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# A Study of Finite Integral Operators Involving the Product of a General Class of Polynomials and the Multivariable H-Function

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***Abstract:*** *In this paper, we evaluate a finite integral involving the product of a general class of polynomials and the multivariable H-function. Also we reduce the H-function of several variables to the product of whittaker function and multivariable H-function to a generalized hyper-geometric function of several variable.*
***Keywords:*** *H-function, Multivariable H-function, Contour Integral, Hyper-geometric function.*

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## I. Introduction

The new modified H-function that is generalization studied by Kantesh Gupta and VandanaAgarwal [12] will be defined and represented in the following manner.

$H(s_1,s_2,\ldots,s_r) = H^{O,N:M^1,N^1;\ldots\ldots;M^{(r)},N^{(r)}}_{P,Q\,:P^1,Q^1\,;\ldots\ldots;P^{(r)},Q^{(r)}}$ [$F_1(x_1, \ldots, x_r)$; $\rho_1,\ldots,\rho_r;\ S_1,\ldots\ldots S_r$]
=

$\int_{d_1}^{\infty}\ldots\ldots\int_{d_r}^{\infty}\prod_{i=1}^{r}(S_i)^{\rho_i-1}\,H^{O,N:M^1,N^1;\ldots\ldots;M^{(r)},N^{(r)}}_{P,Q\,:P^1,Q^1\,;\ldots\ldots;P^{(r)},Q^{(r)}}\left[\begin{matrix}S_1\\ \vdots\\ S_r\end{matrix}\,\begin{matrix}(a_j;\alpha_j{}^1;\ldots\alpha_j{}^{(r)})_{1,P}:(C_j{}^1,\gamma_j{}^1)_{1,P}:\ldots;(C_j{}^{(r)},\gamma_j{}^{(r)})_{1,P^{(r)}}\\ (b_j;\beta_j{}^1;\ldots\beta_j{}^{(r)})_{1,Q}:(d_j{}^1,\eta_j{}^1)_{1,Q}:\ldots;(d_j{}^{(r)},\eta_j{}^{(r)})_{1,Q^{(r)}}\end{matrix}\right]$ $F_1(x_1, \ldots, x_r)dx_1\ldots dx_r$ ……………………………………(1.1.1)

Where $F_1(x_1, \ldots, x_r) = f(a_1\sqrt{x_1{}^2-d_1{}^2U(x_1-d_1)}, \ldots\ldots, a_n\sqrt{x_r{}^2-d_r{}^2U(x_r-d_r)})$

$x_1 > d_1 > 0, \ldots\ldots, x_r > d_r > 0$ ……………………………………………(1.1.2)

Here $U(x_i-d)$ (i= 1, 2, ……, r) is the well known Heaviside's unit function. Further we assume the function $f_1(x_1, \ldots, x_r)$ on $R_+{}^{(n)}$ which are infinitely differentiable with partial derivatives of any order such that

$f_1(x_1, \ldots, x_r) = \begin{bmatrix} o(|x_i|^{w_i})(\max\{|x_1|,\ldots\ldots,|x_r|\}\to 0)\\ o(|x_i|^{-T_i})\quad(\min\{|x_1|,\ldots\ldots,|x_r|\}\to\infty)\end{bmatrix}$ ……………….(1.1.3)

The function defined by (1.1.1) exists provided the following (sufficient) conditions are satisfied:

(i) $|\arg S_i| < \frac{1}{2}\pi\frac{\Omega_i}{k_i}$

Where

$\Omega_i = -\sum_{j=1}^{p}\alpha_j{}^{(i)} - \sum_{j=1}^{Q}\beta_j{}^{(i)} + \sum_{j=1}^{N_i}\gamma_j{}^{(i)} - \sum_{j=N_i+1}^{p_i}\gamma_j{}^{(i)} + \sum_{j=1}^{M_i}\eta_j{}^{(i)} - \sum_{j=M_i+1}^{Q_i}\eta_j{}^{(i)} > 0$

(ii) $\Re(\omega_i) + 1 > 0$

(iii) $\Re(\rho_i-\tau_i) + k_i\,max_{1\le j\le N_r}[\Re\frac{(C_j{}^{(i)}}{\gamma_j{}^{(i)}}] < 0$, $\forall$ i$\in$ {1, …… . r}.

The multivariable H-function has been studied extensively by Srivastava and Panda in their two basic paper on the subject. In this paper, we shall defined and represent it in the following manner[11].

$H(z_1,z_2,\ldots,z_r) = H^{O,N:M^1,N^1;\ldots\ldots;M^{(r)},N^{(r)}}_{P,Q\,:P^1,Q^1\,;\ldots\ldots;P^{(r)},Q^{(r)}}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\,\begin{matrix}(a_j;\alpha_j{}^1;\ldots\alpha_j{}^{(r)})_{1,P}:(C_j{}^1,\gamma_j{}^1)_{1,P}:\ldots;(C_j{}^{(r)},\gamma_j{}^{(r)})_{1,P^{(r)}}\\ (b_j;\beta_j{}^1;\ldots\beta_j{}^{(r)})_{1,Q}:(d_j{}^1,\eta_j{}^1)_{1,Q}:\ldots;(d_j{}^{(r)},\eta_j{}^{(r)})_{1,Q^{(r)}}\end{matrix}\right]$

$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\ldots\int_{L_r}\psi(\xi_1,\ldots\ldots\xi_r)\prod_{i=1}^{r}\{\phi_i(\xi_i)z_i{}^{\xi_i}d\xi_1,\ldots\ldots d\xi_r$ ……………(1.1.4)

Where $\omega = \sqrt{-1}$

$\phi_i(\xi_i) = \frac{\prod_{j=1}^{M_i}\Gamma(d_j{}^{(i)}-\eta_j{}^{(i)}\xi_i)\prod_{j=1}^{N_i}\Gamma(1-c_j{}^{(i)}-\gamma_j{}^{(i)}\xi_i)}{\prod_{j=M_i+1}^{Q_i}\Gamma(1-d_j{}^{(i)}+\eta_j{}^{(i)}\xi_i)\prod_{j=N_i+1}^{P_i}\Gamma(c_j{}^{(i)}-\gamma_j{}^{(i)}\xi_i)}$, $\forall$ i$\in$ {1, …… . r}……..(1.1.5)

$\psi(\xi_1,\ldots\xi_r) = \frac{\prod_{j=1}^{N}\Gamma(1-a_j+\sum_{i=1}^{r}\alpha_j{}^{(i)}\xi_i)}{\prod_{j=N+1}^{P}\Gamma(a_j-\sum_{j=1}^{r}\alpha_j{}^{(i)}\xi_i)\cdot\prod_{j=1}^{Q}\Gamma(1-b_j+\sum_{i=1}^{r}\beta_j{}^{(i)}\xi_i)}$, $\forall$ i$\in$ {1, …… . r}

……………(1.1.6)

For the convergence, existence conditions and other details of the multivariable H-function, we refer to the book mentioned.

In this paper, we shall study the fractional integral operator involving multivariable H-function, which is generalization of an operator studied by Sexena and Kumbhat [9] and dcefined as follows:

$$R^{\eta_1,\ldots\ldots,\eta_n,\alpha}_{x_1,\ldots\ldots,x_n,r_n}\,[f_1(x_1,\ldots\ldots,x_n)] = \prod_{i=1}^{n}(r_i x_i^{-\eta_i - r_i\alpha-1})\int_{t_1=0}^{x_1}\ldots\int_{t_n=0}^{x_n}\{\prod_{i=0}^{n}(t_i^{\eta_i}(x_i^{r_i}-t_i^{r_i})\} = H^{O,N:M^1,N^1;\ldots\ldots;M^{(n)},N^{(n)}}_{P,Q\,:P^1,Q^1\;;\ldots\ldots;P^{(n)},Q^{(n)}}\left[\begin{matrix} K_1\left(\frac{t_1^{r_1}}{x_1^{r_1}}\right)^{m_1}\left(1-\frac{t_1^{r_1}}{x_1^{r_1}}\right)^{n_1} \\ \vdots \\ K_n\left(\frac{t_n^{r_n}}{x_n^{r_n}}\right)^{m_n}\left(1-\frac{t_n^{r_n}}{x_n^{r_n}}\right)^{n_n}\end{matrix}\;\begin{matrix}(a_j;\alpha_j^{1};\ldots\alpha_j^{(r)})_{1,P}:(C_j^{1},\gamma_j^{1})_{1,P}:\ldots;(C_j^{(r)},\gamma_j^{(r)})_{1,P^{(r)}} \\ (b_j;\beta_j^{1};\ldots\beta_j^{(r)})_{1,Q}:(d_j^{1},\eta_j^{1})_{1,Q}:\ldots;(d_j^{(r)},\eta_j^{(r)})_{1,Q^{(r)}}\end{matrix}\right]$$

$f(t_1,\ldots,t_n)dt_1\ldots dt_n$ ……………………………………(1.1.7)

where N , P , Q , $M_i$ , $N_i$ , $P_i$ , $Q_i$ are non negative integrals such that N = P =0 , Q = 0 , $M_i = Q_i = 0$ and $N_i = P_i = 0$ and $|\arg K_i| < \frac{1}{2}\,\Omega`_i\pi$ ($\Omega`_i>0$)

$$\Omega_i = -\sum_{j=N+1}^{p}\alpha_j^{(i)} - \sum_{j=1}^{Q}\beta_j^{(i)} + \sum_{j=1}^{N_i}\gamma_j^{(i)} - \sum_{j=N_i+1}^{p_i}\gamma_j^{(i)} + \sum_{j=1}^{M_i}\eta_j^{(i)} - \sum_{j=M_i+1}^{Q_i}\eta_j^{(i)} > 0$$

Here $r_i$ , $m_i$ , $n_i$ are non-negative integrals. The (sufficient) condition of validity of operator are given below

(1) $\Re(\eta_i) + r_i m_i \min_{1\le j\le M_i}\left\{\Re\left(\frac{d_j^{(i)}}{\eta_j^{(i)}}\right)\right\} + 1 > 0$ (i = 1, ……, n)

(2) $\Re(\alpha) + n_i \min_{1\le j\le M_i}\left\{\Re\left(\frac{d_j^{(i)}}{\eta_j^{(i)}}\right)\right\} + 1 > 0$ (i = 1, ……, n)

**1.2. Main Theorem**

In this section we first prove our main result as detailed below.

Theorem :

If H($s_1,s_2,\ldots,s_r$) =

$$\int_{d_1}^{\infty}\ldots\ldots\int_{d_r}^{\infty}\prod_{i=1}^{r}(S_i)^{\rho_i-1}H^{O,N:M^1,N^1;\ldots\ldots;M^{(r)},N^{(r)}}_{P,Q\,:P^1,Q^1\;;\ldots\ldots;P^{(r)},Q^{(r)}}\left[\begin{matrix}s_1\\ \vdots \\ s_r\end{matrix}\;\begin{matrix}(a_j;\alpha_j^{1};\ldots\alpha_j^{(r)})_{1,P}:(C_j^{1},\gamma_j^{1})_{1,P}:\ldots;(C_j^{(r)},\gamma_j^{(r)})_{1,P^{(r)}} \\ (b_j;\beta_j^{1};\ldots\beta_j^{(r)})_{1,Q}:(d_j^{1},\eta_j^{1})_{1,Q}:\ldots;(d_j^{(r)},\eta_j^{(r)})_{1,Q^{(r)}}\end{matrix}\right] F_1(x_1,\ldots,x_r)dx_1\ldots dx_r$$

And

$$R^{\eta_1,\ldots\ldots,\eta_n,\alpha}_{x_1,\ldots\ldots,x_n,r_n}\,[f_1(x_1,\ldots\ldots,x_n)] = \prod_{i=1}^{n}(r_i x_i^{-\eta_i - r_i\alpha-1})\int_{t_1=0}^{x_1}\ldots\int_{t_n=0}^{x_n}\left\{\prod_{i=0}^{n}(t_i^{\eta_i}(x_i^{r_i}-t_i^{r_i})\right\} = H^{O,N:M^1,N^1;\ldots\ldots;M^{(n)},N^{(n)}}_{P,Q\,:P^1,Q^1\;;\ldots\ldots;P^{(n)},Q^{(n)}}\left[\begin{matrix} K_1\left(\frac{t_1^{r_1}}{x_1^{r_1}}\right)^{m_1}\left(1-\frac{t_1^{r_1}}{x_1^{r_1}}\right)^{n_1} \\ \vdots \\ K_n\left(\frac{t_n^{r_n}}{x_n^{r_n}}\right)^{m_n}\left(1-\frac{t_n^{r_n}}{x_n^{r_n}}\right)^{n_n}\end{matrix}\;\begin{matrix}(a_j;\alpha_j^{1};\ldots\alpha_j^{(r)})_{1,P}:(C_j^{1},\gamma_j^{1})_{1,P}:\ldots;(C_j^{(r)},\gamma_j^{(r)})_{1,P^{(r)}} \\ (b_j;\beta_j^{1};\ldots\beta_j^{(r)})_{1,Q}:(d_j^{1},\eta_j^{1})_{1,Q}:\ldots;(d_j^{(r)},\eta_j^{(r)})_{1,Q^{(r)}}\end{matrix}\right]$$

$f(t_1,\ldots,t_n)dt_1\ldots dt_n$ ………………………………….(1.2.1)

Then

$$R^{\eta_1,\ldots\ldots,\eta_n,\alpha}_{x_1,\ldots\ldots,x_n,r_n}H(s_1,s_2,\ldots,s_r) = \int_{d_1}^{\infty}\ldots\ldots\int_{d_r}^{\infty}\prod_{i=1}^{r}(S_i)^{\rho_i-1} H^{0,r+N:M_1,1+N_1;\ldots;M_r,1+N_r;\,M_1,N_1;\ldots\ldots;M_r,N_r}_{r+P,r+Q\,:P_1+1,Q_1;\ldots;P_r+1,Q_n;P_1,Q_1\;;\ldots;P_r,Q_r}\left[\begin{matrix}K_1 & \\ \vdots & A^*:C^* \\ & K_r \\ & (s_1)^{k_1} \\ \vdots & B^*:D^* \\ & (s_r)^{k_r}\end{matrix}\right] F_1(x_1,\ldots,x_r)dx_1\ldots dx_r \ldots\ldots(1.2.2)$$

Where

$$A^* = \left\{\begin{matrix}\left(\left(1-\left(\frac{\eta_1+\rho_1}{r_1}\right);\,m_1,\underbrace{0,\ldots\ldots,0}_{n-1},\frac{k_1}{r_1},\quad\underbrace{0,\ldots\ldots,0}_{n-1}\right),\left(1-\left(\frac{\eta_2+\rho_2}{r_2}\right);\,0,m_2,\underbrace{0,\ldots\ldots,0}_{n-1},0,\frac{k_2}{r_2},\quad\underbrace{0,\ldots\ldots,0}_{n-2}\right),\right. \\ \ldots\ldots,\left(1-\left(\frac{\eta_n+\rho_n}{r_n}\right);\underbrace{0,\ldots,0}_{n-1},m_n,\underbrace{0,\ldots,0}_{n-1},\frac{k_n}{r_n}\right)\left(a_j;\alpha_j^{1},\ldots,\alpha_j^{n},\underbrace{0,\ldots\ldots,0}_{n}\right)_{1,P} \\ \left(\left(a_j;\underbrace{0,\ldots\ldots,0}_{n},\alpha_j^{1},\ldots,\alpha_j^{n},\right)_{1,P_1}\right)\end{matrix}\right\}$$

$$B^* = \left\{ \begin{matrix} \left(-\alpha - \left(\frac{\eta_1+\rho_1}{r_1}\right);\ (m_1+n_1), \underbrace{0,..,0}_{n-1}, \frac{k_1}{r_1}, \underbrace{0,..,0}_{n-1}\right), \begin{pmatrix} -\alpha - \left(\frac{\eta_2+\rho_2}{r_2}\right);\ 0, (m_2+n_2), \underbrace{0,\dots,0}_{n-1}, 0, \frac{k_2}{r_2}, \\ \underbrace{0,\dots..,0}_{n-2} \end{pmatrix}, \\ \dots\dots, \left(-\alpha - \left(\frac{\eta_n+\rho_n}{r_n}\right); \underbrace{0,\dots,0}_{n-1}, (m_n+n_n, \underbrace{0,\dots,0}_{n-1}, \frac{k_n}{r_n}\right)\left(b_j; \beta_j{}^1, \dots., \beta_j{}^n, \underbrace{0,\dots..,0}_{n}\right)_{1,Q} \\ \left(\left(b_j; \underbrace{0,\dots..,0}_{n-1}, \beta_j{}^1, \dots., \beta_j{}^n,\right)_{1,Q_1}\right) \end{matrix} \right\}$$

$$C^* = \left\{ \begin{matrix} (-\alpha, n_1), \left(c_j{}^1, \gamma_j{}^1\right)_{1,P_1};\ (-\alpha, n_2), \left(c_j{}^2, \gamma_j{}^2\right)_{1,P_2}; \dots\dots;\ (-\alpha, n_n), \left(c_j{}^n, \gamma_j{}^n\right)_{1,P_n};\ \left(c_j{}^1, \gamma_j{}^1\right)_{1,P_1}; \\ (c_j{}^2, \gamma_j{}^2)_{1,P_2}; \dots.;\ (c_j{}^n, \gamma_j{}^n)_{1,P_n} \end{matrix} \right\}$$

$$D^* = \left\{ \left(d_j{}^1, \eta_j{}^1\right)_{1,Q_1};\ \left(d_j{}^2, \eta_j{}^2\right)_{1,Q_2}; \dots\dots.;\ \left(d_j{}^n, \eta_j{}^n\right)_{1,Q_n} \right\}$$

Provided that

(i) $|\arg S_i| < \frac{1}{2}\pi \frac{\Omega_i}{k_i}$ $(\Omega_i > 0)$

(ii) $|\arg K_i| < \frac{1}{2}\grave{\Omega}_i \pi$ $(\grave{\Omega}_i > 0)$

(iii) $r_i$ , $m_i$ , $n_i$ are non-negative integers.

(iv) $\Re(\omega_i) + 1 > 0$

And $\Re(\rho_i - \tau_i) + k_i max_{1 \le j \le N_r}[\Re \frac{(C_j{}^{(i)}}{\gamma_j{}^{(i)}}] < 0$ , $\forall$ i$\in$ {1, …… . r}.

(v) $\Re(\eta_i) + r_i m_i min_{1 \le j \le M_i}\left\{\Re(\frac{d_j{}^{(i)}}{\eta_j{}^{(i)}})\right\} + 1 > 0$ (i = 1, ……, n)

And $\Re(\alpha) + n_i min_{1 \le j \le M_i}\left\{\Re(\frac{d_j{}^{(i)}}{\eta_j{}^{(i)}})\right\} + 1 > 0$ (i = 1, ……, n)

(vi) $\Re(\frac{\eta_i+\rho_i}{r_i}) + m_i min_{1 \le j \le M_i}\left\{\Re(\frac{d_j{}^{(i)}}{\eta_j{}^{(i)}})\right\} > 0$

Proof:

On substituting the value of $H(s_1,\dots\dots,s_r)$ from (1.1.2) and left hand side of (1.2.1), we get

$$R^{\eta_1,\dots\dots,\eta_n,\alpha}_{x_1,\dots\dots,x_n,r_n} H(s_1,\dots\dots,s_r) = \prod_{i=1}^n (r_i x_i{}^{-\eta_i - r_i\alpha - 1}) \int_{t_1=0}^{x_1} \dots. \int_{t_n=0}^{x_n} \{\prod_{i=0}^n (t_i{}^{\eta_i}(x_i{}^{r_i} - t_i{}^{r_i})^{\alpha}\}$$
$$. H^{O,N:M^1,N^1;\dots\dots\dots;M^{(n)},N^{(n)}}_{P,Q:P^1,Q^1;\dots\dots\dots;P^{(n)},Q^{(n)}} \left[K_1\left(\frac{t_1{}^{r_1}}{s_1{}^{r_1}}\right)^{m_1}\left(1-\frac{t_1{}^{r_1}}{s_1{}^{r_1}}\right)^{n_1} \dots. K_n\left(\frac{t_n{}^{r_n}}{s_n{}^{r_n}}\right)^{m_n}\left(1-\frac{t_n{}^{r_n}}{s_n{}^{r_n}}\right)^{n_n}\right]$$
$$. \int_{d_1}^{\infty} \dots\dots \int_{d_r}^{\infty} \prod_{i=1}^{r} (t_i x_1)^{\rho_i - 1} . H^{O,N:M^1,N^1;\dots\dots\dots;M^{(r)},N^{(r)}}_{P,Q:P^1,Q^1;\dots\dots\dots;P^{(r)},Q^{(r)}}[(t_1x_1)^{k_1}, \dots\dots, (t_n x_n)^{k_n}]$$
$$. \{F_1(x_1, \dots., x_r) dx_1 \dots\dots dx_r\} dt_1 \dots\dots dt_r \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1.2.3)$$

Now interchanging the order of $x_i$ and $t_i$ integral which is permissible under the conditions, we obtain

$$R^{\eta_1,\dots\dots,\eta_n,\alpha}_{x_1,\dots\dots,x_n,r_n}[H(s_1,\dots\dots,s_r)] = \prod_{i=1}^n (r_i x_i{}^{-\eta_i - r_i\alpha - 1}) \int_{d_1}^{\infty} \dots. \int_{d_r}^{\infty} \prod_{i=1}^r (x_i)^{\rho_i - 1} . F_1(x_1, \dots., x_r)$$
$$. \left\{\int_{t_1=0}^{s_1} \dots. \int_{t_n=0}^{s_n} \left\{\prod_{i=0}^{n} (t_i{}^{\eta_i + \rho_i - 1}(s_i{}^{r_i} - t_i{}^{r_i})^{\alpha}\right\}\right\}$$
$$. H^{O,N:M^1,N^1;\dots\dots\dots;M^{(n)},N^{(n)}}_{P,Q:P^1,Q^1;\dots\dots\dots;P^{(n)},Q^{(n)}} \left[K_1\left(\frac{t_1{}^{r_1}}{s_1{}^{r_1}}\right)^{m_1}\left(1-\frac{t_1{}^{r_1}}{s_1{}^{r_1}}\right)^{n_1} \dots. K_n\left(\frac{t_n{}^{r_n}}{s_n{}^{r_n}}\right)^{m_n}\left(1-\frac{t_n{}^{r_n}}{s_n{}^{r_n}}\right)^{n_n}\right]$$
$$. H^{O,N:M^1,N^1;\dots\dots\dots;M^{(r)},N^{(r)}}_{P,Q:P^1,Q^1;\dots\dots\dots;P^{(r)},Q^{(r)}}[(t_1x_1)^{k_1}, \dots\dots, (t_n x_n)^{k_n}] dt_1 \dots\dots dt_n\}\ dx_1 \dots\dots dx_n$$

…………………………(1.2.4)

Further, on expressing both the multivariable H-function in terms of their corresponding Mellin-Barnes contour integral with the help of (1.1.4) and changing the order of contour integral and $t_i$ integrals, we arrive at the following result:

$$R^{\eta_1,\dots\dots,\eta_n,\alpha}_{x_1,\dots\dots,x_n,r_n}[H(s_1,\dots\dots,s_r)] = \prod_i = 1^n \left(\frac{r_1}{s_i}\right) \int_{d_1}^{\infty} \dots. \int_{d_r}^{\infty} \prod_{i=1}^r (S_i)^{\rho_i - 1} . F_1(x_1, \dots., x_r)$$

$$\left[\begin{array}{c}\frac{1}{(2\pi\omega)^{2\pi}}\int_{L_1}\dots\int_{L_n}\int_{L'_1}\dots\int_{L'_n}.\psi(\xi_1,\dots\xi_r)\psi'(\xi'_1,\dots\xi'_r)\prod_{i=1}^{n}\{\phi_i(\xi_i)K_i^{\xi_i} \\ .\prod_i = 1^n\left\{\emptyset'_i\left(\xi'_i(s_i)^{K_i\xi'_i}\right)\right\}\left\{\int_{t_1=0}^{s_1}\dots\int_{t_n=0}^{s_n}\prod_{i=0}^{n}\left(\frac{t_i^{r_i}}{s_i^{r_i}}\right)^{\frac{\eta_i+\rho_i}{r_i}+m_i\xi_i+\frac{K_i\xi'_i-1}{r_i}}\left(1-\frac{t_i^{r_i}}{s_i^{r_i}}\right)^{\alpha+n_i\xi_i}dt_1\dots dt_n\right\} \\ .d\xi_1,\dots d\xi_n d\xi'_1\dots d\xi'_n\end{array}\right]$$

$. dx_1 \dots dx_n$ ……(1.2.5)

Next, we transform the $t_i$ integrals to well know multiple Beta integrals by the following transformation:

$1-\frac{t_i^{r_i}}{s_i^{r_i}} = y_i$ or $t_i = S_i(1-y_i)^{1/r_i}$ ……………………………………(1.2.6)

Further, we evaluate the multiple Beta integrals thus obtained and finally on reinterpreting expression thus obtained in terms H-functions of multivariable, we easily arrive at the right hand side of the main theorem after a little simplification.

**1.3 Special Cases:**

On reducing the multivariable H-function involved in (1.1.2) to the product wright generalized Bessel function, Mittag-Leffler function and the multivariable H-function occurring in (1.2.1) to Appell function $F_1$, we arrive at the following after a little simplification.

**Corollary 1**

If

$H(s_1, s_2) = \int_{d_1}^{\infty}\dots\int_{d_n}^{\infty}\prod_{i=1}^{2}(S_i)^{\rho_i-1}.\int_{\lambda}^{\upsilon}(s_1x_1)E_{\gamma,\mu}(-s_2x_2)\,F(x_1,x_2)dx_1dx_2$

And

$R_{x_1,x_2;1,1}^{\eta_1,\eta_2;\alpha}[f(x_1,x_2)] = \prod_{i=1}^{2}(x_i)^{-\eta_i-\alpha-1}\left\{\int_{t_1=0}^{x_1}\dots\int_{t_n=0}^{x_n}\prod_{i=1}^{2}\{t_i^{\eta_i}(x_i-t_i)^{\alpha}\}\right\}$

$.F_1(a, c, e; b; \frac{t_1}{x_1}, \frac{t_2}{x_2})f(t_1t_2)dt_1dt_2$……………………………………(1.3.1)

Then

$R_{s_1,s_2;1,1}^{\eta_1,\eta_2;\alpha}[H_1(s_1, s_2)] = \frac{[\Gamma(1+\alpha)]^2\Gamma(\eta_1+\rho_1)\,\Gamma(\eta_2+\rho_2)}{\Gamma(1+\alpha+\eta_1+\rho_1)\Gamma(1+\alpha+\eta_2+\rho_2)\Gamma(1+\lambda)\Gamma(\mu)}.\int_{d_1}^{\infty}\dots\int_{d_n}^{\infty}\prod_{i=1}^{2}(S_i)^{\rho_i-1}.$

$F_{3;0;0;1;1}^{3;1;1;0;1}\left[\begin{matrix}A_1^*: C_1^*\\ B_1^*: D_1^*\end{matrix};1,1,-s_1x_1-s_2x_2\right]F(x_1,x_2)dx_1dx_2$……………….(1.3.2)

Where

$A_1^* = \{(\eta_1+\rho_1;1,0,1,0),(\eta_2+\rho_2;1,0,1,0),(a,1,1,0,0)\}$

$B_1^* = \{(1+\alpha+\eta_1+\rho_1;1,0,1,0),(1+\alpha+\eta_2+\rho_2;1,0,1,0),(b,1,1,0,0)\}$

$C_1^* = \{(c,1);(e,1);-;(1,1)\}$

$D_1^* = \{-;-;(1+\lambda,\nu);(\mu,\gamma)\}$

The conditions of validity of corollary 1 can be easily derived from the existence condition of the main theorem.

Again, if we reduce H- function of several variable involved in (1.1.2) to the product of Whittaker function and multivariable H-function involved in (1.2.1) to a generalized hyper geometric function of several variable, we easily obtain the following corollary after a little simplification.

**Corollary 2.**

If

$H_2(s_1,\dots,s_r) = \int_{d_1}^{\infty}\dots\int_{d_r}^{\infty}\prod_{i=1}^{r}(S_i)^{\rho_i-1}\prod_{i=1}^{r}e^{-s_i/2}\,W_{\lambda_i,\mu_i}(s_i)F(x_1,\dots x_r)dx_1,\dots dx_r$

And

$R_{2\,s_1,\dots,s_n;1,\dots,1}^{\eta_1,\dots,\eta_n;\alpha}[f(x_1,\dots,x_n)] = \prod_{i=1}^{n}(x_i)^{-\eta_i-\alpha-1}\left\{\int_{t_1=0}^{x_1}\dots\int_{t_n=0}^{x_n}\prod_{i=1}^{n}\{t_i^{\eta_i}(x_i-t_i)^{\alpha}\}\right\}.$

$pFq\left[\begin{matrix}(a_j)_{P;}\\(b_j)_{Q;}\end{matrix}\prod_{i=1}^{n}(1-\frac{t_i}{x_i})\right]f(t_1,\dots,t_n)dt_1\dots dt_n$………(1.3.3)

then

$R_{2\,s_1,\dots,s_n;\frac{1,\dots,1}{n}}^{\eta_1,\dots,\eta_n;\alpha}[H_2(s_1,\dots,s_r)] = \frac{\prod_{j=1}^{Q}\Gamma(b_j)}{\prod_{j=1}^{P}\Gamma(a_j)}\int_{d_1}^{\infty}\dots\int_{d_r}^{\infty}\prod_{i=1}^{r}(S_i)^{\rho_i-1}$

$$R^{0,p\ :1,1;\ldots,1,1;2,1;\ldots,2,1}_{p,n+Q:\underbrace{1,1;\ldots;1,1}_{n-times}\underbrace{2,2;\ldots;2,2}_{n-times}}\left[\begin{matrix} -1 \\ \vdots \\ -1 \\ s_1 \\ \vdots \\ s_n \end{matrix}\ \begin{matrix} A_2^* & : & C_2^* \\ \\ B_2^* & : & D_2^* \end{matrix}\right] F(x_1,\ldots\ldots x_n)dx_1,\ldots\ldots dx_n$$

…………………(1.3.4)

Where

$$A_2^* = \left(1-a_j;\ \underbrace{1,\ldots,1}_{n}\ ;\ \underbrace{0,\ldots,0}_{n}\right)_{1,P}$$

$$B_2^* = \left[\left(-\alpha-\eta_1-\rho_1;1,\underbrace{0,\ldots,0}_{n};\ 1,\underbrace{0,\ldots,0}_{n}\right),\left(-\alpha-\eta_2-\rho_2;0,1,\underbrace{0,\ldots,0}_{n-2};\ 0,1,\underbrace{0,\ldots,0}_{n-2}\right),\ldots\ldots,\right.$$
$$\left.\left(-\alpha-\eta_n-\rho_n;\ \underbrace{0,\ldots,0}_{n-1};\ 1,\underbrace{0,\ldots,0}_{n-1}\right)\left(1-b_j;\ \underbrace{1,\ldots,1}_{n}\ ;\ \underbrace{0,\ldots,0}_{n}\right)_{1,Q},\ldots\right]$$

$$C_2^* = \left[\underbrace{(-\alpha,1);\ldots;(-\alpha,1)}_{n-times};(1-\eta_1-\rho_1-1),(1-\lambda_1,1);\ldots;(1-\eta_n-\rho_n-1),(1-\lambda_1,1)\right]$$

$$D_2^* = \left[\underbrace{(0,1);\ldots.(0,1)}_{n-times};\left(\tfrac{1}{2}\pm\mu_1,1\right);\ldots\ldots;\left(\tfrac{1}{2}\pm\mu_1,1\right)\right]$$

The conditions of validity of corollary 2 follow easily from the conditions of the main theorem. Finally if we reduce both multivariable H-functions involved in the main theorem to the H-functions, we get a known result of Gupta[10]

## II. Conclusion

In the present paper, we investigate the generalization studied by Kantash Gupta and VandanaAgarwal [12]. Also we obtain the number of special cases of our main theorem, which arte related with multivariable H-function.

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