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The Mathematics of Natural Selection and Why Evolution & Speciation Slow Down But Never Stop

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Abstract
This paper shows the role of thermodynamics and energy transfer in ecosystem growth and development and also in evolution and speciation. It also proposes a mathematical model consistent with slowed species biodiversity growth. It also shows the non- existence of a Nash Equilibrium in terms of number of species in an ecosystem before an infinity of time. This is a wonderful example of the interdisciplinarity of the sciences and mathematics.

Background: *All living organisms require energy to live and perform critical processes. Basis the Laws of thermodynamics, this paper proposes and supports the hypothesis that available energy for growth and development in an ecosystem is constantly increasing, slowly, but surely. This amount of energy will increase with the decrease in entropy as the amount of available energy for growth and development will increase. It further correlates this with evolution and speciation.*

Laws Used: *The paper relies on drawing upon the 1^{st}& 2^{nd} law of thermodynamics along with Darwin's theory of natural selection and Competitive exclusion principle to progress the hypothesis. It takes an interdisciplinary approach to prove the hypothesis and arrive at the conclusion.*

Conclusion: *GPP (gross primary productivity) of an ecosystem will tend to approach the original GPP, but this will not happen before an infinity of time.In effect, evolution will never stop mathematically before an infinity of time. A Nash Equilibrium does not exist in terms of species biodiversity and evolution before infinite time. However, with each new species, an ecosystem moves closer to an assumed Nash Equilibrium after an infinity of time.*

Key Words: *Gross primary productivity, Laws of thermodynamics, Nash Equilibrium, Food Chain, Ecosystem.*

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I. Introduction

It is not unknown that all living organisms require energy to live and to perform various vital processes. This energy, of course, is primarily obtained from the Sun in the form of sunlight. Autotrophs (which form a majority of the consumer trophic level apart from chemotrophs), which consist mostly of green plants, blue green algae and a few bacteria, through the process of photosynthesis, convert this light energy to glucose. When consumed by consumers, only a part of this energy is transferred, which keeps decreasing in subsequent trophic levels in lie with the Second Law of Thermodynamics. However, after decomposition, a fraction of this energy is returned to the soil, a fraction of which is used by plants for their growth and development along with sunlight and solar energy.

Therefore, what this paper proposes and supports is the hypothesis that available energy for growth and development in an ecosystem (due to increase in GPP or Gross Primary Productivity and NPP or Net Primary Productivity) is constantly increasing, slowly, but surely. This amount of energy will increase with the decrease in entropy as the amount of available energy for growth and development will increase. I also will correlate this with evolution and speciation.

II. Laws Used

The First Law of Thermodynamics:
This law states that energy can neither be created, nor be destroyed, but can only be transformed from one form into another. This implies that the total amount of energy in a system remains constant, assuming of course particles in the system do not move at velocities comparable to that of light.

The Second Law of Thermodynamics[1](Spakovszky, 5.1 Concept and Statements of the Second Law (Why do we need a second law?), n.d.)
This law shows that for every system in equilibrium, there will always exist entropy, which is in simple terms, the amount of energy lost or degraded during energy transfer. In reversible processes, the changes in entropy

and the surroundings are positive and approach zero with increasing reversibility. However, in isolated systems, entropy changes in the system have no effect on the surroundings.

Darwin's Theory of Natural Selection:

Nature selects only those individuals with advantageous variances or reproductive advantages over others. There is an overproduction of offspring and only the fittest survive.

Competitive Exclusion Principle [2](Khan Academy, n.d.):

No two species can achieve coexistence and stability within the same habitat in an environment or ecosystem. The one without advantageous variances like reproductive advantages will be forced to occupy a smaller assumed niche.

MATHEMATICAL REPRESENTATION OF ENERGY FLOW IN A SIMPLE FOOD CHAIN

While energy flow in ecosystems is assumed to be unidirectional, the hypothesis of this paper is based on the fact that the same is actually cyclic. What follows is the mathematical proof and model for the same.

In the food chain considered, it is assumed that each producer (autotroph) and consumer (heterotroph) is consumed by any subsequent trophic level occupied by a consumer.

Let the sum of entropy and energy wasted as heat during life processes, according to the Second Law of Thermodynamics, be constant and be considered to be equal to a fraction x of the energy transferred between two consecutive trophic levels.

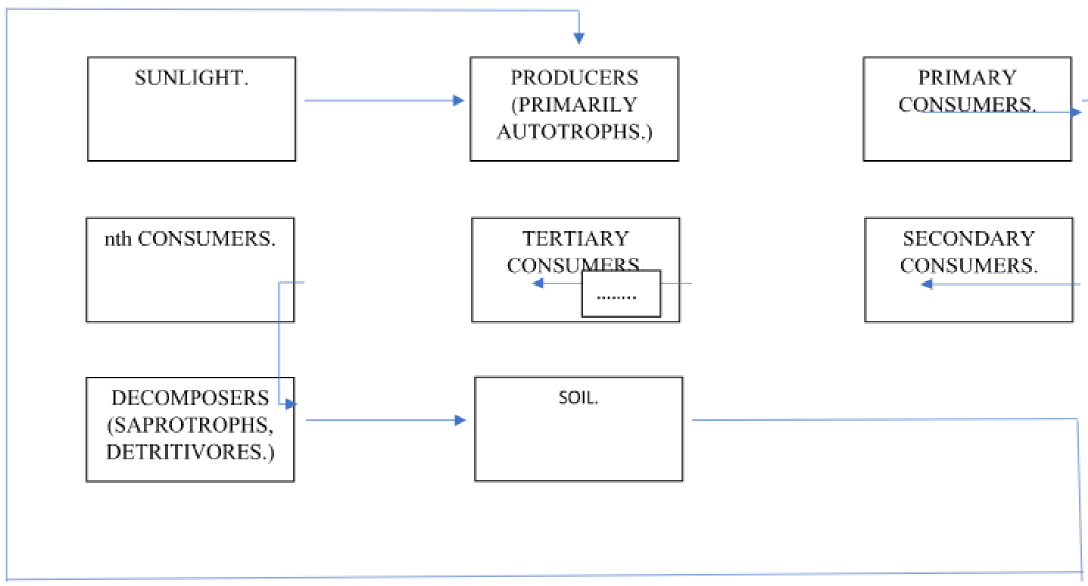

Figure 1-A Simple Food Chain

Given an initialGross Primary Productivity (GPP) of A_0 for plants (autotrophs/ producers), the energy received by primary producers is $A_0(1-x)$.

Subsequently, the following also holds true:

Energy received by Secondary Consumers = $A_0(1-x)^2$.

Energy received by Tertiary Consumers = $A_0(1-x)^3$.

∴Energy received by nth Consumer = $A_0(1-x)^n$.

Energy received by Decomposers (Saprotrophs/ Detritivores) = $A_0(1-x)^{n+1}$.

Energy in decomposed matter or nutrients returned to soil = $A_0(1-x)^{n+2}$.

Energy received by new Consumers from the soil = $A_0(1-x)^{n+3}$.

∴New GPP $A_1$of Primary Consumers = $A_0 + A_0(1-x)^{n+3} = A_0[1+(1-x)^{n+3}]$

Similarly, $A_2 = A_0 + A_1(1-x)^{n+3}$

⇨ $A_2 = A_0 + [A_0 + A_0(1-x)^{n+3}]\,(1-x)^{n+3}$

⇨ $A_2 = A_0 + A_0(1-x)^{n+3} + A_0(1-x)^{2(n+3)}$

Similarly, $A_3 = A_0 + A_2(1-x)^{n+3}$

⇨ $A_3 = A_0 + [A_0 + \{A_0 + A_0(1-x)^{n+3}\}\,(1-x)^{n+3}]\,(1-x)^{n+3}$

⇨ $A_3 = A_0 + [A_0 + A_0(1-x)^{n+3} + A_0(1-x)^{2(n+3)}]\,(1-x)^{n+3}$

⇨ $A_3 = A_0 + A_0(1-x)^{n+3} + A_0(1-x)^{2(n+3)} + A_0(1-x)^{3(n+3)}$

∴It can be clearly noted that the (a+1)th GPP or $A_a = \sum_{t=0}^{a} A_0(1-x)^{t(n+3)}$ _(1)

∴Percentage change in A (%ΔA) = $(A_a - A_0)/A_0 \times 100\% = [\sum_{t=0}^{a} A_0(1-x)^{t(n+3)} - A_0]/A_0 \times 100\% = [\sum_{t=1}^{a} A_0(1-x)^{t(n+3)}]/A_0 \times 100\% = \sum_{t=1}^{a}(1-x)^{t(n+3)} \times 100\%$ **_(2)**

An example Case:
Taking x=0.9, a as 1, n as 3,
From (1),
$A_a = A_1 = A_0 + A_0(0.1)^6$
From (2),
$\%\Delta A = 10^{-4}\% = 0.0001\%$
While such a small increase % in GPP may seem insignificant, when the amount of energy that it represents in an ecosystem is extremely significant.
One thing to note is that with a new species in a food web, it often increases the number of trophic levels in 1 or more food chains.

A FUNDAMENTAL OBSERVATION:

The Theory of Natural Selection, which Charles Darwin proposed, explains the concept of evolution saying that nature selects those species with even the slightest of reproductive advantages. According to the Competitive Exclusion Principle, two species cannot occupy the same environmental or ecological niche in the same environment or ecosystem. The genetically weaker species will eventually be forced to occupy a smaller, assumed niche.

Species evolve or new species are formed according to a number of principles including Darwin's Theory of Natural Selection. However, this happens because species are able to get reproductive and other advantages with the help of increased energy.

For instance, let us consider primary succession. The first species observed are bacteria, followed by other microscopic organisms, up until the larger flora and fauna that occupy forests.

One thing that can be clearly noted from the above is the fact that the evolutionary wheel, or the speed or rapidity at which evolution takes place, or new species are formed (speciation), slows with the increase in the number of species (especially when it increases the number of trophic levels in a food chain or the number of species inside a food web). The increase in biodiversity never stops, but keeps on slowing down.The extinction of a species, in most cases dominant species, is also often detrimental to the environment at first, but then as time goes on, whole new ecosystems, new species develop and flourish and thrive where once previously dominant species used to thrive. For the sake of simplicity, let us consider the example of the dinosaurs. Had they not gone extinct, the doubt arises, would the world be as it is today, or would it be missing many of the species of flora and fauna that inhabit it today, possibly even homo sapiens sapiens?

This observation points to one thing, that the more the number of species, or even trophic levels in an ecosystem, the less likely it is that it will develop even more with the same rapidity, the less likely it is that its species biodiversity will augment itself with the same speed, the less likely it is that evolution goes on at the same rate and speed as it did earlier.

CONSISTENCE OF THE MATHEMATICAL MODEL WITH THIS OBSERVATION AND WITH DARWIN'S THEORY OF NATURAL SELECTION:

A possible reason that can be attributed to the slowing down of biodiversity growth and evolution as time passes is increasing competition. However, this is not the only possible reason behind all these observed phenomena.

In primary succession, initially the number of producers n, in (1) and (2) will be 0, so GPP will register a large growth. Therefore, the GPP increases in an ecosystem initially in primary succession, withproducers and decomposers.

However, as n increases with the increase in the number of consumers and the total number of trophic levels n+3 also increases, the GPP registers a decrease (from hypothetically 1.00001 of initial GPP, it may become 1.000001 of the original GPP and then increase again.) When n is constant for some time, GPP starts increasing slowly again. A new species will form by speciation or a species will evolve only when the increase in GPP is enough to cater to the energy demands for the same. The GPP will decrease once again but will start increasing even more slowly after n becomes constant, until another species originates. In summary, the GPP approaches the original/ initial GPP, but does not reach it before an infinity of time.

This proves consistence with the Theory of Natural Selection, showing that reproductive advantages required for evolution come from the fulfilment of energy requirements for the same by an increase in GPP. This also shows consistence of the mathematical model with the aforementioned observation of slowed biodiversity growth.

THEORETICAL PROOF OF NON-EXISTENCE OF A NASH EQUILIBRIUM IN TERMS OF BIODIVERSITY GROWTH OR IN TERMS OF THE NUMBER OF SPECIES IN AN ECOSYSTEM BEFORE INFINITE TIME:

New species will stop originating or evolution will stop only when species have reached a Nash Equilibrium and gaining any features or favourable characters will cause extinction of that species due to inability to meet energy requirements for the same. However, the mathematical model proposed in (1) and (2) show that no matter how many species have evolved or how much speciation has occurred, the GPP will increase for a constant n infinitely, such that species will be able to acquire advantages extremely slowly due to small increases in GPP. However, when accumulated over a long period of time, these advantages will cause evolution or speciation, which will then again slow the rate of GPP increaseand decrease it once before it starts increasing with that slow rate.Once again on accumulation of advantages or favourable variances over an even longer period of time due to slower growth in available energy, speciation or evolution will take place. While this may take extremely long periods of time (subsequently longer), biodiversity and the number of species will keep increasing.

III. Conclusion

In other words, GPP will tend to approach the original GPP, but this will not happen before an infinity of time. **Therefore, mathematically, evolution will never stop mathematically before an infinity of time.** Therefore, (1) and (2) clearly disprove the existence of a Nash Equilibrium in terms of species biodiversity and evolution before infinite time. However, with each new species, an ecosystem moves closer to an assumed Nash Equilibrium after an infinity of time.

A FEW OTHER IMPLICATIONS OF THE MATHEMATICAL MODEL PROPOSED:

1. GPP of one ecosystem can also be increased by adding manure/ fertilizers. However, if made from material from other ecosystems, the GPP of that ecosystem will move closer to the original/ initial GPP. Evolution or speciation will now take place faster in the ecosystem that has been manured or fertilized. However, too much fertilizer or manure may also cause evolution of pests or weeds as well due to increase in GPP. Therefore, manure or fertilizers must be incorporated, along with pesticides, in a balanced and well contemplated manner, in agricultural or agroforestry systems.
2. Even if competition with or predation upon native species of the ecosystem is not enough to make the native species of the ecosystem extinct in any form, the introduction of invasive species will slow down species evolution or even speciation and therefore, will slow the growth of species biodiversity of an ecosystem, especially if it occupies a new trophic level.
3. Species evolution will never stop or speciation will never stopi.e. a species will never go ‘evolutionarily extinct’ unless it goes biologically extinct. As proven by the mathematical model, evolution and speciation may slow down but will never stop in an ecosystem.

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