IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 6 Ser. I (Nov. – Dec. 2020), PP 23-32
www.iosrjournals.org

Inventory Model for Constant Demand and Two-Parameter Weibull Deterrioration Having Permissible Delayed Payments and Salvage Value under Learning Effect

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Abstract: *Inventories are unrefined supplies, work-in-process commodities and finally completed merchandise that are measured to be the segment of business's material goods that are ready or will be equipped for trade. Formulation of an appropriate inventory model is one of the foremost concerns for a business organization. In the present work, a deteriorating inventory model for constant demand having shortages is developed where shortages are partially backlogged. Two parameter Weibull distribution is assumed as deterioration rate. Permissible delay in payments is considered in two different cases with replenishment cycle. Supplier and retailer both got some profit during the permissible period. The concept of learning and salvage value is also highlighted in the present study. A simple method is used to find out the optimal solution of the total cost. Finally, a numerical example is provided and sensitivity examination of the optimal solution with respect to the parameters is carried out.*

Keywords: *Inventory; shortages; Weibull deterioration; constant demand rate; salvage value; partial backlogging; learning effect; permissible delayed payments.*

Date of Submission: 31-10-2020 Date of Acceptance: 12-11-2020

I. Introduction

Any trader can achieve success and could satisfy the demands of customers only by having sufficient stocks in the trade firm. Therefore, it is necessary to balance the maintenance of the inventory with the various cost and then to find an optimal level where the cost is minimum. To manage the inventory costs mathematical models can provide the best solution. Also, in the real life situation, deteriorating nature of the goods is also a point of major concern for maintaining inventory for the long duration. The development of deteriorating/perishable inventory models begins in 1960s.

A mathematical model with perishable goods following weibull deteriorating rate has been developed by Chakrabarty et. al.[2] Their model permits shortages and demand varies with time. Teng[15] presented a mathematical inventory model for solving the problem related to calculation of the points of replenishment and the amount which is ordered at each replenishment point. A non-linear mathematical model has been formulated by Hsu and Li[4] for studying the supply and demand interaction with consumer socioeconomic behaviour. Time-dependent consumer demand has been assumed in their formulated model. Khanra et al.[6] demonstrated an inventory model for deteriorating goods having time varying quadratic demand rate in which two different cases for permissible delayed payments are discussed and the deterioration rate is constant. An inventory model considering demand as ramp type and deterioration rate as weibull distribution has been studied by Skouri et al.[14]Their model presents two different policies for replenishment and also considers partial backlogging.

In reality, frequently some consumers are agreeable to stay until refill of the stock, particularly if the stay will be short, while some are more eager and walk off elsewhere. To replicate this trend, Chang and Dye [3] demonstrated an inventory model for unpreserved commodities. They focuses on the consequence of the rate of backlogging on the economic order amount decision in their model. An ordering policy for perishable items following three-parameter weibull rate of deterioration has been discussed by Patel and Gor[10] in which demand varies in different time interval. They considered non-instantaneous deterioration in this study. Patro et al.[11] studied both fuzzy and crisp models for deteriorating products. Also, they analyses the learning effect on the optimal solution and imperfect items are considered in this study.

Mishra[8] formulated a mathematical model for perishable goods in which salvage value is also included. Time dependent weibull's distribution is also considered in the model. Mishra and Singh[9] developed an inventory model for deteriorated goods in which carrying cost depends linearly on time and deterioration rate is taken as constant. Variable rate of backlogging is considered in this study. Jayaswal et al.[5] formulated an EOQ model having imperfect quality and perishable goods. In this model, trade-credit financing and concept of learning has also been discussed. A mathematical model for deteriorating goods has been established by Sharma et al.[12] in which two cases of partial backlogging have been discussed.

A production inventory model in which deterioration rate is assumed as time dependent and demand is taken as exponential was formulated by Kumar et. al [7]. Two periods of production are also considered in their presented model. Annudarai [1] explored a deteriorating inventory model where rate of deterioration follows exponential distribution. In this study, completely backlogged shortages are considered with permissible delayed payments. A mathematical inventory model for deteriorating goods has been derived by Singh et al.[13] Demand rate is taken as two-staged and deterministic and time-proportional deteriorating rate is also considered. Shortages are not permitted in their model.

In this paper, an attempt has been made to develop an inventory model with two parameter weibull deterioration rate by assuming constant demand rate with shortages and partial backlogging. The first case discussed in the paper deals with the scenario where the shortages are prohibited and the second case allows for the shortage. Learning effect and the concept of permissible delayed payments has also been studied in this model. Salvage value is also calculated for the considered deteriorated units. Further, numerical example and sensitivity analysis has been provided for the illustration of the developed model.

II. Assumptions And Notations

The assumptions which are applied throughout the manuscript are as follows:

Assumptions

1) Inventory stock is replenished at an infinite rate instantly.
2) Only single item is assumed in the system with infinite scheduling horizon.
3) The two parameter Weibull distribution is taken as rate of deterioration $\theta_0(t) = \alpha\beta t^{\beta-1}$.
4) The demand rate is assumed as constant $A(t) = A$.
5) Salvage value is included with deteriorated items.
6) The concept of permissible delay in payments is also considered.
7) Cost of holding, deterioration and ordering follows the learning curve.
8) For every next replenishment, backlogging rate is dependent on length of waiting line.

Notations:

$Q_0(t)$: Inventory level at time t.

n: Total number of shipments

μ: Learning coefficient

$A(n) = \left(a_0 + \frac{a_1}{n^\mu}\right)$: learning coefficient ordering cost

$H(n) = \left(h_0 + \frac{h_1}{n^\mu}\right)$: learning coefficient holding cost

$DC = \left(d_0 + \frac{d_1}{n^\mu}\right)$: learning coefficient deterioration cost

W:Maximum inventory level

S:Maximum demand backlogged.

C_s: Shortage cost per unit time.

C_0: opportunity cost

C_v: Salvage value parameter

I_e: Interest earned.

I_c: Interest charged.

M: permissible delay period

T_l: length of ordering cycle

t_d: time when inventory level comes down to zero, $0 \leq t_d < T_l$.

III. Mathematical formulation and Solution

The rate of change of inventory during positive stock period [0, td] and shortage period [td, Tl] is governed by the following differential equations:

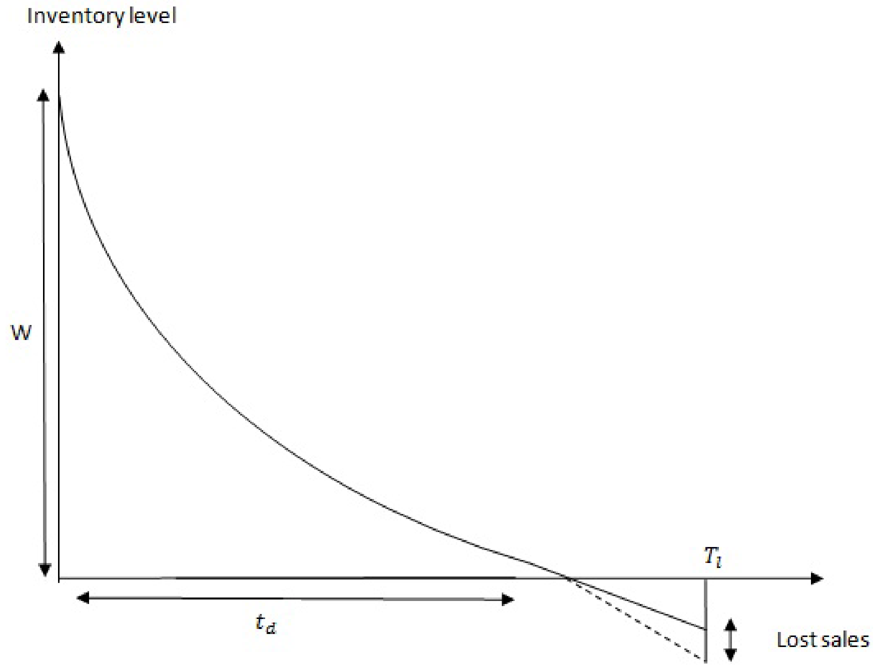

Figure 1: Graphical representation of inventory system

$$\frac{dQ_{01}(t)}{dt} + \theta_0(t)Q_{01}(t) = -A(t), \qquad 0 \leq t \leq t_d \tag{1}$$

$$\frac{dQ_{02}(t)}{dt} = \frac{-A}{1+\xi_0(T_l - t)}, \qquad t_d \leq t \leq T_l \tag{2}$$

The governing boundary conditions for the equation (1) and (2) are: $Q_{01}(t) = Q_{02}(t) = 0$ at $t_d = t$.

Case 1: Inventory level without shortages:
In this case, at initial time t = 0, the inventory level starts with zero level stock. Due to market demand and deterioration, the inventory level diminishes gradually and falls to zero at t =t_d.

Hence, the inventory level at any time during [0, t_d] is described by the differential equation-

$$\frac{dQ_{01}(t)}{dt} + \theta_0(t)Q_{01}(t) = -A(t), \qquad 0 \leq t \leq t_d \tag{3}$$

Where $\theta_0(t) = \alpha\beta t^{\beta-1}$

With boundary conditions $Q_{01}(t) = Q_{max}$ and $Q_{01}(t_d) = 0$.

Under these boundary conditions and neglecting the higher powers of ${\theta_0}^2$, the solution of equation (3) is:

$$Q_{01}(t) = A\left[(t_d - t) + \frac{\alpha t_d^{\beta+1}}{\beta+1} + \frac{\alpha\beta t^{\beta+1}}{\beta+1} - \alpha t_d t^{\beta}\right] \tag{4}$$

Maximum inventory level for each cycle is obtained by putting the boundary condition $Q_{01}(0) = Q_{max}$ in equation (4):

Therefore,

$$Q_{01}(0) = Q_{max} = A\left[t_d + \frac{\alpha t_d^{\beta+1}}{\beta+1}\right] \tag{5}$$

Case 2: Inventory level with Shortages:
In this case, we develop a model for deteriorating products when shortages are permitted and backlogged partially. Shortages start occurring in the interval [td; Tl] and the demand at any time t is partially backlogged at the fraction $\frac{1}{1+\xi_0(T_l-t)}$.

Therefore, the differential equation governing the amount backlogged is

$$\frac{dQ_{02}(t)}{dt} = \frac{-A}{1+\xi_0(T_l - t)}, \qquad t_d \le t \le T_l \tag{6}$$

with the boundary condition $Q_{02}(t_d) = 0$.

The solution of the equation (6) is:

$$Q_{02}(t) = \frac{A}{\xi_0}\left[\ln\left(1 + \xi_0(T_l - t)\right) - \ln(1 + \xi_0 - t_d))\right] \tag{7}$$

Maximum amount of demand backlogged per cycle is obtained by putting t = T_l in equation (7).
Therefore,

$$S_{max} = -Q_{02}(t) = \frac{A}{\xi_0}\ln(1 + \xi_0 - t_d)) \tag{8}$$

The total cost per replenishment cycle consists of the following components:

Ordering cost

$$OC = \left(a_0 + \frac{a_1}{n_s^{\mu}}\right) \tag{9}$$

Inventory holding cost

$$HC = H(n) = \left(h_0 + \frac{h_1}{n_s^{\mu}}\right)\int_0^{t_d} Q_{01}(t)dt$$

$$HC = \left(h_0 + \frac{h_1}{n^{\gamma}}\right) A\left[(t_d - t) + \frac{\alpha t_d^{\beta+1}}{\beta+1} + \frac{\alpha\beta t^{\beta+1}}{\beta+1} - \alpha t_d t^{\beta}\right] dt$$

$$HC = \left(h_0 + \frac{h_1}{n^{\gamma}}\right) A\left[\frac{t_d^2}{2} + \frac{\alpha t_d^{\beta+2}}{(\beta+1)(\beta+2)}\right] \tag{10}$$

Deterioration cost

$$DC = \left(d_0 + \frac{d_1}{n^{\gamma}}\right)\left[Q_{max} - \int_0^{t_d} A(t)\right] dt$$

$$DC = \left(d_0 + \frac{d_1}{n^{\gamma}}\right) A\left(\frac{\alpha t_d^{\beta+1}}{\beta+1}\right) \tag{11}$$

Shortage cost

$$SC = C_s \int_{t_d}^{T_l} Q_{02}(t)\, dt$$

$$SC = C_s A\left[\frac{T_l - t_d}{\xi_0} - \frac{1}{\xi_0^2}\ln(1 + \xi_0(T_l - t_d))\right] \tag{12}$$

Opportunity cost

$$OCLS = C_0 \int_{t_d}^{T_l} A\left(1 - \frac{1}{1 + \xi_0(T_l - \mathrm{t})}\right) dt$$

$$OCLS = C_0 A\left[T_l - t_d - \frac{1}{\xi_0}\ln(1 + \xi_0(T_l - t))\right] \quad (13)$$

Salvage value

$$SV = C_v\left(d_0 + \frac{d_1}{n^\gamma}\right)A\left(\frac{\alpha t_d^{\beta+1}}{\beta+1}\right) \quad (14)$$

Thus objective function of this system, total cost function per time unit

$$TC = \frac{1}{T}[IHC + DC + OC + OCLS + SC - SV] \quad (15)$$

Now we discuss two major cases which arise in each order cycle regarding interest charged and interest earned in detail:

Case1: $M \leq t_d$ (Payment at or before total depletion of inventory)
For this case, the credit time expires on or before the inventory depleted completely to zero. In this case, the interest earned at an annual rate I_e and the interest paid with an annual rate I_c is given by

$IE_1 = pI_eA\left(\frac{t_d^2}{2}\right)$.
$IC_1 = pI_cA(t_d - M)$.

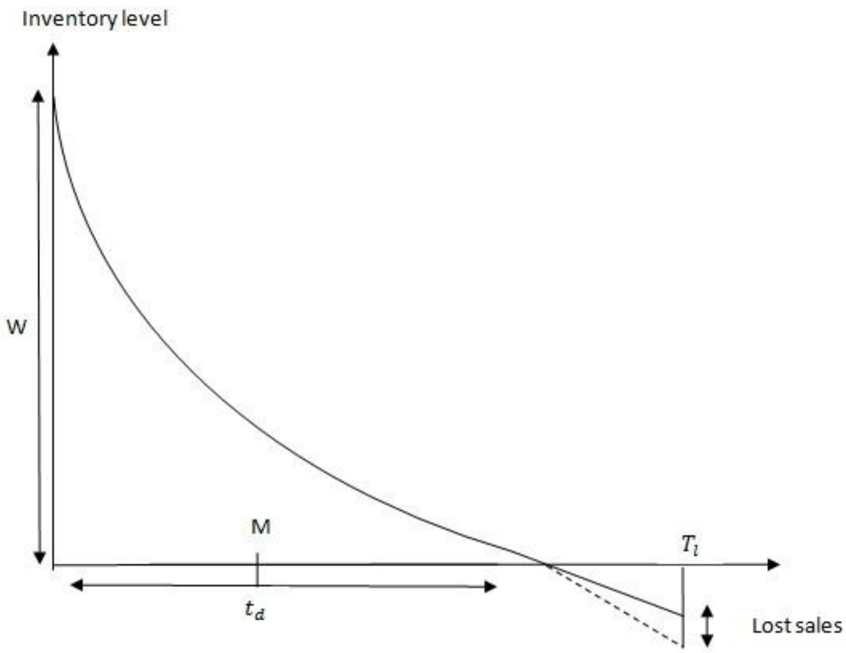

Figure 2: Inventory level for case (1)

Equation (16) gives the total average inventory cost:

$$C_M^1(t_1, T) = \frac{1}{T}[OC + HC + DC + SC + OCLS - SV + IC_1 - IE_1]$$

$$C_M^1(t_d,T_l) = \frac{1}{T_l}[\left(a_0 + \frac{a_1}{n_s^{\mu}}\right) + \left(h_0 + \frac{h_1}{n^{\gamma}}\right)A\left[\frac{t_d^2}{2} + \frac{\alpha t_d^{\beta+2}}{(\beta+1)(\beta+2)}\right] + \left(d_0 + \frac{d_1}{n^{\gamma}}\right)A\left(\frac{\alpha t_d^{\beta+1}}{\beta+1}\right)$$

$$+C_s A\left[\frac{T_l - t_d}{\xi_0} - \frac{1}{\xi_0^2}\ln(1 + \xi_0(T_l - t_d))\right] + C_0 A\left[T_l - t_d - \frac{1}{\xi_0}\ln(1 + \xi_0(\mathrm{T_l} - \mathrm{t}))\right] - C_v \left(d_0 +\right.$$

d1nγAαtdβ+1β+1 *+pIcAtd −M−pIeAtd22*

(16)

Our purpose is to obtain the minimal cost of the system. The necessary conditions for cost minimization are:

$$\frac{\partial C_M^1(t_d,T_l)}{\partial t_d} = 0 \qquad and \qquad \frac{\partial C_M^1(t_d,T_l)}{\partial T_l} = 0 \qquad (13)$$

Provided that this equation satisfy the given condition

$$\frac{\partial^2 C_M^1(t_d,T_l)}{\partial t_d^2} > 0, \qquad \frac{\partial^2 C_M^1(t_d,T_l)}{\partial T_l^2} > 0 \ \ and \left(\frac{\partial^2 C_M^1(t_d,T_l)}{\partial t_d^2}\right)\left(\frac{\partial^2 C_M^1(t_d,T_l)}{\partial T_l^2}\right) - \frac{\partial^2 C_M^1(t_d,T_l)}{\partial T \partial t_1} > 0$$

Case II. $M > t_1$ (Payment after Depletion)

For this case, paid interest is zero and earned interest is given by $IE_2 = pI_e A\left(Mt_d + \frac{t_d^2}{2}\right)$.

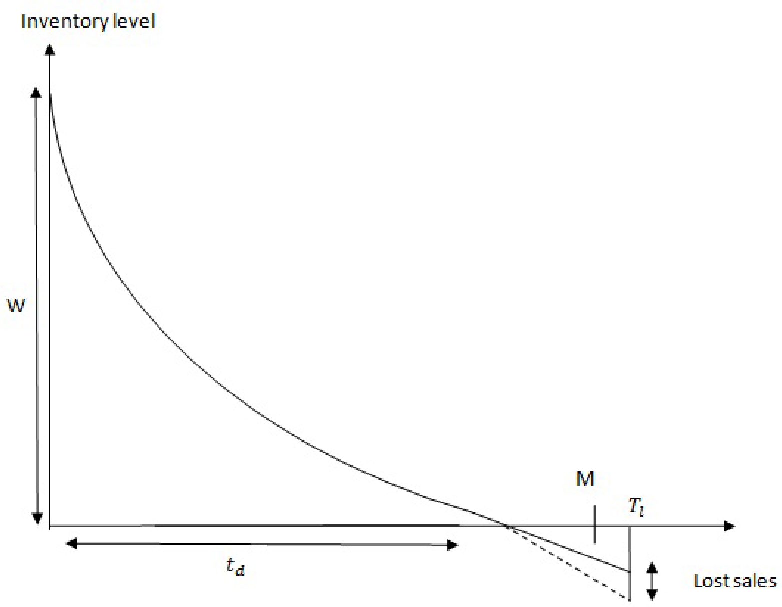

Figure 3: Graphical representation of inventory system for case 2

Equation (18) gives the total average inventory cost:

$$C_M^2(t_d,T_l) = \frac{1}{T_l}[OC + HC + DC + SC + OCLS - SV + IC_2 - IE_2]$$

$$C_M^2(t_d,T_l) = \frac{1}{T_l}[\left(a_0 + \frac{a_1}{n_s^{\mu}}\right) + \left(h_0 + \frac{h_1}{n^{\gamma}}\right)A\left[\frac{t_d^2}{2} + \frac{\alpha t_d^{\beta+2}}{(\beta+1)(\beta+2)}\right] + \left(d_0 + \frac{d_1}{n^{\gamma}}\right)A\left(\frac{\alpha t_d^{\beta+1}}{\beta+1}\right)$$

$$+C_sA\left[\frac{T_l-t_d}{\xi_0}-\frac{1}{\xi_0^2}\ln(1+\xi_0(T_l-t_d))\right]+C_0A\left[T_l-t_d-\frac{1}{\xi_0}\ln(1+\xi_0(\mathrm{T_l}-\mathrm{t}))\right]$$
$$-C_v\left(d_0+\frac{d_1}{n^\gamma}\right)A\left(\frac{\alpha t_d^{\beta+1}}{\beta+1}\right)-pI_eA\left(\frac{t_d^2}{2}\right) \tag{18}$$

Our purpose is to obtain the minimal cost of the system. The necessary conditions for cost minimization are:

$$\frac{\partial C_M^2(t_d,T_l)}{\partial t_d}=0 \qquad and \qquad \frac{\partial C_M^2(t_d,T_l)}{\partial T_l}=0 \tag{13}$$

Provided that this equation satisfy the given condition

$$\frac{\partial^2 C_M^2(t_d,T_l)}{\partial t_d^2}>0, \qquad \frac{\partial^2 C_M^2(t_d,T_l)}{\partial T_l^2}>0 \quad and \left(\frac{\partial^2 C_M^2(t_d,T_l)}{\partial t_d^2}\right)\left(\frac{\partial^2 C_M^2(t_d,T_l)}{\partial T_l^2}\right)-\frac{\partial^2 C_M^2(t_d,T_l)}{\partial T\partial t_1}>0$$

IV. Numerical Example

In this section, a numerical example is given for the illustration of the obtained results of the model.
The considered values of the parameters are given as:

A= 30, ξ_0= 10, α =0.15, β = 1.5, C_s = 5, C_0= 2, C_μ = 0.1, h_0 = 4, h_1 = 3, n = 3, μ= 2, a_0= 3, a_1= 0.2, d_0= 2, d_1= 0.1, M = 0.020, I_c= 0.18, I_e= 0.10, p = 15.

Solving the equations, we get t_1= 0.652344 unit time and T = 0.91521 unit time and TC = 57.3126.

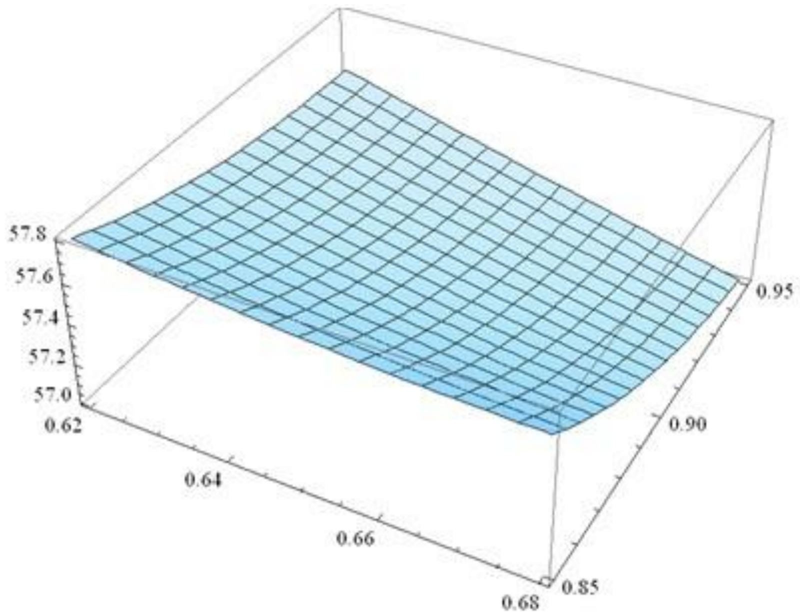

Fig 4: Convexity of Total Cost function

V. Sensitivity Analysis

The purpose of sensitivity analysis is to identify the parameters to the changes of which the solution of the model is sensitive. To study the reliability of the model, we have analysed the sensitiveness on some parameters given below:

Effect of α on t_d, T_l and Total Cost:

Parameter	t_d	T_l	Total cost (TC)
0.10	0.652344	0.91521	56.6817
0.12	0.652344	0.91521	56.9340
0.15	0.652344	0.91521	57.3126
0.18	0.652344	0.91521	57.6911
0.20	2.678440	2.74348	39.3593

Effect of β on t_d, T_l and Total Cost:

Parameter	t_d	T_l	Total cost (TC)
1.0	0.652344	0.91521	58.3259
1.2	0.652344	0.91521	57.8539
1.5	0.652344	0.91521	57.3126
1.7	0.652344	0.91521	57.0330
2.0	0.652344	0.91521	56.7014

Effect of ξ_0 on t_d, T_l and Total Cost:

Parameter	t_d	T_l	Total cost (TC)
6	0.652344	0.91521	56.0927
8	0.652344	0.91521	56.7725
10	0.652344	0.91521	57.3126
12	0.652344	0.91521	57.7551
14	0.888701	0.90588	58.5571

Effect of A on t_d, T_l and Total Cost:

Parameter	t_d	T_l	Total cost (TC)
10	5.27144	10.5388	20.3833
15	3.39656	3.40301	19.6187
20	2.43093	2.58708	26.9606
25	0.652344	0.91521	48.9535
30	0.652344	0.91521	57.3126

Effect of C_s on t_d, T_l and Total Cost:

Parameter	t_d	T_l	Total cost (TC)
3	1.30843	1.38318	49.7804
4	3.53680	3.6270	23.3867
5	0.652344	0.91521	57.3126
6	0.652344	0.91521	57.7518
7	0.652344	0.91521	58.8384

Effect of C_0 on t_d, T_l and Total Cost:

Parameter	t_d	T_l	Total cost (TC)
2	0.652344	0.91521	57.3126
4	0.888701	0.905881	58.5749
6	0.888701	0.905881	58.6627

8	0.888701	0.905881	58.7506
10	0.888701	0.905881	58.8384

Effect of P on t_d, T_l and Total Cost:

Parameter	t_d	T_l	Total cost (TC)
5	0.888701	0.905881	28.0444
7	0.888701	0.905881	34.1330
10	0.888701	0.905881	43.2657
12	0.888701	0.905881	49.3543
15	0.652344	0.91521	57.3126

All the observations are based on the computational results. Following tables shows the effect of several parameters on the optimal values of total cost, total cycle length and length of positive stock period.

VI. Conclusion

In this work, we have presented a mathematical model in which the demand of the product is constant and the weibull two parameter distribution is taken as rate of deterioration. Shortages are also considered with partial backlogging. Delay in payment is also considered to motivate the buyers to purchase more products. Salvage value is calculated for the considered perishable products. Various cost components follows the learning curve. A simple solution procedure is given to calculate the optimal values of the parameters. Numerical example is provided to illustrate the model and the solution procedure. Sensitivity analysis is performed with respect to key parameters to obtain interestial managerial insights.

Further several extensions of this paper are to consider the multi-item system, variable lead time, rate of inflation and we may also extend the model for stochastic demand pattern too.

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Jyoti, et. al. "Inventory Model for Constant Demand and Two-Parameter Weibull Deterrioration Having Permissible Delayed Payments and Salvage Value under Learning Effect." *IOSR Journal of Mathematics (IOSR-JM)*, 16(6), (2020): pp. 23-32.