IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 5 Ser. V (Sep. – Oct. 2020), PP 27-33
www.iosrjournals.org

Economic Order Quantity Model for Deteriorating Multi-Item when Constraints of Storage Capacity Are Active

Reza Budiman, Ihda Hasbiyati, and Moh Danil Hendry Gamal
Department of Mathematics, University of Riau, Pekanbaru 28293, Indonesia
Corresponding author: Reza Budiman

Abstract:
This paper discusses the Economic Ordering Quantity (EOQ) for multi-item deterministic inventory. Deterioration happens to each item when the constraints of shortage capacity are active. Then, Lagrange method is used to determine the EOQ of this case. The main purpose of this model is to determine the time cycle with and without backorder, as well as the amount of optimal shortage so as to minimize the total inventory cost. It is show that the developed model has the effective time cycle with minimum cost.
Keywords*: EOQ; deteriorating item; multi-item; shortage; Lagrange method.*

Date of Submission: 26-10-2020 Date of Acceptance: 05-11-2020

I. Introduction

Industrial era of 4.0 is an era that maximizes production and order where the deteriorating items are taken into account in order to satisfy consumers and maximize profit. Deteriorating can be define as changing, defecting, and losing a certain quality or original value of a commodity so that the original product decreases its value. The common examples of deteriorating products are vegetables, fruits, breads, and others.

Ghare and Schrader [7] is the first to develop the effect of defecting to inventory. They observe that certain items can be dwindling as the time goes by with proportion close to negative exponential function. The observation creates inventory item model with deteriorating process stated by differential equation

$$\frac{dI(t)}{dt} + \theta I(t) = -f(t).$$

Covert and Philip [3] develop Ghare and Schrader's model by using Weibull's distribution. Philip [11] simplifies the model using Weibull's distribution from Covert and Philip. Misra [10] develops it by using exponential distribution in completion of product. Dave and Patel [4] develop the deteriorating model for fixed demand. Goyal and Giri [8], and also Bakker et al. [1] elaborate the inventory model for deteriorating items taken by research in 1990s.

Holding to inventory model is one of the main components to determine the optimal total cost. Taha [12, p.428] defines holding costs as all of expense incurred because having inventory in which consideration to time, with inventory component are capital investment, warehouse, and handling. The beginning of integration of demands and unit costs is proposed by Cheng [2]. Maity et al. [9] optimize multi-item production to deteriorating items which is solved by Chebyshev's approximation. Yalcn et al. [14] solve the multi-product dynamic pricing problems by using stochastic dynamic program and analyzing the optimal price. Furthermore, Ghafour and Rashid [6] develop multi-item models with constraint the numbers of ordering take effect with demand.

Ghafour et al. [7] develop the classical EOQ model with storage that has effect to numbers of space used for storage. This paper develops classical EOQ model as deteriorating model with storage constraint to numbers of space used for storage. This paper is organized as follows: section 1 presents a literature review, section 2 gives assumptions and notations needed, and section 3 introduces a developed model to determine the solution of time cycle with and without backorder and the numbers of optimal shortage. The simulation of model is also given to show the usefulness of the model. Finally, section 4 provides some conclusions.

II. Notations and Assumptions

This research develops EOQ deteriorating model with storage active constraint. Assumptions and notations are given to make it easier to classify.

Assumptions

The following assumption are made in developing the model:

(i) Deteriorating possibility rate to constant item.
(ii) No lead time or lead time is 0.
(iii) Constant order.
(iv) Shortages are allowed and backorder that occurs for shortages is allowed.
(v) Backorder occurs for deteriorating multi-item EOQ model when the constraints of storage capacity are active.

Notations

The following notations are used in developing the model:

O_i := Ordering cost for item i, $i = 1, 2, \dots, n$.
h_i := Holding cost for every item i, $i = 1, 2, \dots, n$.
S_i := Deteriorating item quality cost for every item i, $i = 1, 2, \dots, n$.
R_i := Backorder cost for every item i, $i = 1, 2, \dots, n$.
D_i := Demand for every item i, $i = 1, 2, \dots, n$.
Q_i^* := Optimum ordering numbers for every item i, $i = 1, 2, \dots, n$.
T_i := Length of ordering time for each item i, $i = 1, 2, \dots, n$.
T_i^* := Length of optimum ordering time for each item i, $i = 1, 2, \dots, n$.
K_i := Percentage of demand which available from the stock for every item i, $i = 1, 2, \dots, n$.
K_i^* := Percentage of optimum demand which available from the stock for every item i, $i = 1, 2, \dots, n$.
θ_i := Deteriorating quality rate for every item i, $i = 1, 2, \dots, n$.
B_i^* := Maximum backorder for every item i, $i = 1, 2, \dots, n$.
A := Numbers of storage capacity.
f_i := Space needed for storage for every item i, $i = 1, 2, \dots, n$.

III. Development Of Model

This paper is about deteriorating EOQ model with and without backorder with active storage capacity constraint. The detailed model is explained as follows.

EOQ Model for Deteriorating Multi-Item without Backorder when the Constraint of Storage Capacity Are Active

Figure 1 shows deteriorating graph without backorder and the decreasing of inventory to time *t*. When the time cycle of time*T* occurs, demand can be fulfilled. Inventory equation for inventory rate is

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \qquad 0 \le t \le T\,. \tag{1}$$

The solution of equation (1) is

$$I(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right), \qquad 0 \le t \le T\,. \tag{2}$$

Figure 1 shows the total inventory $\int_0^T I(t)\,dt$. The exponent value of total inventory is calculated by using Taylor series approximation. Thus, the equation for inventory rate is

$$I = \frac{1}{2}(DT^2). \tag{3}$$

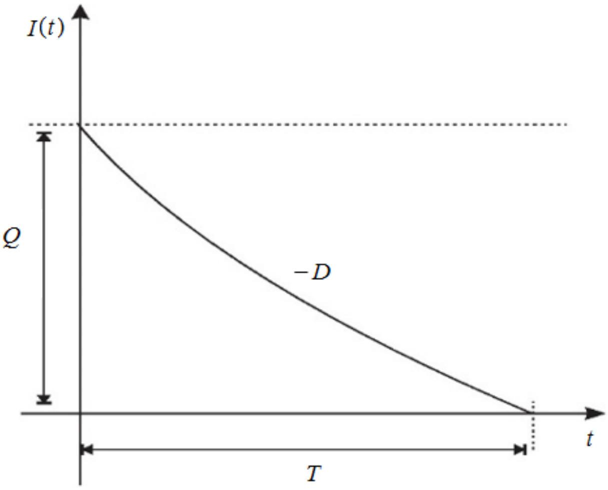

Figure 1: Model EOQ without Backordering

The total cost for every single item is the total of order cost, holding cost, and deteriorating cost. The total cost is

$$TC(T) = \frac{O}{T} + h\frac{I}{T} + s\theta\frac{I}{T},$$

$$TC(T) = \frac{O}{T} + h\frac{1}{2}(DT) + s\theta\frac{1}{2}(DT). \quad (4)$$

The total single cost is modified in form of multi-item, thus the total multi-cost becomes objective function. The storage limit in inventory is the number of ordering item and space required needs to be smaller or equal to storage limit available. Moreover, the storage limit is the given constraint functionsand it can be formulated as follows:

$$\sum_{i=1}^{n} \frac{f_i D_i}{T_i} \leq A. \quad (5)$$

Determining the optimum value for multi-item of active storage uses Lagrange method. The total cost for all of the first constraints are

$$TC(T,\lambda) = \frac{O_i}{T_i} + h_i\frac{1}{2}(D_iT_i) + S_i\theta\frac{1}{2}(D_iT_i) - \lambda\left(\sum_{i=1}^{n}\frac{f_iD_i}{T_i} - A\right). \quad (6)$$

Determining optimal time T_i* we set the first derivative of $TC(T_i, \lambda)$with respect to T equals zero and then solve for T; *i.e*,

$$\frac{dTC(T,\lambda)}{dT} = \frac{-O_i}{{T_i}^2} + \frac{h_iD_i}{2} + \frac{S_iD_i\theta}{2} + \lambda\frac{f_iD_i}{{T_i}^2} = 0,$$

$${T_i}^2 = \frac{2(O_i - \lambda f_i D_i)}{D_i(h_i + S_i\theta)}.$$

Taking ${T_i}^*$of every $i$as a positive value gives

$${T_i}^* = \sqrt{\frac{2(O_i - \lambda f_i D_i)}{D_i(h_i + S_i\theta)}}. \quad (7)$$

There are two ways in determiningλ. The first by using trial and error of compensation method with the initial number is$\lambda = 0$to the closest condition, either$+$or$-$. Second, by using Saint Method which proposed by Ghafour et al. [7] that is more accurate by considering derivation method. Substitute the value of$T$to constraint equation to get

$$\sum_{i=1}^{n} \frac{f_i D_i}{\sqrt{\frac{2(O_i - \lambda f_i D_i)}{D_i(h_i + S_i\theta)}}} = A,$$

$$2\lambda f_i D_i = 2O_i - \left(\frac{f_i D_i \sqrt{D_i(h_i + S_i\theta)}}{A}\right)^2,$$

$$\lambda = \frac{O_i}{f_i D_i} - \frac{f_i D_i (D_i(h_i + S_i\theta))}{2A^2}. \qquad (8)$$

The total optimum costof every i is

$$TC(T_i^*, \lambda^*) = \frac{O_i}{T_i^*} + h_i \frac{1}{2}(D_i T_i^*) + S_i\theta \frac{1}{2}(D_i T_i^*) - \lambda^* \left(\sum_{i=1}^{n} \frac{f_i D_i}{T_i^*} - A\right). \qquad (9)$$

EOQ Model for Deteriorating Multi-Item with Backorder when the Constraints of Storage Capacity Are Active

Figure 2 shows EOQ graph with backorder in general. The graph shows a decreasing inventory to time t with demand and deteriorating item. $KT$is the time when demands occur in general.Furthermore, the deficiency time is the cycle minus the order without shortage, that is $T - KT$.The inventory rate without shortage occurs to $t = 0$ and during KT time. The backordering rate occurs from the end of KT time to the end of time T. The differential equation of inventory rate without shortage and the differential equation of backordering rate respectively are

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \qquad 0 \le t \le KT\,. \qquad (10)$$

$$\frac{dB(t)}{dt} = -D, \qquad KT \le t \le T\,. \qquad (11)$$

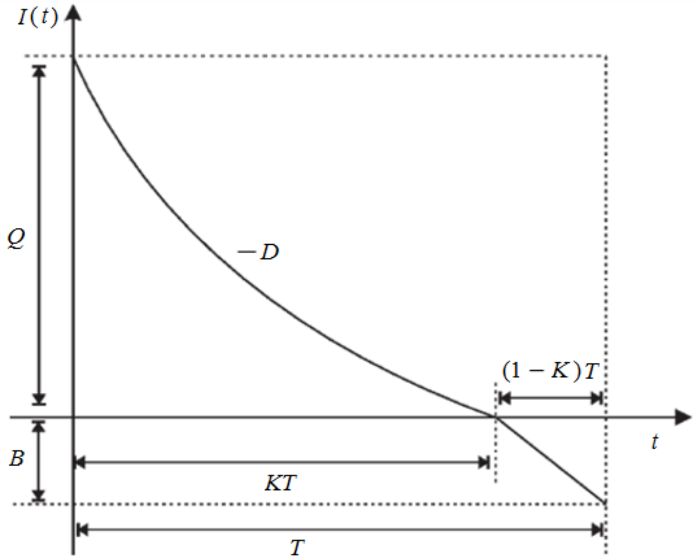

Figure 2: Model EOQ with Backordering

The solution of differential equation for equation (10) and (11) is

$$I(t) = \frac{D}{\theta}\left(e^{\theta(KT-t)} - 1\right), \qquad 0 \le t \le KT\,. \qquad (12)$$

$$B(t) = Dt, \qquad KT \le t \le T\,. \qquad (13)$$

The total for single item is the total of ordering cost, holding cost, and deteriorating cost. The total of all is

$$TC(T, K, \lambda) = \frac{O}{T} + \frac{D}{2}R(1-K)^2T + \frac{D}{2}hK^2T + S\theta\frac{D}{2}K^2T. \qquad (14)$$

The total single cost with backorder is modified in form of multi-item, thus the total multi cost becomes an objective function. The storage limit in inventory is the number of ordering item and space required needs to be smaller or equal to storage limit available. Moreover, the storage limit is the given constraint function and it can be formulated as follows:

$$\sum_{i=1}^{n} \frac{f_i D_i}{T_i} \leq A. \tag{15}$$

Lagrange method is used to determine the optimal value for multi-item of active storage. The total cost of all for the first constraint is

$$TC(T_i, K_i, \lambda) = \frac{O_i}{T_i} + \frac{D_i}{2} R_i (1 - K_i)^2 T_i + \frac{D_i}{2} h_i {K_i}^2 T_i$$
$$+ S_i \theta_i \frac{D_i}{2} {K_i}^2 T_i - \lambda \left(\sum_{i=1}^{n} \frac{f_i D_i}{T_i} - A \right). \tag{16}$$

To determine the optimumT_i^* and K_i^* in an easier way, we refer to Talazaideh [13]. The total cost of equation (16) is given as

$$TC(T_i, K_i, \lambda) = \frac{\alpha_1}{T_i} + (\alpha_2 {K_i}^2 - 2\alpha_3 K_i + \alpha_4) T_i - \frac{\alpha_5}{T_i} + \alpha_6 \tag{17}$$

where

$$\alpha_1 = o_i > 0,$$
$$\alpha_2 = \frac{(D_i h_i + S_i \theta_i + D_i R_i)}{2} > 0,$$
$$\alpha_3 = \frac{D_i R_i}{2} > 0,$$
$$\alpha_4 = \frac{D_i R_i}{2} > 0,$$
$$\alpha_5 = \lambda \sum_{i=1}^{n} \frac{f_i D_i}{T_i} \leq 0,$$
$$\alpha_6 = \lambda A \leq 0.$$

Let$\gamma(K_i)$ be the quadratic function ${K_i}^2$ in equation(17). Thus, the total cost equation is given by

$$TC(T_i, K_i, \lambda) = \frac{\alpha_1}{T_i} + \gamma(K_i) T_i - \frac{\alpha_5}{T_i} + \alpha_6. \tag{18}$$

Determining optimal time T_i* we set the first derivative of$TC(T_i, K_i, \lambda)$with respect to T equals zero and then solve for T; *i.e*,

$${T_i}^* = \sqrt{\frac{\alpha_1 - \alpha_5}{\gamma(K_i)}}. \tag{19}$$

The minimum global value of K is determined in two steps. The first step is to substitute${T_i}^*$ value to equation (18)to get a new total cost. The second step is to show that the total cost equation is global minimum by taking the derivative of total cost equation twice with respect to K,

$$\frac{dTC(K_i, TC(K_i), \lambda)}{dK} = \frac{\sqrt{\alpha_1 - \alpha_5}[\gamma'(K_i)]}{\sqrt{\gamma(K_i)}}. \tag{20}$$
$$\frac{d^2TC(K_i, TC(K_i), \lambda)}{dK^2} = \frac{\sqrt{\alpha_1 - \alpha_5}[2\gamma''(K_i)\gamma(K_i) - \gamma'(K_i)^2]}{2\gamma(K_i)^{\frac{3}{2}}},$$
$$\frac{d^2TC(K_i, TC(K_i), \lambda)}{dK^2} = \frac{\sqrt{\alpha_1 - \alpha_5}[\alpha_2 \gamma(K_i) + (\alpha_2 - \alpha_3)\alpha_3]}{\gamma(K_i)^{\frac{3}{2}}} > 0. \tag{21}$$

Since $(\alpha_2 - \alpha_3) = ((D_i h_i + S_i \theta_i + D_i R_i)/2) - \left((D_i R_i)/2\right) > 0$, then it is true that$TC(K_i, T(K_i), \lambda)$ is convex. Determining optimum K_i can be done by setting the first derivative in equation (20)equalszero. As $\sqrt{\alpha_1 - \alpha_5}$ and $\sqrt{\gamma(K_i)}$ is bigger than zero, then $\gamma'(K) = 0$. Then the value of$K$ for each$i$ is obtained as follows:

$$\gamma'(K_i) = 2\alpha_2 K_i - 2\alpha_3 = 0$$

$$K_i^* = \frac{\alpha_3}{\alpha_2}$$

$$K_i^* = \frac{D_i R_i}{(D_i h_i + S_i\theta_i + D_i R_i)}. \qquad (22)$$

SubstitutingT^* value to equation (5) by using Saint Method to get λ, we have the following:

$$\sum_{i=1}^{n} \frac{f_i D_i}{\sqrt{\frac{2(O_i - \lambda f_i D_i)(D_i h_i + S_i\theta_i + D_i R_i)}{D_i R_i (D_i R_i - 1)}}} = A,$$

$$2\lambda f_i D_i (D_i h_i + S_i\theta_i + D_i R_i) = 2O_i (D_i h_i + S_i\theta_i + D_i R_i) - \frac{(f_i D_i)^2 \big(D_i R_i (D_i R_i - 1)\big)}{A^2},$$

$$\lambda = \frac{O_i}{f_i D_i} - \frac{(f_i D_i)\big(D_i R_i (D_i R_i - 1)\big)}{2{A_1}^2 (D_i h_i + S_i\theta_i + D_i R_i)}. (23)$$

The overall total optimal cost is obtained by substituting the values of T^*, K^* and λ to equation (16); *i.e.*,

$$TC(T_i^*, K_i^*, \lambda^*) = \frac{O_i}{T_i^*} + \frac{D_i}{2} R_i (1 - K_i^*)^2 T_i^* + \frac{D_i}{2} h_i {K_i^*}^2 T_i^* + S_i\theta_i \frac{D_i}{2} {K_i^*}^2 T_i^* - \lambda^* \left(\sum_{i=1}^{n} \frac{f_i D_i}{T_i^*} - A \right). \qquad (24)$$

Numerical Simulations

Numerical simulations are given to show the use of EOQ model for deteriorating multi-item without backorder when the constraints of storage capacity are active and EOQ model for deteriorating multi-item with backorder when the constraints of storage capacity are active. The data is taken from a biggest supermarket in the city of Pekanbaru which has 100 items maximum display storage ability. The displayed items are 20 items of fish, 10 items of beef, and 20 items of chicken. Table 1 shows the parametersto determine the time cycle, shortage, and optimum total cost.

Table 1: Parameter of three items

i	*D*	*H*	*O*	*S*	*θ*	*R*
1	70	IDR10,000	IDR20,000	IDR2,000	0.08	IDR5,000
2	60	IDR11,000	IDR80,000	IDR8,000	0.10	IDR13,000
3	50	IDR10,000	IDR35,000	IDR3,000	0.05	IDR6,000

Illustration of EOQ model for deteriorating multi-item without backorder when the constraint of storage capacity are active is given to determine the optimum time cycle and the optimum total cost. Substituting each parameter into equation (7), (8), and (9) we obtain the optimum time cycle and the optimum total cost as seen in Table 2

Table 2: EOQ model without backorder

I	*T*	*TC*
1	33	IDR11,818,227
2	22	IDR7,713,451
3	33	IDR8,429,403

Determining a better time cycle and total cost for EOQ model for deteriorating multi-item without backorder when the constraint of storage capacity are active is by increasing each parameter by 50% and 25%, and decreasing it by -25% and -50%. The results for the optimum time cycle and total cost value whendemands decrease by-50% are shown in Table 3.

Table 3: 50 percent daily demand decreasing without backorder

i	*T*	*TC*
1	17	IDR2,971,805
2	11	IDR1,942,657
3	17	IDR2,116,797

Illustration of EOQ model for deteriorating multi-item with backorder when the constraints of storage capacity are active is given to determine the optimum time cycle, demand being fulfilled from the stock, and optimum total cost. Substituting each parameter into equation (19), (22), and (24) we getthe optimum time cycle and the optimum total cost as shown in Table 4.

Table 4: EOQ model with backorder

I	*T*	*K*	*TC*
1	36	0.33	IDR4,206,991
2	19	0.54	IDR3,526,452
3	34	0.38	IDR3,185,166

Determining a better time cycle, demand being fulfilled from the stock, and total cost for EOQ model for deteriorating multi-item without backorder when the constraint of storage capacity are active is by increasing each parameter by 50% and 25%, and decreasing it by -25% and -50%. Table 3 represents the result for the optimum time cycle and total cost when demand parameters decrease by -50%.

Table 5: 50 percent daily demand decreasing with backorder

I	*T*	*K*	*TC*
1	18	0.33	IDR1,048,495
2	10	0.54	IDR887,741
3	17	0.38	IDR794,282

IV. Conclusion

This paper develops and analyzes the deterministic inventory model for the deteriorating multi-item with the active storage constraints. The solution ofEOQ model for deteriorating multi-item without and with backorder when the constraints of storage capacity are active solution is evaluated by using Lagrange method. The benefits of this model are those one can have the optimum time cycle and minimum total cost. The results of numerical simulations provide the minimum total cost and lower time cycle. The 50% decrease of demand causes about 50% reduction in time cycle and total cost. This simulation applies to models without backorder and with backorder.

References

[1] Bakker M, Riezebos J and TeunterRH. Review of inventory systems with deterioration since 2001, European Journal of Operational Research. 2012; 221: 275-284.

[2] Cheng TCE. An economic order quantity model with demand-dependent unit cost. European Journal of Operational Research. (1989); 40: 252-256.

[3] Covert RB and Philip GS. An EOQ model with Weibull distribution deterioration. AIIE Transactions. 1973; 5: 323 - 326.

[4] Dave U and PatelLK. (T,S_i) policy inventory model for deteriorating items with time promotional demand. Journal of the Operational Research Society. 1981; 32: 137-142.

[5] Ghafour KM, Ramli R, and Zaibidi NZ. Development of multi-items EOQ when the constraints of investment and shortage capacity are active.AIP Conference Proceedings. 2015; 1691: 030005.

[6] Ghafour KM and RashidRH. Optimizing multi-item EOQ when the constraint of annual number of order is active. Modern Applied Science. 2017: 11: 55-61.

[7] Ghare PM and Schrader GH. A model for exponentially decaying inventory system. International Journal of Production and Research. 1963: 21: 449-460.

[8] Goyal SK and Giri BC. Recent trends in modeling of deteriorating inventory. European Journal of Operational Research. 2001; 134:1-16.

[9] Maity AK, Maity K, Mondal Sand Maiti M. A Chebyshev approximation for solving the optimal production inventory problem of deteriorating multi-item. Mathematical and Computer Modelling. 2007; 45: 149-161.

[10] Misra RB. Optimum production lot size model for a system with deteriorating inventory. International Journal of Production Research. 1975: 13: 495-505.

[11] Philip GC. A generalized EOQ model for items with Weibull distribution deterioration. A I I E Transactions. 1974: 6:159-162.

[12] TahaHA. Operations Research: An Introduction, Eighth Edition. New Jersey:Pearson Prentice Hall; (2007).

[13] Taleizadeh AA. An Economic Order quantity model for a deteriorating item in a purchasing system with multiple prepayments. Applied Mathematical Modelling.2014; 38:5357-5366.

[14] Yalcn A, NatarajanHP and XuSH. Joint dynamic pricing of multiple perishable products under consumer choice. Management Science.2010; 56:1345-1361.

Reza Budiman, et. al. "Economic Order Quantity Model for Deteriorating Multi-Item when Constraints of Storage Capacity Are Active." *IOSR Journal of Mathematics (IOSR-JM)*, 16(5), (2020): pp. 27-33.