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Generalized a New Class of Harmonic Univalent Functions

NAGALAXMI NAKEERTHA

Abstract
In this present paper , we defined complex valued functions that are univalent of the form f = h+$\bar{g}$ where h and g are analytic in the open unit disk Δ . we obtain several sufficient coefficient conditions for normalized harmonic functions that are starlike of order $((\lambda,\alpha,k)\ \ 0 \le \lambda < 1, 0 \le \alpha < 1, 0 \le k < 1$ *there coefficients condtions are also shown to necessary when h has negative and g has postive coefficients.*
***Key Words**: Harmonic function ,univalent function sense-preserving; starlike.convex combination*

I. INTRODUCTION

A continuous function f = u+iv is a complex-valued harmonic function in a complex domain $\mathfrak{C}$ if both u and v are real harmonic in $\mathfrak{C}$. In any simply connected domain $\mathfrak{D} \in \mathfrak{C}$ we can write f= h +$\bar{g}$ where h and g are analytic in $\mathfrak{D}$. We call h the analytic part and g the co-a$\mathfrak{C}$nalytic part of f. A necessary and sufficient condition for f to be locally univalent and sense-preserving in $\mathfrak{D}$ is that in $\mathfrak{D}$. See Clunie and Sheil-Small [2] .

Denote by $\mathcal{H}$ the class of functions f= h +$\bar{g}$ that are harmonic univalent and sense-preserving in the unit disk $\Delta = \{ z: |z| < 1 \}$for which

h(0) = f(0) = $f_z(0) - 1$ Then for f= h +$\bar{g}$ $\in \mathcal{H}$ we may express the analytic functions h and g as ` `

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad g(z) = \sum_{n=1}^{\infty} b_n z^n \qquad (1)$$

Note that $\mathcal{H}$ reduces to the class of normalized analytic univalent functions if the co-analytic part of its members is zero. In 1984 Clunie and Sheil-Small [2] investigated the class $\mathcal{H}$ as well as its geometric subclasses and obtained some coefficient bounds. Since then, there have been several related papers on $\mathcal{H}$ and its subclasses. For more references see Duren [3] . w x In this note, we look at two subclasses of $\mathcal{H}$ and provide univalence criteria, coefficient conditions, extreme points, and distortion bounds for functions in these classes.

For $0 \le \alpha < 1$ we let $\mathcal{G}_{\mathcal{H}}(\alpha)$. a denote the subclass of $\mathcal{H}$ consisting of $\mathcal{H}$ harmonic starlike functions of order α. A function f of the form (1) is harmonic starlike of order α, $0 \le \alpha < 1$ for $|z| = r < 1$ if

$$\frac{\partial}{\partial\theta} \ (\arg f (r\, e^{i\theta})) \ge \alpha, |z| = r < 1 \qquad (2)$$

We further denote by $\mathcal{G}_{\mathcal{H}}(\alpha)$.. a the subclass of $\mathcal{G}_{\mathcal{H}}(\alpha)$.. a such that the functions h and g in f= h +$\bar{g}$ are of the form

$$h(z) = z - \sum_{n=2}^{\infty} |a_n| z^n \quad , \quad g(z) = \sum_{n=1}^{\infty} |b_n| z^n \qquad (3)$$

II. MAIN RESULTS

It was shown by Sheil-Small [4, Theorem 7] that $|a_n| \le (n+1)$ (2n+1)/6 and $|b_n| \le (n-1)$ (2n-1)/6 if f= h +$\bar{g}$ $\in \mathcal{G}_{\mathcal{H}}{}^0(0)$.

The subclass of $\mathcal{G}_{\mathcal{H}}(\alpha)$. where $\alpha = b_1 = 0$ is denoted by $\mathcal{G}_{\mathcal{H}}{}^0(0)$. These bounds are sharp and thus give necessary coefficient conditions for the class $\mathcal{G}_{\mathcal{H}}{}^0(0)$.

Avci and Zlotkiewicz [1] proved that the coefficient condition is sufficient for functions f= h +$\bar{g}$ to be in $\mathcal{G}_{\mathcal{H}}{}^0(0)$.Silverman proved that this coefficient condition is also necessary if $b_1 = 0$ and a_n if a and b in 1 are negative.

We note that both results obtained in are subject to the restriction that $b_1 = 0$. The argument presented in this paper provides sufficient coefficient conditions for functions $\mathcal{G}_{\mathcal{H}}(\alpha)$ f= h +$\bar{g}$ of the form (1) to be in $\mathcal{G}_{\mathcal{H}}(\alpha)$ where 0≤ $\alpha < 1$ and b_1 not necessarily zero. It is shown that these conditions are also necessary when f $\in \mathcal{G}_{\mathcal{H}}(\alpha)$. a .

THEOREM 1.

Let f= h +$\bar{g}$ be given by (1). Furthermore, let `$\sum_{n=1}^{\infty}[n^2(\beta+1)-(\beta+\alpha)(\lambda^n-\lambda+1)]\,|a_n|+\sum_{n=1}^{\infty}[n^2(\beta+1-(\beta+\alpha)(\lambda n-\lambda+1)]\,b_n \leq 2-\alpha$ .Where $a_1=1$ and $0\leq\lambda<1$, $0\leq\alpha<1$, $\beta\geq0$, then f is harmonic univalent in Δ and f $\in \mathcal{G}_{\mathcal{H}}(\lambda, \alpha, \beta)$

Proof is later

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