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A new application of Adomian Decomposition Method

Somaia Ali Alaqel[2], Yahya Qaid Hasan[1]
Department of Mathematics, Sheba Region University

Abstract
In this work, we applied a powerful technique to approximate the solutions of singular initial value problems of second order this technique is based on a new application of Adomian decomposition method. We presented Few illustrative examples which indicate that the approach is accurate, reliable and efficient.

Key words:*Ordinary differential equations, Adomian Decomposition Method, second-order singular initial value problems.*

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I. Introduction

The singular initial value problems of second order ordinary differential equations occur very frequently in many important scientific applications. These problems have been received a significant attention of many mathematicians and physicists. Several effective methods[6-8,11] were presented to solve different scientific models of these equations, one of the influencing method is Adomian Decomposition Method (ADM) [1-3] and its modifications [4,5,10] which are provided to treat linear and nonlinear singular value problems. The aim of this paper is to use ADM for solving singular initial value problems of the form

$$y'' + (\frac{2}{x} + \alpha)y' + \frac{\alpha}{x}y + f(x) = g(x, y), \tag{1}$$

were f(x,y) and g(x) are given real functions, α is real constant. We offer a new differential operator to deal with this type of equations

II. Basic ideas of ADM

We consider the nonlinear equation in the form (1) with the following initial conditions

$y(0) = a_1, y'(0) = a_2,$

Eq.(1) can be rewritten as

$$Ly = f(x, y), \tag{2}$$

we propose a new differential operator as

$$L(.) = L(.) = x^{-1}\frac{d}{dx}e^{-\alpha x}\frac{d}{dx}e^{\alpha x}x(.), \tag{3}$$

then the inverse operator L^{-1} is given by

$$L^{-1}(.) = x^{-1}e^{-\alpha x}\int_0^x e^{\alpha x}\int_0^x x(.)dxdx, \tag{4}$$

Taking L^{-1} to both sides of (2) gives

$$y = \gamma(x) + L^{-1}g(x, y), \tag{5}$$

such that

$$L\gamma(x) = 0.$$

The ADM decompose the solution $y(x)$ in to an infinite series

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{6}$$

and the nonlinear term $g(x, y)$ into a series

$$g(x, y) = \sum_{n=0}^{\infty} A_n, \tag{7}$$

where the components $y_n(x)$ of the solution $y(x)$ will be determined recurrently, and the A_n are the Adomian polynomials, specific algorithms were seen in [3] to formulate Adomian polynomials . The flowing algorithm:

$$\begin{aligned} A_0 &= Z(y_0), \\ A_1 &= Z'(y_0)y_1, \\ A_2 &= Z'(y_0)y_{(2)} + \frac{1}{2}Z''(y_0)y_1^2, \\ A_3 &= Z'(y_o)y_{(3)} + Z''(y_0)y_1y_2 + \frac{1}{3!}Z'''(y_0)y_1^3, \end{aligned} \tag{8}$$

from (5) , (6) and (7) we have

$$\sum_{n=0}^{\infty} y_{(n)} = \gamma(x) + L^{-1}\sum_{n=0}^{\infty} A_n. \tag{9}$$

To determine the components $y_n(x)$,we use Adomian decomposition method by using the relation

$$\begin{aligned} y_0 &= \gamma(x) + L^{-1}f(x), \\ y_{n+1} &= -L^{-1}A_n, n \geq 0, \end{aligned} \tag{10}$$

therefore

$$\begin{aligned} y_0 &= \gamma(x) + L^{-1}f(x), \\ y_1 &= -L^{-1}A_0, \\ y_2 &= -L^{-1}A_1, \\ y_3 &= -L^{-1}A_3, \\ &\dots . \end{aligned} \tag{11}$$

Using the equation (8) and (11) we can determine the components $y_n(x)$, and therefore, we can directly obtain series solution of $y(x)$ in (9). In addition, and for numerical reasons, we can be the n-term approximate

$$\Psi_n = \sum_{n=0}^{n-1} y_n(x),$$

in order to approximate the exact solution.

III. Numerical examples

To explain the effectiveness of the ADM we study three test examples for the singular initial value problems of second order.

Example 1. We assume the following nonlinear equation

$$y'' + (\frac{2}{x} + 3)y' + \frac{3}{x}y = \frac{e^x\ (5 + 4\,x)}{x} - e^{2x} + y^2, \qquad (12)$$

$$y(0) = 1, y'(0) = 1$$

with the exact solution is $y(x) = e^x$.

Where

$$L(.) = x^{-1}\frac{d}{dx}e^{-3x}\frac{d}{dx}e^{3x}x(.),$$

the inverse operator L^{-1} is given by

$$L^{-1}(.) = x^{-1}e^{-3x}\int_0^x e^{3x}\int_0^x x(.)dxdx,$$

the equation (12) written in an operator form becomes

$$Ly = \frac{e^x\ (5 + 4\,x)}{x} - e^{2x} + y^2. \qquad (13)$$

Taking L^{-1} to both sides of (13) yields

$$y = e^x - L^{-1}e^{2x} + L^{-1}y^2,$$

Now we use the recursive relation

$$y_0 = e^x - L^{-1}e^{2x},$$

$$y_{n+1} = L^{-1}A_n,\ n \geq 0, \qquad (14)$$

where the nonlinear term y^2 has the first few Adomian polynomials A_n are given by

$$A_0 = y_0^2,$$

$$A_1 = 2y_0y_1,$$

leads to

$$y_0 = 1 + x + \frac{x^2}{3} + \frac{x^3}{8} - \frac{x^4}{30} + \frac{x^5}{720} + ...,$$

$$y_1 = \frac{x^2}{6} + \frac{x^3}{24} + \frac{7\,x^4}{120} + \frac{x^5}{720} + \ldots,$$

$$y_2 = \frac{x^4}{60} + \frac{x^5}{180} + \ldots,$$

etc. this yields the approximate solution of the problem in a series form is

$$y(x) = y_0 + y_1 + y_2 = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \ldots,$$

which is quite close to Taylor expansion of exact solution.

Example 2. Consider the linear equation

$$y'' + (\frac{2}{x} - 15)y' - \frac{15}{x}y = x\left(12 - 40\,x - 75\,x^2\right), \qquad (15)$$

$$y(0) = 0, y'(0) = 0$$

where

$$L(.) = x^{-1}\frac{d}{dx}e^{15x}\frac{d}{dx}e^{-15x}x(.),$$

then the inverse operator L^{-1} is given by

$$L^{-1}(.) = x^{-1}e^{15x}\int_0^x e^{-15x}\int_0^x x(.)dxdx.$$

We write Eq.(15) by ADM operator form as

$$Ly = x\left(12 - 40\,x - 75\,x^2\right). \tag{16}$$

Taking L^{-1} to (16) we get the exact solution

$$y(x) = x^4 + x^3.$$

Note that, the exact solution is easily obtained by ADM.

Example 3. We consider the following nonlinear equation

$$y'' + (\frac{2}{x} + 10)y' + \frac{10}{x}y = \frac{e^{x^3}\left(10 + 12\,x^2 + 30\,x^3 + 9\,x^5\right)}{x} + 2x^3 - \log y^2, \tag{17}$$

$y(0) = 1, y'(0) = 1$ Where

$$L(.) = x^{-1}\frac{d}{dx}e^{-10x}\frac{d}{dx}e^{10x}x(.),$$

the inverse operator L^{-1} is given by

$$L^{-1}(.) = x^{-1}e^{-10x}\int_0^x e^{10x}\int_0^x x(.)dxdx,$$

the equation (17) written in an operator form becomes

$$Ly = \frac{e^{x^3}\left(10 + 12\,x^2 + 30\,x^3 + 9\,x^5\right)}{x} + 2x^3 - \log y^2. \tag{18}$$

Taking L^{-1} to both sides of (18) yields

$$y = e^{x^3} + L^{-1}(2x^3) - L^{-1}(\log y^2),$$

by modified ADM in [9] we have

$$y_0 = e^{x^3},$$

We use the recursive relation

$$\begin{aligned} y_1 &= L^{-1}(2x^3) - L^{-1}A_0, \\ y_{n+1} &= L^{-1}A_n,\; n \geq 1, \end{aligned} \tag{19}$$

where the nonlinear term $\log y^2$ has the first few Adomian polynomials A_n are given by

$$A_0 = \log y_0^2,$$
$$A_1 = \frac{2y_1}{y_0},$$

which gives

$$y_0 = e^{x^3}$$
$$y_1 = 0,$$
$$y_{n+1} = 0,\ n \geq 1$$

this yields the solution of the problem is given by

$$y(x) = y_0 + y_1 + y_2 = e^{x^3}.$$

Which is equal the exact solution

IV. Conclusion

We present a new application of ADM which has a very high ability to solve singular value problems with initial conditions. The results obtained from three examples show that the method is effective and useful in finding the exact solution for initial value problems. We see that series solution converges very rapidly in these problems by proposed method.

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