IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 16, Issue 5 Ser. V (Sep. – Oct. 2020), PP 01-06
www.iosrjournals.org

Factorization of Wreath Product of Permutation Groups

ShuaibuGarbaNgulde[1] Maryam Mustapha Mohammed[2]BukarGoni Ahmadu[3]
[1]Department of Mathematical Sciences, University of Maiduguri, Nigeria
[2] Department of Mathematical Sciences, University of Maiduguri, Nigeria
[3]Department of Mathematical Sciences, University of Maiduguri, Nigeria

Abstract:*In Mathematics, the wreath product of groups is a specialized product of two groups, based on a semi direct product. Factorization of wreath product seems to be nice source of examples, it may be interesting to investigate their nature in a systematic way and also its basic properties connected with the properties of factorization of each constituent.*

Key Words:*Wreath Product, Factorization, Constituent, Zappa-Szip product, Exact.*

Date of Submission: 15-10-2020 Date of Acceptance: 31-10-2020

I. Introduction

Wreath product of groups have been used to explore some useful characteristics of finite group in connection with permutation designs and construction of lattices (Praeger and scheider, 2002). As well as in the study of interconnection networks. In Mathematics, the wreath product of group theory is a specialized product of two groups, based on a semi direct product. Wreath products are used in the classification of permutation of groups and provide a way of constructing interesting examples of groups.

We say that a group G is factorized by its subgroups $G_{1,\dots}$, G_n. if

G=$G_1, \dots, G_n$={$g_{1,}g_2, \dots, g_n$ } $\forall$ $g_i \in G_i i = 1, \dots, n$…………………………… ……………1.0

ie G is a product of $G_{i's}$called a factorization of G. We called G factorizable if there exist a natural number n≥ 2 and subgroups G =$G_1 \dots G_n$ of G satisfying (1.0)

The factorization of a wreath product G is called exact if each pair of $G_{i's}$intersects trivially, that is $G_i \cap G_j$= {1}, i ≠ j. One can examine the nature of that factorization. Let us consider an exact factorization of a group by two subgroups namely G=KH. If both K and H are normal, then G is just their direct product. If one is normal, then G is a zappa - szip product of subgroups K, H.

Since factorization of wreath product seems to be nice source of examples, it may be interesting to investigate their nature in a systematic way and also its basic properties connected with the properties of factorization of each constituent.

Wreath product of two permutation groups, G¸ ≤Sym(Γ)and H ≤Sym(Γ) can be considered as permutation group acting on the set Π of functions from Δ to Γ(Cheryl and Csaba 2013). The action, usually called the product action of wreath product plays a very important role in the theory of permutation groups as several classes of primitive or quasi primitive groups can be described as subgroup of such wreath products. In addition, subgroup of wreath products in product action arise as automorphism groups of graph products and codes. In their paper, they consider subgroups X of full wreath products SymΓwrSymΔ in product action. A suitable conjugate of X the subgroup ofSymΓ induced by a stabilizer of a coordinate S∈Δ only depends on the orbit of δ under the induced action of X on Δ. Hence if the action of X and Δ is transitive, then X can be embedded into a much smaller wreath product. Further, if this X action is transitive, then X can be embedded into a direct product of such wreath product where the factor of the direct product corresponds to the X-orbits in Δ.

Definition 1.1: The wreath product of C by D denoted by W = C wr D, is the semi-direct product of P by D, so that, W = {(f,d)|f∈P, d∈D}, with multiplication in W defined as;

$$(f_1,d_1)\ (f_2,d_2) = [(f_1 f_2^{d_1^{-1}},\ d_1d_2)$$

For all $f_1, f_2 \in$ P and $d_1, d_2 \in$ D, we can write fd instead of (f,d) for element of W. Where P is the Base group(Audu, M. S. 2001).

Definition 1.2:Orbit:- Let G ≤Sym(Ω). Then G acts faithfully on Ω. We define a relation R on Ω by setting αRβ if and only if α, β∈Ω and there is an element g in G such that αg = β. Then R is an equivalence relation on

Ω and partition Ω into a disjoint equivalence classes. These equivalence classes are called the orbit of transitivity classes of action.

Definition 1.3: **The constituents** G^{Ω_i} **:** Let G ≤Sym(Ω). Let $\Omega_1, \Omega_2, ..., \Omega_k$ be the orbits of G. Then each g∈G induces a permutation on Ω which we denoted by g^{Ω_i}. The totality of all g^{Ω_i} formed for all g ∈G is called the constituent of G^{Ω_i} of G on Ω. We can easily see that G^{Ω_i} is a permutation group on Ω..

Definition 1.4: Transitivity of Wreath product on semi-direct product of r and Δ. Let $(\alpha_1\delta_1)$ and (α_2,δ_2) be two arbitrary points in Γ× Δ. Then W will be transitive on Γ × Δ if and only if there exits fd∈ W i.e f∈P, d∈Δ such that (α_1, δ_1) fd = $(\alpha_2\delta_2)$, if and only if $(\alpha_1 f(\delta_1), \delta_1 d) = (\alpha_2\delta_2)$, if and only if $\alpha_1(\delta_1) = \alpha_{2,}\delta_1 d = \delta_2$. Thus (f,d) exist if C and Δ are transitive in Γ and Δ respectively which is necessary condition for W to be transitive on Γ× Δ.

Definition 1.5: The center of wreath product W denoted by Z (W) is defined by

$$Z(W) = \{f\,d \mid (f, d)(f_{1,}d_1) = (f_{1,}d_1)(f\,d) \text{ for all } f_1 \in P,\ d_1 \in D\}. \text{ Hence, } f\,d \in Z(W)$$

if and only i$fdd_1 = f_1 f^{d_1^{-1}} d_1 d$ for all f_1∈P, d_1∈D

Definition 1.6: The stabilizer W (α, δ) of a point (∝,δ)∈Γ×Δ under the action of W on Γ× Δ, the stabilizer of any point (α, δ) in Γ× Δ denoted by W (α, δ) is given by

$$\begin{aligned} W(\alpha, \delta) &= \{f\,d \in W \mid (\alpha, \delta)\, f\, d = (\alpha, \delta)\} \\ &= \{fd \in W \mid (\alpha f(\delta), \delta d) = (\alpha, \delta)\} \\ &= \{fd \in W \mid \alpha f(\delta) = \alpha_{,}\ \delta d = \delta\} \\ &= \{F(\delta)_\alpha \Delta_\delta\} \end{aligned}$$

where F(δ)αis the set of all f(δ) that stabilize α, and Δα is the stabilizer of δ under the action of Γ and Δ

Definition 1.7: Faithfulness of W on Γ× Δ

W is faithful on r× Δ if and only if the identity element of W is its only element that fixes every point of Γ× Δ. If the identity element of W is 1 and thus, if W is to be faithful on r then for any (α,δ) in Γ× Δ, the stabilizer of W on Γ× W (α,δ) must be F(δ)α Δα = 1

Hence, F (δ)α = 1 and Δδ = 1 for all α∈ D and αF (δ) = α, δd = δ imply that f(δ) = 1 and d = 1. Thus we deduce that W would be faithful on Γ× Δ, if the stabilizer of any α∈F and δ∈Δ are the identity element in P and Δ respectively. Therefore, we conclude that W is faithful on Γ× Δ, if P or C and Δ are faithful on r and Δ respectively.

Definition 1.8: The primitivity of W on Γ× Δ

The product W would be primitive on Γ× Δ, if and only if given any (α,δ) in Γ× Δ, W (α,δ), the stabilizer of (α,δ) is a maximal subgroup of W.

II. Methodology

The methods employed in the construction of group to investigate the factorization of wreath product of permutation group is to use a software program known as Group Algorithm Program (GAP).

http://www.gap-system.orgmversion 4.3, 2002

This is the program use in order to generate the require group.

```
gap> a:=SymmetricGroup(3);
Sym( [ 1 .. 3 ] )
gap> Elements(a);
[ (), (2,3), (1,2), (1,2,3), (1,3,2), (1,3) ]
gap> Size(a);
6
gap>b:=Group((),(1,2)(3,4),(1,3)(2,4),(1,4)(2,3));
Group([ (), (1,2)(3,4), (1,3)(2,4), (1,4)(2,3) ])
gap> c:=WreathProduct(a,b);
<permutation group of size 5184 with 12 generators>
```

Meta Cyclic Wreath Product

A wreath product G is called metacyclic if it has a normal cyclic subgroup k with cyclic quotients by taking generators a of k and b if k of G/K whichhas a presentation of the form $G = \langle a, b / a^m = 1, b^s = a^t, b^{-1}ab = a \rangle$,where the integers m, s, t and r satisfy $r^s \equiv 1(mod m)$ and $m|t(r - 1)$. If we let $L = \langle b \rangle$, then the wreath product Gcan be written as $G = LK$. this decomposition is called a Meta cyclic factorization of$G$. If $L \cap K = 1$ then the Meta cyclic factorization is called split.

Let G be a wreath product of group which acts transitively on a set Ω. We say that the wreath product is primitive if G has no nontrivial blocks on Ω; otherwise G is called imprimitive. Note that we only use the term "primitive" and "imprimitive" with reference to a transitive group.

To describe the relation between blocks and subgroups we shall require the following notation which extends the notation for a point-stabilizer. Suppose G is a wreath product of groups acting on a set Ωand $\Delta \subseteq \Omega$. Then the pointwise stabilizer of Δ in G is

$$G_{(\Delta)} = \{x \in G \mid \delta^x = \delta \ \ for\ all\ \delta \in \Delta\}$$

and the setwise stabilizer of Δ in G is

$$G_{\{\Delta\}} = \{x \in G \mid \Delta^x = \Delta\}$$

It is readily seen that $G_{\{\Delta\}}\ \ and\ G_{(\Delta)}$ are both subgroups of G and that $G_{(\Delta)} \Delta\ G_{\{\Delta\}}$. Note that $G_{\{\alpha\}} = G_{(\alpha)} = G_\alpha\ \ for\ \ each\ \alpha \in \Omega$. More generally for a finite $\Delta = \{\alpha_1, ..., \alpha_k\}$ we shall often write $G_{\alpha_1,...,\alpha_k}\ \ in\ \ place\ of\ \ G_{(\Delta)}$.

Base and orders of 2-transitive Wreath Product of groups

Lemma: Let n, d and t be positive integers. Let Ω be a set of size n, and suppose that F is a family of subsets of Ω such that each γ∈Ω lies in exactly t subsets from F. Then

for each Γ⊆Ω there exist Δ∈Ω such that |Γ∩Δ| ≤ |Γ||Δ|/n,

if each Δ∈F has at least d elements, then for each real c > 1 there exist a subfamily Fc ⊆ F such that | Fc | < (nlogc)/d+1 and $|\bigcup_{\Delta \in F} \Delta| > (1 - \frac{1}{c})n$

Proof (i) Let F(γ) denote the set of Δ∈ F with γ∈Δ, and not that | F(γ)| = t by hypothesis. Then $\sum_{\Delta \in F} |\Gamma \cap \Delta| = \sum_{\gamma \in \Gamma} |$ F(γ)| = t|Γ|. In particular, substituting Ω for Γ gives $\sum_{\Delta \in F} |\Delta| = tn$. Hence, for general Γ we have $t|\Gamma| = \sum_{\Delta \in F} \frac{|\Gamma \cap \Delta|}{|\Delta|} |\Delta| \geq \frac{|\Gamma \cap \Delta^{\bullet}|}{|\Delta^*|} \sum_{\Delta \in F} |\Delta|$ for some $\Delta^* \in F,$ and (i) follows.

(ii)Define subsets $\Gamma_0, \Gamma_1, ...\ \ of\ \ \Omega$ as follows. Put $\Gamma_0 = \phi$ for each $i \geq 0$ we use (i) to choose $\Delta_i \in$ F such that $|\Gamma_i \cap \Delta_i| \leq \frac{|\Gamma_i||\Delta_i|}{n}$ and put $\Gamma_{i+1} = \Gamma_i \cup \Delta_i\ \ and\ \ g_i = |\Gamma_i|.\ Clearly\ \ g_{i+1} > g_i$ as long as $\Gamma_i \neq \Omega$. We claim that if we stop at the index k where $g_k > (1 - \frac{1}{c})n \geq g_{k-1}$ then $k < (\frac{n \log c)}{d+1}$. Since the latter inequality is trivial for k = 1, we can suppose that k ≥ 2. The choice of Δ_i shows that for each i

$n - g_{i+1} \leq n - g_i - |\Delta_i|(1 - \frac{g_i}{n}) \leq (n - g_i)(1 - \frac{d}{n})$ since g0 = 0 and k ≥2 this shows that

$\frac{n}{c} \leq n - g_{k-1} \leq n(1 - \frac{d}{n})^{k-1} < n\ \exp\{\frac{-d(k-1)}{n}\}$.Therefore $-\log c = \log \frac{1}{c} < \frac{-d(k-1)}{n}$.

Lemma: Suppose that G ≤Sym(Ω) has degree n≥ 2 and that k ≥ 5. If G does not have a section Isomorphic to A_k, then there exists $\Delta \subseteq \Omega$ with |Δ| ≤ 2k such that every orbit of $G_{(\Delta)}$ has length less than 0.63n.

Proof: Suppose that no such set Δ exists. To simplify notation, put b = 0.63. Then we can define a sequence of subgroups G_i (i = 0,…,2k) of G such that G(0) = G and for each i≥ 1, the group G_i is a point stabilizer of $G_{(i-1)}\ \ with\ |G_{(i-1)} : G_{(i)}| \geq bn$ (choose the point to lie in the largest orbit of $G_{(i-1)}$. Then $G_{(2k)} = G_{(\Delta)}$ for some subset Δ of size 2k, and $|G : G_\Delta| \geq (bn)^{2k}$. On the other hand, considering the action of G on the set $\Omega^{\{2k\}}$ of 2k-subsets we have

$$|G : G_{\{\Delta\}}| \leq |\Omega^{\{2k\}}| = \binom{n}{2k}$$

and so

$$|G_{\{\Delta\}}:G_{(\Delta)}|\geq \binom{n}{2k}^{-1}(bn)^{2k}=\frac{(2k)!n^{2k}b^{2k}}{n(n-1)...(n-2k+1)}\geq (2k)!b^{2k}$$

Now the restriction map gives a homomorphism of $G_{\{\Delta\}}\to Sym(\Omega)\cong S_{2k}$ with kernel $G_{(\Delta)}$ *and image* $H=G^{\Delta}_{\{\Delta\}}\cong G_{\{\Delta\}}/G_{(\Delta)}$. By the hypothesis on G the group H cannot contain a subgroup Isomorphic to A_k. Since k > 4, this implies that the index of H in Sym(Δ) is at least $\binom{2k}{k}$. Therefore $|H|=|G_{\{\Delta\}}:G_{(\Delta)}|\leq \frac{(2k)!}{\binom{2k}{k}}$. As we have $!\binom{2k}{k}\geq\frac{2^{2k}}{2k}$. Using this together with the last two inequalities for $|G_{\{\Delta\}}:G_{(\Delta)}|$ we conclude that $(2k)^{\frac{1}{2k}}\leq 10^{\frac{1}{10}}<1.26=2b$ *for all* $k\leq 5$ this gives a contradiction. Thus there exist a set Δ for which $G_{\{\Delta\}}$ has all orbits of size < bn.

Theorem: 4.1

Let $G=\wr_{i=1}^{n}G_i$ be a finite wreath product of permutation groups $((G_i,\Omega),i\in N$, each of which is factorized by at most m subgroups, that is $\forall i\in NG_i=G_{i1},G_{i2},...G_{im}$, with some G_{ik} possibly trivial. Now, let $G^{[1]},G^{[2]},...,G^{[m]}$ be groups defined in the following way: $g^{[k]}\in G^{[k]}$ if and only if

$$g^{[k]}=\left[g_1^{(k)},g_2^{(k)}(x_1),g_3^{(k)}(x_1,x_2),...,g_n^{(k)}(x_{n-1},x_n)\right],g_i^{(k)}\in G_{ik}^{\Omega_1\times...\times\Omega_{i-1}}$$

Then $G=G^{[1]}G^{[2]}...G^{[m]}$……………………………………………………………………..(1)

Proof:

Observe first that $G^{[k]},k=1,...,m$ are subgroups of$G$, which is obvious since $G_{ik,}k=1,...,m$ are subgroups of G_i. Taking $\forall i\in NG_i=G_{i1},G_{i2},...G_{im}$, under consideration for every, $g\in G$, we have $[g]_i=g_1^{(1)},...,g^{(m)}$, $[g]_n=g_n^{(1)}(\bar{x}_{n-1})...g_n^{(m)}(\bar{x}_{n-1}),n\geq 2$ ------------------------(2)

Now, for a given $g\in G$we define $A^{(k)}\in G^{[k]}$ for $k=1,2,...,m$ in the following way:

$[A^{(k)}]_1=g_1^{(k)}$and for every natural $n\geq 2$.

$[A^{(1)}]_n=g_n^{(1)}(\bar{x}_{n-1}),[A^{(k)}]_n=g_n^{(k)}(\bar{x}_{n-1}^{\left(g_{n-1}^{-(1)}...g_{n-1}^{-(k-1)}\right)^{-1}})$----------------------------- (3)

Note that the definition of $[A^{(k)}]_n$ is correct since $x_{n-1}^{\bar{u}_{n-1}}\in\Omega_1\times...\times\Omega_{n-1}$ for every $u\in G$. Thus by the rule of multiplication in the finite wreath product we have $[A^{(1)}...A^{(m)}]_1=g_1^{(1)},...,g^{(m)}=[g]_1$ and for every natural $n\geq 2$ $[A^{(1)}...A^{(m)}]_n=g_n^{(1)}(\bar{x}_{n-1})g_n^{(2)}\left((\bar{x}_{n-1}^{\left(g_1^{(1)}\right)^{-1}})^{g_1^{(1)}}\right)...g_n^{(m)}\left((\bar{x}_{n-1}^{\left(g_{m-1}^{(1)}...g_{m-1}^{(m-1)}\right)^{-1}})^{\bar{g}_m^{(1)}...\bar{g}_{m-1}^{(m-1)}}\right)=$ $g_n^{(1)}(\bar{x}_{n-1})g_n^{(2)}(\bar{x}_{n-1})...g_n^{(m)}(\bar{x}_{n-1})=[g]_n$ -- (4)

Where $g=A^{(1)}A^{(2)}...A^{(m)}$ which proves (1) as required.

Corollary 4.2

Let $G=\wr_{i=1}^{n}G_i$ be a finite iterated wreath product of permutation groups $(G_i,\Omega_i),i\in N$, each of which is factorized by at most m permutation groups, that is $\forall i\in N \quad G_i=G_{i1}G_{i2}...G_{im}$

If the factorization of each $G_i$is exact, then the factorization of G is exact.

If each $G_i$is a zappa-Szep product of its subgroups, then G is a zappa-Szep product of its subgroups.

Proof:

By theorem 4.1, G = $G^{[1]}G^{[2]}...G^{[m]}$.

Take $g=[g_1,g_2(x_1),...g_n(x_n)]\in G^{[j]}\cap G^{[i]},j\neq i.$ Then g_1 belongs both to $G_{1j}\,and G_{1i}$ for every $\bar{x}_{i-1}\in\Omega_1\times...\times\Omega_{i-1}$ and every natural $i\geq 2.$ that means that the only possible value of each function g_i is 1, so$g=e,$ which means that the factorization is exact.

To prove that it is enough to show that for eachi,k the group G_{ik} intersects trivially with the group generated by G_{ij}, j≠ $k.$ the proof is analogue to that of (1).

Corollary 4.3
Let $G = \wr_{i=1}^{n} G_i$be finitely iterated wreath product of permutation groups (G_i, Ω_i), i∈ N each which is factorized by at most on permutation groups. Then $\wr_{i=1}^{n} fG_i$ can be factorized by m subgroups. Moreover, if each G_i is zappa-Szep product of its subgroups, so is $\wr_{i=1}^{n} fG_i$
Proof:
By theorem 1, $G = G^{[1]}G^{[2]} \dots G^{[m]}$, where each $G^{[k]}$ consist of

$$g^{(k)} = \left[g_1^{(k)}, g_2^{(k)}(x_1), g_3^{(k)}(x_1, x_2), \dots, g_n^{(k)}(x_{n-1}, x_n)\right], g_i^{(k)} \in G_{ik}^{\Omega_1 \times \dots \times \Omega_{i-1}}$$

Now consider any element if $\wr_{i=1}^{n} fG_i$ then it can be written as a product of those elements each $G^{[k]}$ which have only a finite number of non-trivial components. Thus if $G_f^{[k]}$is subset of $G^{[k]}$ consisting of all elements with only a finite number of nontrivial components, then it is actually a subgroup of $G^{[k]}$, which gives the required factorization. The last part follows from corollary 4.2, part 2).

Corollary 4.4.
Let $\wr_{i=1}^{n} fsG$ be a finite state iterated wreath power of a permutation group (G,Ω), which is factorized by m permutation subgroups. Then $\wr_{i=1}^{n} fsG$ can be factorized by m subgroups. Moreover, if G is a zapper-Szep product of its subgroups, so is $\wr_{i=1}^{n} fsG$
Proof:
If $G = G_i \dots G_m$, then by theorem 1,$\wr_{i=1}^{n} G = G^{[1]}G^{[2]\cdots}G^{[m]}$, where each $G^{[k]}$, consists of elements of the form

$$g^{(k)} = [g_1^{(k)}, g_2^{(k)}(x_1), g_3^{(k)}(x_1, x_2), \dots g_n^{(k)}(x_{n-1}, x_n)]$$
$$g_i^{(k)} \in G_k^{\Omega_{i-1}}$$

Now consider any element of $\wr_{i=1}^{n} fsG_i$then it can be written as a product of those elements from each $G^{[k]}$ which have only a finite number of states. Thus if $G_{fs}^{[k]}$ is a subset of $G^{[k]}$ consisting of all elements with only a finite number of states, then it is actually a subgroup of $G^{[K]}$, which gives the required factorization. The last part follows from corollary 4.2, part 2).

Example 1:
Take $\wr_{i=1}^{n} (S_3, \Omega)$ where $\Omega = \{1,2,3\}$ since S_3 is soluble of order 6, it can be factorized by its sylow 2 and 3-subgroups, which are both cyclic. Although S_3 is a semi direct product of $(\langle(1,2,3)\rangle, \Omega)$, a cyclic group of order 3 (which is normal), by $(\langle(1,2)\rangle, \Omega)$, a cyclic group of order 2, the group $\wr_{i=1}^{n} (S_3, \Omega)$ is not a semi direct of $\wr_{i=1}^{n} ((< (1,2,3) >, \Omega)$ and $\wr_{i=1}^{n} (\langle(1,2)\rangle, \Omega)$. But it is a zappa-Szep product, which follows from 2. In corollary 4.2, that means $\wr_{i=1}^{n} (S_3, \Omega) \cong (\wr_{i=1}^{n} (C_3, \Omega)) \bowtie (\wr_{i=1}^{n} (C_2, \Omega))$.

Example 2:
Take under consideration$\wr_{i=1}^{n} (A_4, \Omega)$, $\Omega = \{1,2,3,4\}$. Each of sylow 3-subgroups of A_4 has order 3 (there are 4 of them), which makes it cyclic. Choose any and denote it by C_3. Sylow 2-subgroups has order 4 and there is only such subgroup (where it is normal), namely $\{(1), (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$, which is the Klein 4-group. Thus
$A_4 \cong C_3 \times V_4$ Since $|C_3||V_4| = |A_4|$ and A_4 is generated by these two subgroups. Therefore $\wr_{i=1}^{n} (A_4, \Omega) \cong \left(\wr_{i=1}^{n} (V_4, \Omega)\right) \bowtie \left(\wr_{i=1}^{n} (C_3, \Omega)\right)$.

III. Conclusion

Based on this finding so for, on thiswork we have seen that wreath product of group can be factorized using the basic properties of factorization and also retained the properties of permutation group. More so we have observed that the factorized wreath product is either an exact factorization or zappa-Szep.

Reference

[1]. Achaku, D. T and Abam A. O, (2016). Properties of Permutation Groups using Wreath Product. *International Journal of Mathematical Research*, 5(1): 58-62
[2]. Alice, M. (2007). Distinguishing sets under Group Actions, The wreath product Action. *Paper on Algebra*1-10.
[3]. Apine E. (2015). On Application of Transitive permutation Groups to wreath product. *Mathematical theoryand modelling* 5(1):2224-5804.
[4]. Audu, M. S. (2001). Wreath Product of Permutation Group. A research Oriented Course in Arithmetic of Elliptic Curves, Groups and Loops, Lecture Notes Series, National Mathematical Centre, Abuja.
[5]. Beata B. and Vitality S, (2014). On a factorization of an iterated Wreath Product of Permutation Groups. *Journal of Algebra and Discrete Mathematics*,18(1): 14-26
[6]. BodnarchukYu .V (1994). On the isomorphism of wreath product of groups. *Ukrainian mathematical Journal* 46(6):725-734.
[7]. Bodnarchuk Yu. V (1984). The structure of the group of automorphisms of non standard wreath product of groups. *Ukrainian mathematical journal* 36(2):143-148.
[8]. CaiHeng Li and Cheryl E Praeger, (2012). On Finite Permutation Groups with a Transitive

[9]. Cyclic Subgroup. *Journal of Algebra*, 349, 117-127.

[10]. Charles W. (Wilbur), Holland Jr and Stephen H M Cleary,(1969). Wreath Product of Ordered Permutation Group.*Pacific Journal of Mathematics*, 31(3), 703-716.

[11]. Cheryl, E. P. and Saba, S. (2013). Embedding Permutation Groups in to Wreath Product in Product Action. *Researched Paper in Mathematics*, 34(2) :1-9.

[12]. Chun P. B, Gau P. D, Gyemang D. G and Nimyel T. N, (2016). An Enumaeration of the Transitivity, Primitivity and Faithfulness of the Wreath Products of Permutation Groups. *IOSR, Journal of Mathematicis*, 12(2): 35-38

[13]. Chun P. B, Choji N. M, Adidi C. S and Ajai P. T, (2014). The Transitivity, Primitivity and Faithfulness of Wreath Products of Permutation Groups. *IOSR Journal of Mathematics*, 10(3): 1-4

[14]. Cornulier Yves. (2018). Locally compact wreath products. *Journal of the Australian mathematical society.* 107(1):26-52.

[15]. Dias-Barriga A, Gonzalez-Acuna F,Marmolejo F and Romero N. (2014). Finite Metacyclic Groups AsActive Sums Of Cyclic Subgroups. Instituto de Mathematicas, UNAM1980(2):1-7

[16]. Evans S.(2002). Eigenvalues of Random wreath products. *Eletronic journal of probability* 9(7):1-15.

[17]. Eudokimov S. A and Poromarenko I. N. (2013). Schurity of S-ring over a cyclic group and generalizedwreath product of permutation groups. *St. Petersburg MathematicalJournal*.24:431-460.

[18]. Grigorchuk R. I. Nekrasheich V. V. and Sushchanskii V. I, (2000). Automata, dynamical systems, and Groups. *TR. Mat. Inst. Stekiova*.231(4):128-203.

[19]. Paula A. S. and Ian Stewart. (2000). Invariant theory for wreath product groups. *Journal of Pure and Applied Algebra*.150:61-84.

[20]. Peter, J. C. (2000). Counting Problems Related to Permutation Group. *Journal of Discrete Mathematics* 225:77-92.

[21]. Praeger C. E. and Scheider C. (2002).Ordered Triple Designs and Wreath Product Of Groups. *Australian Research Council*

[22]. Reuben G.(2002). Cellular structure of wreath product algebras. *Journal of pure and Applied algebra*.224(2):819-835.

[23]. Riedl J. M. (2009). Automorphism of Regular wreath product p-Groups. *Internation journal of mathematics and mathematical science.* 2009(245617):1-12

[24]. Soichi Okada (1990). Wreath product by symmetric Groups and product posets of youngslattices. *Journal of combination theory.series*A55:14-32

[25]. http://www.gap-system.orgmversion 4.3, 2002

ShuaibuGarbaNgulde, et. al. "Factorization of Wreath Product of Permutation Groups."*IOSR Journal of Mathematics (IOSR-JM)*, 16(5), (2020): pp. 01-06.