the density function and the Reliability function (Adolfo,2007,p.51).The failure rate and repair were calculated for each machine as follows.

4.1.1Results of machine (Mill troup):

Table1
Calculate failure rate and repair rate for machine (Mill troup)

Month	Failure no	Repair time	PDF $\hat{f}(t)$	Reliability function $\hat{R}(t)$	Failure rate $\hat{\lambda} = \frac{f(t)}{R(t-1)}$
1	3	2.1	0.05	0.95	0.05
2	6	4.8	0.10	0.85	0.11
3	9	5.7	0.15	0.70	0.18
4	6	6.3	0.10	0.61	0.14
5	9	8.5	0.21	0.39	0.34
6	6	3.9	0.10	0.30	0.26
7	2	3.5	0.03	0.26	0.10

8	2	2.3	0.03	0.23	0.12
9	5	5.5	0.08	0.15	0.35
10	1	5.8	0.02	0.13	0.13
11	2	1.3	0.03	0.11	0.23
12	6	3.2	0.10	0.01	1
Total	**61**	**52.9**			**3.01**

From above table :

1.Failure rate for machine (Mill troup):

$$\lambda_1 = \frac{\sum_{i=1}^{12} \hat{\lambda}_i}{12} = \frac{3.01}{12} = 0.2508$$

2.Repairrate for machine (Mill troup):

$$\mu_1 = \frac{\text{Repair time}}{\text{Failure no}} = \frac{52.9}{61} = 0.8721$$

3.The mean time to failure:

$$\text{MTTF} = \frac{1}{\lambda_1} = \frac{1}{0.2508} = 3.9872 \approx 4 \text{ month}$$

3.The mean downtime:

$$\text{MDF} = \frac{1}{\mu_1} = \frac{1}{0.8721} = 1.1467 \approx 1 \text{ hour}$$

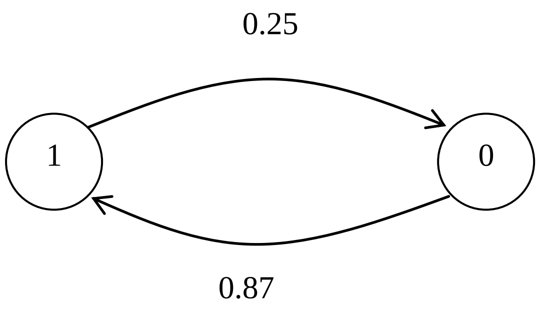

Figure 2: Markov Chain Model diagram for a machine (Mill troup).

From table. no (1) andfigure. no (2), the number of failures ofmachine(Mill troup) is (61) failure with failure rate (0.25) and repairrate (0.87). The mean time to failureis (4) month and mean downtime (1) hour.

4.1.2 Results of machine (Boiler):

Table 2
Calculate failure rate and repair rate for machine (Boiler)

Month	Failure no	Repair time	PDF $\hat{f}(t)$	Reliability function $\hat{R}(t)$	Failure rate $\hat{\lambda} = \frac{f(t)}{R(t-1)}$
1	2	2.8	0.04	0.96	0.04
2	3	4.2	0.06	0.90	0.06
3	6	9.7	0.12	0.78	0.13
4	5	4.8	0.10	0.68	0.13
5	4	5.1	0.08	0.61	0.12
6	5	3.9	0.10	0.51	0.16
7	3	1.8	0.06	0.45	0.12
8	2	1.2	0.04	0.41	0.09
9	3	2.8	0.06	0.35	0.15
10	4	1.8	0.08	0.27	0.23
11	8	7.8	0.16	0.12	0.59
12	6	2.9	0.12	0.00	1

Total	**51**	**48.8**			**2.82**

From above table :

1.Failure rate for machine (Boiler):

$$\hat{\lambda}_2 = \frac{\sum_{i=1}^{12} \hat{\lambda}_i}{12} = \frac{2.82}{12} = 0.2350$$

2.Repairrate for machine (Boiler):

$$\hat{\mu}_2 = \frac{\text{Repair time}}{\text{Failure no}} = \frac{48.8}{51} = 0.9687$$

3.The mean time to failure:

$$\text{MTTF} = \frac{1}{\lambda_2} = \frac{1}{0.2350} = 4.2553 \approx 4 \text{ month}$$

4.The mean downtime:

$$\text{MDF} = \frac{1}{\mu_2} = \frac{1}{0.9687} = 1.0323 \approx 1 \text{ hour}$$

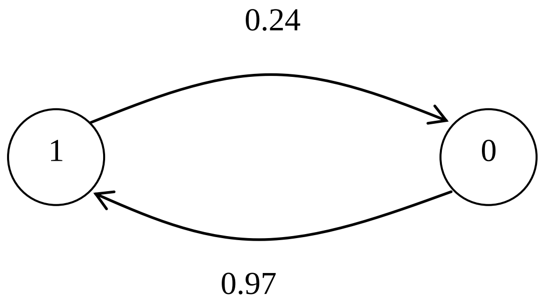

Figure 3: Markov Chain Model diagram for a machine (Boiler).

From table. no (1) andfigure. no (3), the Number of failures ofmachine(Boiler) is (51) failurewith failure rate (0.24) and repairrate (0.96).The mean time to failureis (4) month and mean downtime (1) hour.The four states of system according to the failure rate and repairrate in following figure:

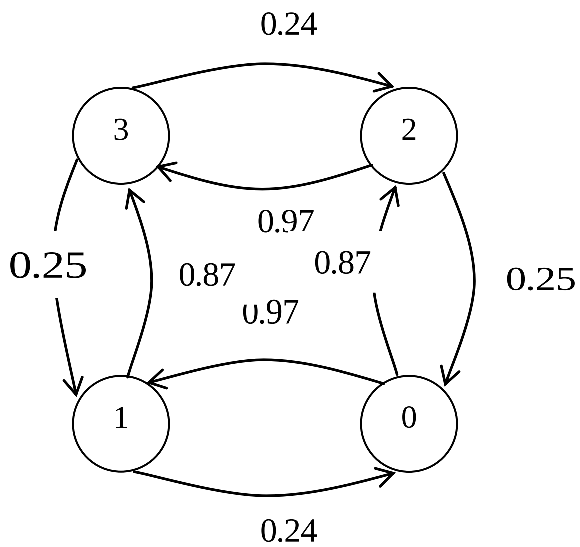

Figure 4: Markov Chain Model diagram for two machines.

From above figure ,It is clear that The failure rate of failures ofmachine(Mill troup) is greater than the failure rate of the machine(Boiler).That explains length of the repair time of machine (Mill troup).

4.2 Estimate Markov chain:
1**.State (0):** Both machines are working.

$$P_0 = \frac{1}{1+\frac{\lambda_1}{\lambda_2+\mu_1}+\frac{\lambda_2}{\lambda_1+\mu_2}} = \frac{1}{1+\frac{0.2508}{0.2350+0.8721}+\frac{0.2350}{0.2508+0.9687}} = 0.7046$$

2**. State (1):** Machine (Mill troup) down -machine (Boiler) up. .

$$P_1 = \frac{\lambda_1}{\lambda_2+\mu_1}P_0 = \frac{0.2508}{0.2350+0.8721}0.7046 = 0.1596$$

3. State (2): Machine (Boiler) up- machines (Mill troup**)** down**.**

$$P_2 = \frac{\lambda_2}{\lambda_1+\mu_2}P_0 = \frac{0.2350}{0.2508+0.9687}0.7046 = 0.1358$$

The initial condition equation $P_0 + P_1 + P_2 = 1$
as:
$0.7046+0.1596+0.1358 = 1$
Overall failure rate of machines:

$$\lambda sys = \lambda_2 P_1 + \lambda_1 P_2 = (0.2350)(0.1596) + (0.2508)(0.1358) = (0.0375) + (0.0340)$$
$= 0.0715 \approx 0$

Through above results:
- Probability of both machines are working is (0.7046).that means %70of the available operating time of the both machine are in working condition.
- Probability of Machine (Mill troup) non- working -machine (Boiler) workingis (0.1596).that means %16of the available operating time machine (Mill troup) non- working and machine (Boiler) working.
- Probability of Machine (Boiler) working - machines (Mill troup**)** non- workingis (0.1358).that means %14of the available operating time machine (Boiler) working and machine (Mill troup) non- working.
- The probability of the overall failure rate of the machines (0.0715) is negligible probability for the machines which isa good indicator as it is unlikely that both machines will fail at the same time. That means a maintenance work is taking place immediately for the machine that suffers a malfunction.

Table 3
Compare between machine (Mill troup) and machine (Boiler) in rate failure and repair

Rate	Type of machine	
	Mill troup	Boiler
Failure rate (λ)	0.2508	0.2350
Repairrate (μ)	0.8721	0.9687
The mean time to failure (MTTF)	3.9872 $\approx$ 4 month	1.2553 $\approx$ 4 month
The mean downtime (MDF)	1.1467 $\approx$ 1 hours	1.0323 $\approx$ 1 hours

From the above table : The failure probability for both machines is close, but the failure probability of machines (Mill troup) (0.2508**)** is greater than the failure probability of machine (Boiler) (0.2350).The mean time to failure and The mean downtime of two machines are equal.

4.2.1 The steady-state probability of Machines:
1**. State (0):**

$$P_0 = \frac{\lambda_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{(0.2508)(0.2350)}{(0.2508+0.8721)(0.2350+0.9687)} = \frac{0.0589}{1.3516} = 0.0436 \text{ year}$$

Mean hours in state (0) per year: 0.0436*8760 =381.934 ≈ 382 hour/year

In the long run the machines will stay in state (0) approximately 382 hours per year

2. State (1):

$$P_1 = \frac{\lambda_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} = \frac{(0.2508)(0.9687)}{(0.2508 + 0.8721)(0.2350 + 0.9687)} = \frac{0.2429}{1.3516} = 0.1797$$

Mean hours in state (1) per year: 0.1797*8760 =1574.172 ≈ 1574 hour/year

In the long run the machines will stay in state (1) approximately 1574 hours per year.

4**. State (2):**

$$P_2 = \frac{\lambda_2 \mu_1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} = \frac{(0.2350)(0.8721)}{(0.2508 + 0.8721)(0.2350 + 0.9687)} = \frac{0.2049}{1.3516} = 0.1516$$

Mean hours in state (2) per year: 0.1516*8760 =1328.016 ≈ 1328 hour/year

In the long run the machines will stay in state (2) approximately 1328 hours per year.

3. State (3):

$$P_2 = \frac{\mu_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} = \frac{(0.2350)(0.8721)}{(0.2508 + 0.8721)(0.2350 + 0.9687)} = \frac{0.8448}{1.3516} = 0.6250$$

Mean hours in state (3) per year: 0.6250*8760 =5475.000 ≈ 5475 hour/year

In the long run the machines will stay in state (3) approximately 5475 hours per year.

V. Conclusion:

From the reality of the Markov chains machines conducted for the failure time of two machines in Asalaya Sugar Company, it became clear that: the probability of both machines in working condition is high, which indicates the efficiency and ability of the maintenance unit in the factory by placing the machines in the operating state. thatpromotes increased production in factory. The ability of the Markov chains model to assess the operating condition of machines and their maintenance needs. It became clear from the results of the calculated failure rate and repair, probability of machines (Mill troup) has more failure which requiresmore effort in maintenance than machine (Boiler**)**.The mean amount of time the machine operates to failure is 4 hours for both machines and the mean time to return a non-working machine to its working condition is 1 hour,which indicates the efficiency of the factory maintenance unit. Thestudy recommends:improving the operational efficiency of the machines through total maintenance based on the results obtained by the Markov Chain, it provides a more accurate measure of the operational condition of machines.

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