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Investigating the performance of some parametric survival models with different censoring proportions

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Abstract:
The study used simulated datato investigate the performance of parametric survival models of Weibull, lognormal and logistic distributions in estimating the parameters with different random censoring percentages.Data of 300 observations were used for each of the distributionswith different censoring proportions, of 5%, 10%, 20%, 25%, and 50%. The parameters were evaluated and compared with the postulated parameters used in data generation. All three distributions performed well, where Weibull distribution slightly underestimated theparameters at 50% censored of both scale and shape parameter, showed a better estimate of the parameters at 25%, 20%, 10% and 5% censored. Lognormal distribution overestimated at all the five censoring percentages. While logistic distribution overestimatedat 50%, 25%, 20% and slightly gave a better estimates at 10% and 5% censoring. In general this showed that Weibull distribution performed better than lognormal and logistic distribution. That is the estimated parameters of Weibull distribution at 50%, 25%, and 20% censoring are more close to the postulated parameters used in the data generation, then lognormal and logistic distribution. But Logistic distribution showed a slight better performance at 10% and 5% censored observation. The consistency of distributions in estimating the parameters at different censoring proportionswas investigated by repeating the process 300 times. The MSE and RMSE were employed to assess the constancy of the models in estimating the parameters. TheWeibull performed better in all the five censoring percentages, and showed consistency in estimating the parameters. Hence Weibull distribution better performance than lognormal and logistic distribution at 50%, 25%, 20%, and 10%, censored, with lognormal showing a slight better performer at 5% censored.

Keywords: *Weibull, Lognormal,Logistic, Likelihood, Random censoring and Survival analysis;*

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I. Introduction

Survival analysis is a collection of statistical procedures which accommodate time-to-event censored data. Prior to these new procedures, incomplete data were treated as missing data and are often omitted from the analysis[1]This resulted in the loss of partial information obtained and introducing serious systematic error (bias) in estimated quantities. This, of course, lowers the efficacy of the analysis. It should be noted that survival analysis is employed in a variety of different fields, which include medicine and some other fields [2]. For example, survival analysis can be very useful to determine the “lifespan” of a product or piece of equipment. While this is more commonly referred to as reliability analysis, it is in essence the same as survival analysis. Survival analysis techniques can also be used for subjects that are not human. For example, it can be used in a variety of biological studies to determine the lifespan of different animals, or how long plants survive when living in different conditions [3].

The main variable of interest in survival analysis as stated early is time – to – event censored data and there are different types of censoring in survival analysis, such as: right censoring, interval censoring and left censoring. Random right censoring arises simply because some individuals are still surviving at the time that the study is terminated and the analysis is done [4][17]. That is a patient survived at least longer than the follow up time, but we are not sure how long. Right hand censoring is the common type of censoring, and they are three main reasons for this type of censoring to occur: firstly; the patient is still alive at the end of the study secondly; the patient withdraws from the study and thirdly; the researcher loses track of the patient[5].

The most common of these reasons is the first one, that a patient is still alive at the end of the study. So, at the end of the study, the researcher can only report that a patient lived at minimum, the length of time the patient was in the study, and after that, we don’t know what happened. Because right censoring is so common, many survival analysis techniques are able to account for this in their predictions [6].

Some researchers such as [3] and [7] have discussed and looked at the problem of censored data using non parametric and parametric mixture models respectively and it was observed that it overestimates at high levels of censoring, and underestimates at lower levels of censoring. But one of the biggest issues with survival analysis over the years is still the problem of censoring. [8] Censoring is the issue of incomplete data or lost information. The nonparametric methods are frequently used to analyze survival data. Pure classical parametric survival models are very powerful methods in survival analysis; they perform better than the nonparametric methods when the chosen distribution fit the data properly [9].

Hence this paper investigates how some parametric survival models performances at different censoring proportion, and it is arrange in the following order. Section two; outlines some important functions in survival analysis and properties of Weibull, lognormal and logistic distribution are discussed. Section three; highlights the method and procedure used to simulate and analysis the survival data at different censoring proportion of the propose survival models. Section four;discussthe sample size and censored simulation Procedure for the models. Section five; it analysis the data and estimate the parameters of the proposed model and compare the different censoring proportions. Section six is used for summary and conclusion.

II. Survival Functions and Probability Distributions

From the early definition of survival analysis, the survival analyst makes statements about the survival distribution of the failure times. This distribution allows questions about such quantities as; survivability, expected life time, and mean time to failure to be answered.

Let T be the elapsed time until the occurrence of a specified event. The event may be death, occurrence of a disease, disappearance of a disease, appearance of a tumor, etc. The variable T may be specified by distribution function, probability distribution, survival function and hazard function[8], [9]. Once one of these functions has been specified, the others may be derived using the mathematical relationships presented.

Cumulative distribution function, denoted by F(t) is the probability that an individual survives until time t.

$F(t) = \int_0^t f(x)dx$

Probability density function, denoted by f(t) is the probability that an event occurs at time t. Survival function, denoted S(t)is the probability that an individual survives beyond time t. It may be estimated using the nonparametric Kaplan-Meier curve or one of the parametric distribution functions.

$S(t) = \int_t^\infty f(x)dx = 1 - F(t)$

Hazard function, denoted by h(t) is the probability that an individual at time t experiences the event in the next instant. It is a fundamental quantity in survival analysis [9]. It is also known as the conditional failure rate in reliability, the force of mortality in demography, the intensity function in stochastic processes, the age specific failure rate in epidemiology, and the inverse of Mill's ratio in economics. The empirical hazard rate may be used to identify the appropriate probability distribution of a particular mechanism, since each distribution has a different hazard function. Some distributions have a hazard function that decreases with time, others have a hazard function that increases with time, some are constant, and some exhibit all three behaviors at different points in time. $H(t) = \frac{f(t)}{S(t)}$[10],[11].

Weibull Distribution

The Weibull distribution is named after Professor WaloddiWeibull whose papers led to the wide use of the distribution. He demonstrated that the Weibull distribution fit many different datasets and gave good results, even for small samples. The Weibull distribution has found wide use in industrial fields where it is used to model time to failure data[11].

The two parameter Weibull distribution is indexed by a shape (γ) , and a scale (λ) parameter. Using these symbols, the two parameter Weibull density function may be written as

$f(t/, \gamma, \lambda) = \frac{\gamma}{\lambda}(\frac{t}{\lambda})^{\gamma-1} e^{-(\frac{t}{\lambda})^{\gamma}}$

The symbol t represents the random variable (usually elapsed time).The shape (or power) parameter controls the overall shape of the density function. Typically, this value ranges between 0.5 and 8.0. One of the reasons for the popularity of the Weibull distribution is that it includes other useful distributions as special cases or close approximations. For example, if $\gamma = 1$ The Weibull distribution is identical to the exponential distribution, $\gamma = 2$ The Weibull distribution is identical to the Rayleigh distribution, $\gamma = 2.5$ The Weibull distribution approximates the lognormal distribution, $\gamma = 3.6$ The Weibull distribution approximates the normal distribution [11].

The scale parameter only changes the scale of the density function along the time axis. Hence, a change in this parameter has the same effect on the distribution as a change in the scale of time—for example, from days to months or from hours to days. However, it does not change the actual shape of the distribution. The parameter λ is known as the characteristic life [11].

The cumulative distribution function for the two parameter Weibull distribution is F(t)=$e^{-(\lambda t)^{\gamma}}$. The survivorship function, S(t), gives the probability of surviving beyond time t. For the Weibull distribution, the survival function is
S(t)=$e^{-(t/\lambda)^{\gamma}}$. The survival function is defined to be one minus the cumulative distribution function. That is,S(t)=1- F(t). The hazard function represents the instantaneous failure rate. For this distribution, the hazard function is H(t)=$\gamma/\lambda(t/\lambda)^{\gamma-1}$
Depending on the values of the distribution's parameters, the Weibull's hazard function can be decreasing (when γ<1), constant (when γ=1), or increasing (when γ>1) over time.

Lognormal Distribution
In its simplest form the lognormal distribution can be defined as the distribution of a variable whose logarithm follows the normal distribution [9].
The probability density function is defined as f(t) =$\frac{1}{t\sigma\sqrt{2\pi}}\exp[-\frac{1}{2\sigma^2}(logt-\mu)^2]$.

The shape parameter of the lognormal distribution of t is the standard deviation in the normal distribution of ln(tσ) or log(t-σ). That is, the scale parameter of the normal distribution is the shahpe parameter of the lognormal distribution[11]. The scale parameter of the lognormal distribution t is the mean in the normal distribution.The survivorship function, S(t), gives the probability of surviving beyond time t. For the Lognormal distribution, the survival function isS(t)= $\frac{1}{\sigma\sqrt{2\pi}}\int_t^{\infty}\frac{1}{x}\exp[-\frac{1}{2\sigma^2}(logt-\mu)^2]$ dx. Where G(y) is the cumulative distribution function of a standard normal variableG(y)=$\frac{1}{\sqrt{2\pi}}\int_0^{y}e^{u^2/2}$du.The lognormal distribution is specified completely by the two parameters μ andσ^2. Time T cannot assume zero values since (log T) is not defined for T=0
The hazard function is obtain from (2.4.0) and (2.4.1), and has the form

$$h(t)=\frac{\frac{1}{t\sigma\sqrt{2\pi}}exp\frac{[-\frac{1}{2\sigma^2}(logt-\mu)^2]}{2\sigma^2}}{1-G([-\frac{1}{2\sigma^2}(logt-\mu)/\sigma])\quad]}=\frac{f(t)}{S(t)}$$

The Logistic Distribution
The logistic distribution is the location (shape) -scale family associated with the standard logistic distribution. Thus, if z has the standard logistic distribution, then for any α∈ℝ and any γ> 0,[10]
t=α +γz has the logistic distribution with location (shape) parameter α and scale parameterγ.
Let t be a random variable with probability density function

$$f(t)=\frac{e^{-z}}{(1+e-t)2}, =\frac{\exp(-\frac{t-\alpha}{\gamma})}{\gamma(1+\exp(-\frac{t-\alpha}{\gamma})2}\ ,\quad t\in R,$$

F(t) is the cumulative probability of failure from time zero till time t. Very handy when estimating the proportion of units that will fail over a warranty period, for example.

$$F(t) = \exp\left(\frac{\frac{t-\alpha}{\gamma}}{\left(1+exp\left(\frac{t-\alpha}{\gamma}\right)\right)}\right),\quad t\in\mathbb{R}$$

The survival S(t) is the chance of survival from time zero till time t. Instead of looking for the proportion that will fail the survival function determines the proportions that are expected to survive [12].

$$S(t) =\frac{1}{1+\exp(\frac{t-\alpha}{\gamma})}$$

This is the instantaneous probability of failure per unit time.

$$h(t)=\frac{1}{\gamma(1+\exp\ -\left(\frac{t-\alpha}{\gamma}\right))} = \frac{f(t)}{S(t)}$$

The logistic distribution is characterized by two parameters α, and γ[13].

III. Methodology

The study simulated samples ofsize 300 observations at different censoring proportion of each of the Weibull, Lognormal and Logistic distributions. The focus is to investigate how the survival models perform as we loss the amount of information from the survival data. Different censoring proportion of the parametric survival models is tested. We compare and test the consistency of each of the parametric survival models at different percentage censored. All these investigations are done through data simulation which is a fictitious representation of reality and the method uses repetition, each sample three hundred times to determine the consistency and stability of the models in estimating the parameters. The MSE and RMSE were used to assess the distributions

IV. Sample size and censored simulation Procedure for the models

The study adopt model based simulation method or approach for each of Weibull, lognormal and logistic distributions, in which thedata were generated and the model postulated parameters are specified, along with a sample size[14],[15],[16]. The Data generated were used to estimate the parameters for each distribution at different censoring percentage. We compare the performance of Weibull and lognormal, Weibull and logistics, lognormal and logistic distributions at different censoring proportion, of 5%, 10%, 20%, 25%, and 50% censored by looking at how close estimated parameters are to the postulated parameter. This process is repeated 300 times, in order to test the consistency of Weibull, lognormal and logistics distribution in the parameter estimation , and records are kept for each of the propose models, allowing us to examine how the statistical procedure performs in estimating the true parameter values. MSE and RMSE are obtained by taking the average of the estimated parameters when the sample size of three hundred (300), is repeated three hundred times (300).

V. Data analysis

The performance of Weibull, lognormal and logistic distributions was investigated through simulated data generated from the survival models of the distributions. The censoring percentages of 5%, 10%, 20%, 25%, and 50% censored are investigated. The survival data of sample size 300 observations were generated and each of the censoring proportions was determined through model based simulation method in which the data generation model and the model parameters are specified, along with a sample size. The postulated parameter values of 2 and 3 for the scale and shape parameter, respectively, wereused for each of the percentage censored of all the three distributions.

Table 1 displays the result of the estimated and postulated parameters values of the models for the five different censoring percentages in descending order, it compares the performance of distributions at different censoring proportion, of 50%, 25%, 20%, 10%, and 5% censored.

Table 1: Estimated Parameters of the Simulated Data

Weibull distribution of sample size 300 observation at different censoring proportion										
Percentage	50%		25%		20%		10%		5%	
Parameters	Λ	ɣ	Λ	ɣ	Λ	ɣ	Λ	ɣ	Λ	ɣ
Postulated	2	3	2	3	2	3	2	3	2	3
Estimated	2.49	2.82	2.16	3.21	2.13	3.23	2.06	3.40	2.03	3.41
Lognormal distribution of sample size 300 observation at different censoring proportion										
Percentage	50%		25%		20%		10%		5%	
Parameters	Λ	ɣ	Λ	ɣ	Λ	ɣ	Λ	ɣ	λ	ɣ
Postulated	2	3	2	3	2	3	2	3	2	3
Estimated	3.17	4.33	2.41	3.39	2.31	3.29	2.09	3.14	2.01	3.08
Logistic distribution of sample size 300 observation at different censoring proportion										
Percentage	50%		25%		20%		10%		5%	
Parameters	Λ	ɣ	Λ	ɣ	Λ	ɣ	Λ	ɣ	λ	ɣ
Postulated	2	3	2	3	2	3	2	3	2	3
Estimated	3.06	4.79	2.21	3.42	2.10	3.28	1.93	3.05	1.84	2.96

The table 1 showed the estimated parameters of all the three distribution at glances with a constant postulated value of 2 and 3for scale and shape parameter respectively. Weibull distribution shows a slight good estimation of parameters at 50% censored of both scale and shape parameter, and a better estimation of parameters at 25%, 20%, 10% and 5% censored. Lognormal distribution overestimates at high percentage censored show a better estimate as we move to lower censored percentages. While logistic distribution overestimates at 50%, 25%, 20% and slightlygive a better estimates at 10% and 5% censoring. This showed that estimated parameters of Weibull distribution performances better than lognormal and logistic distribution in general. That is the estimated parameters of Weibull distribution at 50%, 25%, and 20% censored are more close to the postulated parameters used in the data generation, then lognormal and logistic distribution. Also Logistic distribution showed a good performance at 10% and 5% censored observation. At 50%, 25% and 20% censoredWeibull distribution performance better, then lognormal and logistic distribution. At 10% and 5% censored lognormal and logistic distribution show more better performance thenWeibull distribution.

The Repeated Simulation of 300 Observations

The simulation of Weibull, lognormal and logistic distributions at different censoring proportion of 50%, 25%, 20%, 10%, and 5% censored observations were repeated 300 times to check the consistency, stability and reliability of the survival models. The averages, the mean square errors and root mean square errors of the estimated parameters were determined and displayed for Weibul, log normal and logistic distributions in table 3,4, and 5 respectively..

Table 3: Replication of Weibull distribution

Weibull distribution of repeated simulation of set of 300 observation at different censoring proportion										
Percentage	50%		25%		20%		10%		5%	
Parameters	Λ	ɣ	Λ	ɣ	Λ	ɣ	Λ	ɣ	λ	ɣ
Postulated	2	3	2	3	2	3	2	3	2	3
Estimated	2.57	2.48	2.18	2.79	2.14	2.83	2.08	2.91	2.03	2.97
MSE	0.3343	0.2877	0.0369	0.0639	0.0226	0.0457	0.0076	0.0268	0.0024	0.0196
RMSE	0.5782	0.5364	0.1920	0.2529	0.1503	0.2138	0.0870	0.1638	0.0489	0.1401

The averages of the repeated simulation of the estimated parameters of Weibull distribution of five different censoring proportions are closed to the postulated parameters of the model with **MSE** and **RMSE** relatively small as we go down from the higher to the lower percentage censored. This suggests that Weibulldistribution performances better as we move from the higher to lower percentage censored.

Table 4: replication of lognormal distribution

Lognormal distribution of repeated simulation of set of 300 observation at different censoring proportion										
Percentage	50%		25%		20%		10%		5%	
Parameters	Λ	ɣ	Λ	ɣ	Λ	ɣ	Λ	ɣ	λ	ɣ
Postulated	2	3	2	3	2	3	2	3	2	3
Estimated	3.17	4.33	2.41	3.39	2.31	3.29	2.09	3.14	2.01	3.08
MSE	1.8068	2.0653	0.2210	0.1438	0.1357	0.0837	0.0317	0.0226	0.0204	0.0175
RMSE	1.3442	1.4371	0.4693	0.3793	0.3683	0.2893	0.1781	0.1505	0.1428	0.1322

The estimated parameters of lognormal distribution of five different censoring proportions showedan overestimation at 50% censored proportion and slight better overestimating at 25%, 20%, 10% and 5% censored. Also the **MSE** and **RMSE** are greater than one (>1) at 50% censored of both parameters, and relatively small that is, less than one (<1) as we go down from 25% to 5% censored. This suggests that lognormal distribution showed good performances as we move from 25% to the lower percentage censored.

Table 5: replication of logistic distribution

Logistic distribution of repeated simulation of set of 300 observation at different censoring proportion										
Percentage	50%		25%		20%		10%		5%	
Parameters	Λ	ɣ	Λ	ɣ	Λ	ɣ	Λ	ɣ	λ	ɣ
Postulated	2	3	2	3	2	3	2	3	2	3
Estimated	3.06	4.79	2.21	3.42	2.10	3.28	1.93	3.05	1.84	2.96
MSE	2.0738	4.6715	0.3636	0.5843	0.1689	0.2586	0.0441	0.0794	0.0217	0.0493
RMSE	1.4401	2.1613	0.6030	0.7644	0.4109	0.5086	0.2121	0.2818	0.1474	0.2220

The mean averages of the estimated replicated parameters oflogistic distribution of five different censoring proportions showed overestimating at 50% and slightly better estimating at 25%, 20%, 10% and 5% censoring, i.ethe estimated parameters are more closed to the postulated parameters at 25%, 20%,10% and 5% censored of the model. Hence the **MSE** and **RMSE** are greater than one (>1) at 50% and relatively small as we go down from 25% to 5% censored. This suggests that logistic distribution performances well as we move from 25% to 5% censored.

The table 3, 4 and 5 of repeated simulation observations at different censoring proportion with constant postulated values of 2 and 3 for scale shape parameter respectively. It showed that Weibull performances better at all the five censoring percentages. Also, MSE and RMSE showed the consistency in the performance of Weibull distribution as a better performer than lognormal and logistic distribution at 50%, 25%, 20%, and 10%, censored, with lognormal showing a better performer at 5% censored.

VI. Conclusion

The simulation of Weibull, lognormal and logistic distributions at different censoring proportion of 50%, 25%, 20%, 10%, and 5% censored observations were successfully computed. The performances of the distributions were compared, by looking at how close estimated parameter is to the postulated parameter. All three distributions perform well.Weibull distribution, showed a slight underestimation of parameters at 50% censored of both scale and shape parameter, and better estimate at 25%, 20%, 10% and 5% censored. Lognormal distribution slightly overestimates at all the five censored percentages. While logistic distribution overestimates at 50%, 25%, 20% and slightly give a better estimates at 10% and 5% censoring. The process was repeated 300 times, and it shows a consistency in estimating the parameters, of the survival models. Weibull distribution showed a better performance then lognormal and logistic distribution at50%25%, 20%, while at 10% and 5% censored lognormal and logistic distribution show slightly better performance thanWeibull distribution. The averages of the repeated simulation of the estimated parameters at different censoring proportion with constant postulated values of 2 and 3 for scale and shape parameterwere also obtained. And the services of MSE and RMSE were employed, which show consistency, and stability of Weibull distribution as a better performer than lognormal and logistic distribution at 50%, 25%, 20%, and 10%, censored, with lognormal showing a better performer at 5% censored.

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