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Solution of Some Optimization Problems by Numerical Methods

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Abstract
From the start the problem solution was designed to allow the design of the superconductive magnets using mathematical optimization techniques. Due to the principle of features, the complex coil assemblies can be generated in 2 and 3 dimensions using just a limited amount of engineering data that can then be viewed as optimization design variables. The development of operating research has been motivated by mathematical optimization, including numerical methods such as linear and nonlinear programming, integer programming, theory of network flow and dynamic optimization. Most real-world problem optimization involves multiple competing goals, and so-called vector optimization problems must be taken into account simultaneously. The solution cycle is three times based on decision-making methods, nonlinear constraint methods and algorithms of optimization to minimize objective function.

Keywords: *Optimization Problems, Numerical Methods, Solution, mathematical optimization, techniques, programming, network, nonlinear programming, algorithms*

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I. Introduction

For decades now, theoretical electromagnetic have been used with mathematical optimization techniques. Halbach introduced 1967 with Finite Element (FE) field calculations the approach to optimize the spiral structures and the pole shapes of magnets. Armstrong, Fan, Trowbridge and Simkin 1982[2] merged optimization algorithms with an integral volume approach for the H-magnet pole profile optimization. The electrode profile was designed by Girdinio, Molfino, Molinari and Viviani in 1983. However, these attempts were usually applied. Numerical field calculation packages have not been in an optimization environment since the end of 80th both for 2d and 3d applications. Reasons for this delay included computer power constraints, discontinuity problems, non-differential functional problems resulting from FE mesh, precision of the solution field, and problems with the implementation of software. The sources contain a small selection of papers.

Decision-making methods were developed and used for a wide variety of problems in the field of economy based on the optimal criterion set by Pareto in 1896. In 1951 Kuhn and Tucker developed the principle of nonlinear programming with limitations. Zoutdendijk 1960, Fiacco and McCormick 1968[13], among others, developed methods of handling non-linear constraints.During the sixties various optimization algorithms were developed and covered, using both deterministic and stochastic components. Researchers continue to come back to genetic and evolutionary algorithms recently because they are ideal for parallel processing and are able to find a global optimum.

There is no general method for solving nonlinear problems of optimization in machine electromagnetics in the same way as the simplex algorithm exists to solve linear problems. For machine electromagnetics, there are several different applications, and each requires a particular procedure. In the following sections, other optimization techniques for computer electromagnetic problems have proven successful.

Pareto-optimality

Most issues with optimization in the real world include multiple competing goals, which need to be reconciled. Characterized by the existence of an objective conflict in these so-called vector optimization problems, where there are different solutions for each individual objective function, and no solution exists where all the objectives meet their absolute minimum.

A structured mathematical problem of vector optimization reads:

$$"min"\vec{F}(\vec{X}) = "min"(f_1(\vec{X}), f_2(\vec{X}), .., f_K(\vec{X})) \tag{1}$$

$\vec{F} : R^n \to R^K$, $g_i, h_j : R^n \to R$ subject to

$$g_i(\vec{X}) \leq 0 \qquad (i = 1, 2, ..., m) \tag{2}$$

$$h_j(\vec{X}) = 0 \qquad (j = 1, 2, ..., p) \tag{3}$$

$$x_{ll} \leq x_l \leq x_{la} \qquad (l = 1, 2, ..., n) \tag{4}$$

with the design variable vector $\vec{X} = (x_1, x_2, ...x_n)$ and the in general nonlinear objective functions f_k arranged in the vector $\vec{F}(\vec{X})$. The x_{ll} and x_{lu} are the lower respectively upper bounds for the design variables. For the definition of the optimal solution of the vector-optimization problem we apply the optimality criterion by Pareto originally introduced for problems in economics Pareto [48], Stadler [65]. A Pareto-optimal solution $\vec{X}^*$ is given when there exists **no** solution $\vec{X}$ in the feasible domain $M = \{\vec{X} \in R^n \mid g_i(\vec{X}) \leq 0;\ h_j(\vec{X}) = 0;\ x_{ll} \leq x_l \leq x_{lu} \forall\, i = 1, ..m;\ j = 1, ..p;\ l = 1, ..n\}$ for which

$$f_k(\vec{X}) \leq f_k(\vec{X}^*) \quad \forall\, k \in [1, K] \tag{5}$$

$$f_k(\vec{X}) < f_k(\vec{X}^*) \quad for\, at\, least\, one\, k \in [1, K] \tag{6}$$

6) An aspect of the Pareto optimal solution set is a design where improvement of one goal causes at least one another to degrade. This description explicitly offers a set of solutions rather than a single one.Fig. 1.

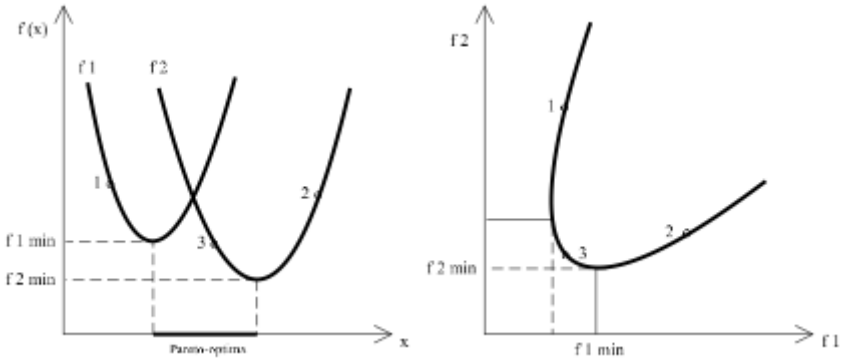

Fig. 1: Pareto-optimal solutions

Objective weighting

The objective weighting function, Kuhn and Tucker is the sum of the weighted objectives and results in the minimization problem:

$$min\{u(\vec{F}(\vec{X})) = \sum_{k=1}^{K} t_k \cdot f_k(\vec{X}) \mid \vec{X} \in M\} \tag{7}$$

with the weighting factors i.K representing the users preference. For convex optimization problems where for all Fandel [12], the eq can be demonstrated indirectly. (7) aParetooptimal solution minimization problem. The challenge is, particularly if the targets have different quantitative values and sensitivity, to find the correct weighting factors. Objective weighting thus leads to an iterative solution process, in which many optimizations with modified weighting factors have to be performed.

Distance function

Even when the distance function process, Charnes and Cooper [6] is used, the question of selecting weighting factors correctly happens. A minimum square objective function is most common.They f_k^* are the requirements for the optimum design. The minimization problem reads for $\vec{X} \in M$ and the norm $\| \vec{x} \|_p = (| x_1 |^p + ... + | x_n |^p)^{1/p}$

$$min \| \vec{F}^*(\vec{X}) - \vec{F}(\vec{X}) \|_2^2 = min \| \vec{z}(\vec{X}) \|_2^2 =$$

$$min \sum_{k=1}^{K} \iota_k (f_k^*(\vec{X}) - f_k(\vec{X}))^2. \qquad (8)$$

For convex functions and for f_k^* MK taken as the insignificant individual arrangements it very well may be demonstrated, in a similar way with respect to the target weighting capacity, that (8) has a special Pareto-ideal arrangement. The weakness of least squares target functions with the Euclidean standard $\| \cdot \|_2$ is the low affectability for residuals littler than one. Along these lines adequately high weighting factors i.K must be presented. On the off chance that the outright worth standard is applied, the disservice is the non-differentiable target work in the ideal.

Constraint formulation

Through defining the problem in the constraint formulation, the weighting factor problem can be solved. Just one target is reduced and the other is taken into account by constraints. The resulting optimization problem reads:

$$min f_i(\vec{X}) \qquad (9)$$

$$f_k(\vec{X}) - r_k \leq 0 \qquad (10)$$

$\forall k = 1, K; k \neq i$ More limitations, etc. (1)-(2)-(2). The gYK is the minimum user-specified request value for the k-th goal. Combining (10) and (2), omitting the limits of the template variables (4) results in a vector notation since they can be handled separately $\vec{g}', \vec{c} \in R^{m+K-1}, \vec{h}', \vec{d} \in R^p$: :

$$min f_i(\vec{X}) \qquad (11)$$

$$\vec{g}'(\vec{X}) - \vec{c} \leq \vec{0} \qquad (12)$$

$$\vec{h}'(\vec{X}) - \vec{d} = \vec{0} \qquad (13)$$

Sensitivity analysis

The benefit of restricting the formulation is that a sensitivity analysis can be conducted optimally using the correct optimal conditions. $\vec{X}^*$ which read, see Luenberger:

$$\nabla_{\vec{X}} L = \nabla_{\vec{X}} f_i(\vec{X}^*) + \vec{\alpha} \nabla_{\vec{X}} \vec{g}'(\vec{X}^*) + \vec{\beta} \nabla_{\vec{X}} \vec{h}'(\vec{X}^*) = \vec{0} \qquad (14)$$

$$\vec{g}'(\vec{X}^*) - \vec{c} = \vec{0} \qquad (15)$$

$$\vec{h}'(\vec{X}^*) - \vec{d} = \vec{0} \qquad (16)$$

$$\vec{\alpha} > \vec{0} \qquad (17)$$

The $\vec{\alpha}, \vec{\beta}$ The related multipliers are the vectors of Lagrange. The KuhnTucker equations are Equations (14)-(17). The Lagrange function's gradient must be zero and the Lagrange multipliers of active inequality constraints will take more than zero values; then the value of a constraint may be decreased without increasing the objective function, which is not indicative of optimal values, of course. Via the respective Lagrange function L it can also be shown that(11)-(13) is a question of minimisation, if all constraints are active, with a single Pareto optimal solution. A non-active restriction in the weighting feature will be equal to null weight.
The Lagrange multipliers are calculated by means of the resolution of the linear equation method (14)

$$min_{\vec{\alpha},\vec{\beta}} \parallel \nabla L \parallel = min_{\vec{\alpha},\vec{\beta}} \parallel \nabla_{\vec{X}} f(\vec{X}^*) + \mathbf{A}\vec{\alpha} + \mathbf{B}\vec{\beta} \parallel \qquad (18)$$

with the gradients of the constraints arranged in the matrices $\mathbf{A}$ and $\mathbf{B}$. Lagrange multipliers are a calculation of the price to be paid if the cap is lowered. This connection is expressed mathematically

$$\nabla_{\mathbf{c}} f(\vec{X}^*) = -\vec{\alpha}, \qquad (19)$$

$$\nabla_{\mathbf{d}} f(\vec{X}^*) = -\vec{\beta}. \qquad (20)$$

Payoff table The payoff table is a device that tells the decision-maker of the hideous tools of a concept. The table K offers a solution to individual optimization problems such that the best answer is found to all of the K goals (X^i being the minimizer of the problem $min f_i(\vec{X})$.)

Table 1: Payoff table for K objectives

$f_1(\vec{X}^1)$	$f_2(\vec{X}^1)$	.....	$f_K(\vec{X}^1)$	$\vec{X}^1$
$f_1(\vec{X}^2)$	$f_2(\vec{X}^2)$	.....	$f_K(\vec{X}^2)$	$\vec{X}^2$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$f_1(\vec{X}^K)$	$f_2(\vec{X}^K)$	.....	$f_K(\vec{X}^K)$	$\vec{X}^K$

Best compromise solutions can then be sought by decreasing the distance from the "complete" solution on the payoff table diagonal, in general, cf. Fig. 1. by using various criteria, e.g. L_1, L_2 and L_∞ Optimal forms of compromise can be found to standard. The payoff table can also help to set up limit problems with Pareto-optimal solutions (i.e. to find feasible solutions to active restricted problems).

Fuzzy sets
Considering the sometimes vague complexity of evaluations in multi-target programming problems, the fuzzy approach provided by Bellman and Zadeh[3] seems promising. X is described as a fuzzy subset by its membership function

$$\mu_A : X \to [0,1] \qquad (21)$$

which assigns to each element $x \in X$ a real number $\mu_A(x)$ in the interval [0,1] where the value of $\mu_A(x)$ The degree of association of x in A. Therefore, the concept of the fuzzy set theory is not whether a part is in or not the subset, but whether it is more or less a subset member. The membership function may be associated with a constraint defined by, e.g. "The value must be substantially greater than 10."

$$\mu_A(x) = \begin{cases} 0 & x < 10 \\ 1 - \frac{1}{1+(0.1(x-10))^2} & x \geq 10 \end{cases} \qquad (22)$$

Bellman and Zadeh [3] introduced three basic concepts: Fuzzy goal, fuzzy constraint and fuzzy decision. Let be $G1, G2, ..Gm$ the m fuzzy goals represented by their membership functions $\mu_{G1}, ..\mu_{Gm}$ and $C1, C2, ..Cm$

the m fuzzy constraints represented by their membership functions $\mu_{C1}, .. \mu_{Cm}$ then the fuzzy decision is the element with the maximum degree of membership of the intersection of the fuzzy goals and constraints.

$$max_x \mu_D = max_x \, min(\mu_{G1}, .., \mu_{Gm}, \mu_{C1}, .., \mu_{Cm}) \qquad (23)$$

However, the approach has two disadvantages. The first is the best option of membership functions to be paired with fuzzy statements such as medium, high, incredibly large, smaller, slightly lower, etc. Secondly, there is a flat-function topology where the member function of the crossing is zero and the use of stochastically optimized algorithms is therefore required.

Constrained optimization

Now we consider the constrained optimization problem

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \geq 0 \quad i \in I \qquad \text{(NLP)} \\ & h_j(x) = 0 \quad j \in E. \end{aligned}$$

We define the feasible region *F* of this optimization problem to be the set of all points that satisfy the constraints, so

$$\begin{aligned} \mathcal{F} = \{ & x \in \mathbb{R}^n \mid g_i(x) \geq 0, \ \ i \in I; \\ & h_j(x) = 0, \ \ j \in E \}. \end{aligned}$$

Just as for the unconstrained optimization problem, we can define global and local solutions. Of course, a desired optimal solution x^* to this optimization problem satisfies $f(x^*) \leq f(x)$ for all $x \in \mathcal{F}$. A point x^* is a local minimizer if it satisfies $f(x^*) \leq f(x)$ for all $x \in \mathcal{N}(x^*) \cap \mathcal{F}$ for some neighbourhood $\mathcal{N}(x^*)$ of x^*. The vector x is an isolated local minimizer if there exists a neighbourhood $\mathcal{N}(x^*)$ in which it is the only local minimizer.

Such concepts are somewhat unnecessary to determine optimal solutions – with one significant exception – like their counterparts in the case of unconstrained optimization problems. The simple way to overcome linear programming problems is by comparing objective functional values in some different areas in the feasible field. Nevertheless, most other functional numerical approaches rely on optimum conditions again. Penalty methods turn the problem into uncontrolled problems with optimization (a sequence) and rely on their conditions. For constraints on optimization, several methods rely directly on optimal conditions. Such conditions of optimum behaviour, which do not degenerate, require potential minimizers. Conditions except these degenerate points are referred to as 'control qualifications.' These are essential factors that are not always properly taken into account in economy, but are specifically dealt with by Simon and Blume (1994). There are several such constraints; we only list one such condition here.

Theorem

Suppose the functions f, g_i, $i \in I$, and h_j, $j \in E$, are continuously differentiable. Further suppose x^* is a local solution of the problem (NLP) that satisfies the (LICQ). Then there exist unique Lagrange multipliers v_i^*, $i \in I$, and λ_j^*, $j \in E$, such that the following conditions are satisfied.

$$\nabla f(x^*) - \sum_{i \in I} v_i^* \nabla g_i(x^*) - \sum_{j \in E} \lambda_j^* \nabla h_j(x^*) = 0,$$

$$g_i(x^*) \geq 0, \quad \text{for all } i \in I,$$

$$h_j(x^*) = 0, \quad \text{for all } j \in E,$$

$$v_i^* g_i(x^*) = 0, \quad \text{for all } i \in I,$$

$$v_i^* \geq 0, \quad \text{for all } i \in I.$$

II. Conclusion

At the existing limit of numerical analysis, mathematical programs with limits of equilibrium. Objective weighting, distance function and fuzzy set decision allow an algorithm to be implemented immediately in order to evaluate the minimum value of an uncompressed target function. It is very important to find the right method of minimization in order to suit the decision-making process and handle the nonlinear constraints. The special problem in electro-magnetism optimization is time-consuming evaluation by the system of finite elements of the objective function (electromagnetic field). The benefits of stochastic algorithms are the potential for addressing problems with a large number of design variables, the ability to answer non-convex and discrete questions and the ease of use. When a carefully chosen specification starts the quest the deterministic algorithms converge even faster.

The optimization algorithm involves one-dimensional minimizations by Powell's main search directions (user supplied) and a Gram-Schmidt orthogonal direction evaluated. After the single-dimensional search (end of the search step) a vector from the original outline is modified to the minimum of the search step in the main search direction. This user friendly algorithm is suitable for almost all applications, including scalar, central, penalty and Lagrange enhanced functions. There are no user parameters provided that can affect the rate of convergence in addition to the initial phase size.

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