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Review on Estimation and Prediction of Population Using MathematicalModels in Tanzania

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Abstract:
The purpose of this review study focused on identifying and verifying an effective and reliable mathematical growth model for prediction of future Population size of United Republic of Tanzania. To specify the best model out of the three models, two criteria's, that have been used using numerical and graphical techniques, were in order to compare and contracts the models. The Exponential, Logistic growth model and Method of Least square (MLS) used to estimate the model parameters with previous census data from 1980 to 2016 inclusive. During estimation, different software used such as MATLAB (R2015a) and R version 3.6.3 software. The review proposed exponential model via the estimation of method of least square is most effective and reliable model with the least SEE (241,806) and MAD (188,413) as well as the highest R^2(99.95%) relative to the other models.
Keyword: *least square, estimation, Diagnosis, parameters, prediction, statistical models, Exponential.*

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I. Introduction

The population projection has become one of the most important problems in the world. Population sizes and growth in a country directly influence the situation of economy, policy, culture, education and environment of that country, determine exploring, and cost of natural resources. No one wants to wait until those resources exhausted by the population incensement. Population has been a controversial subject for ages.

Every government and collective sectors always require accurate idea about the future size of various entities like population, resources, demands and consumptions for their planning activities. To obtain this information, the behavior of the connected variables analyzed based on the previous data by the statisticians and mathematicians at first and using the conclusions drawn from the analysis they make future projections of the variable aimed. There are enormous concerns about the consequences of human population growth for social, environment and economic development. Intensifying all these problems is population growth. Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in life sciences. Thus, it is a process of mimicking reality by using the language of mathematics. Many people examine population growth through observation, experimentation or through mathematical modeling.

Like other country in Africa, United Republic of Tanzania is a developing country in East Africa situated just south of the Equator. It has an area of 945, 087km^2. Bounded by Uganda, Lake Victoria, and Kenya to the north, by the Indian Ocean to the east, by Mozambique, Lake Nyasa, Malawi, and Zambia to the south and southwest, and by Lake Tanganyika, Burundi, and Rwanda to the west. Tanzania as a country its population is growing as fast as the way technology grows [Larsen, P. O., & von Ins, M. 2010]. Tanzania has a high population among the East African Community (EAC) Countries. This continuous and constant increase has great influence to the national resources and demand especially land utility, settlements and basic needs.

Population grows exponentially that means it increases as the birth rate rises due to the dynamic population we have in the world. There are countries that have managed to reduce the birth rate in order to manage the resources they have for the future generation [Nargund G. ,2009.], to mention a few: Switzerland, Greece, Italy and Lithuania.

According to http://www.worldometers.info (2019) Tanzania's share of the World population is 0.79% with approximately 60 million-population size. Population is the vital element of the nation rendering its projection has become one of the most serious problems in third world countries because when not well addressed it significantly affects planning, decision making for the socio-economic and demographic development [Wali.A. et al , 2011].

II. Objective of the Study

The objective of this paper is to review the results, which were present by Mussa A. and Jung I. 2019 to projected the population of Tanzania using exponential growth model, Logistic growth model and Method of Least Square for the period offrom 1980 to 2016 inclusivelyin order to select the best model for the prediction of population of Tanzania.

III. Literature Review

Studies are being done upon population growth in order to reflect on economic growth, employment, savings and environment, conservation of assets, investments and environmental impacts [Richard and Robert, 1997; Ehrlin and Lui, 1997; Walker,2016]. Recent studies about population foster for solving problems that are presumed to be happening in future that will lead to some difficulties in resource allocation especially in developing countries [Guria, 2015; Gao, 2015]. In order to obtain good and reliable estimates we need to employ mathematical models for the sake of obtaining precise and accurate estimations of parameters and predictions as stated in [Kapur and Khan, 1979; Vankatesha,2017]. Ali et al. (2015) studied census data and predicted population of Bangladesh by using logistic model. They used a curve fitting method and tried to compare the prediction between the cases when carrying capacity known and unknown. They used a curve fitting method for the population of a certain period of consecutive 11 years. The method gave better predictions, which were close to the actual values of population. Ofori et al. (2013) developed mathematical model for Ghana's population growth. They applied the exponential and logistic growth models to project the population. Their study found that the exponential model gave a better projection compared to the logistic model. However, Ghana’s population properties are different from Tanzania, including growth rate, initial population and therefore carrying capacity estimations. Kulkarni et al. (2014) estimated the population growth of India from 2009 to 2012 using the logistic model approach and gave a comparison with actual population of India for the same period. An error equation also deduced based on the trend line for the specified period and the population of India for the year 2013 and onwards until 2025 estimated. The time required for India to reach its carrying capacity also discussed.

Few studies concerning Tanzania population conducted. For example, Madulu (2004) discussed the linkages between population growth and environmental degradation in Tanzania. Tanzania’s major environmental problems, demographic characteristics, and the linkages between environmental change and rapid population growth both at national and regional levels discussed. A conclusion made is that, environmental change is as an important factor to demographic and economic factors as it is population growth to economic development and environmental conservation.

As [Kapur and Khan, 1979; Vankatesha, 2017] argued that in order to obtain good and reliable estimates we need to employ mathematical models for the sake of obtaining precise and accurate estimations of parameters and prediction of population growth. Therefore, in this paper, we review the study of Mussa and Jung, 2019 using Exponential model, Logistic model and Method of Least the square used to estimate the model parameters and predict the future population size of the Tanzania.

IV. Material and Methods

The study used secondary data directly from the paper Mussa and Jung, 2019 which was obtained from the National bureau of Statistics in Tanzania from 1980 to 2016.The study simulated and used the real data to estimate and fit them in two models namely exponential and Logistic growth model and Method of Least Square. The results obtained in the previous step used to predict the future population of the united republic of Tanzania. The experimental processes done in a windows machine installed with MATLAB version (R2015a) and R studio version 3.6.3. The results presented in terms of numerical, tabular and graphical forms. These softwares were powerful tool to analyze statistical models.

To determine the model having the highest performance to predict population of Tanzania, the models were selected using numerical and graphical diagnosis. The numerical technique of diagnosis performed by using SEE, which is standard deviation of the predicted population, MAD is the mean absolute difference of the actual population and predicted population by the model and R^2 is the proportion variation of the population in a unit change of time. A model is best to predict the population having the smallest SEE and MAD and the highest R^2 of the model.

V. Development of the models

a. Exponential Model

Thomas R.Malthus (1798), proposed a mathematical model of population growth. The exponential model is the model that relies on the assumption that population grows at a constant rate proportional to the original population size. This assumption is reasonable for the ideal condition that unlimited environment, adequate nutrition, absence of predators and immunity from diseases are excluded in the model and the model is expressed in simple differential equation as follows;

$$\frac{dp}{dt} = \beta P \tag{1.1}$$

Where P the total population size and β is the constant growth rate defined as the differencebetween the birth rate and death rate for a certain population size. We rearrange and integrate both side of equation (1.1)

$$\int_{P_0}^{P(t)} \frac{dP}{P} = \int_{t_0=0}^{t} \beta dt$$

Thus, the exponential model is

$$P(t) = P_0 e^{\beta t} \tag{1.2}$$

Where, P_0 represents the population at some specified time, $t = 0$. The population increase if $\beta > 0$, population decrease if $\beta < 0$, or no change if $\beta = 0$. The model may be impractical when the number of generations gets large enough for other factors to come into play.

The constant growth rate parameter β can estimated as follows

$$\beta = \frac{lnP\,(t) - \ln P_0}{t} \tag{1.3}$$

b. Logistic growth model

Logistic model was developed by Belgian mathematician Pierre Verhulst in 1838 (Brauer and Castillo-Chavez 2001). He commented that the population growth depends on carrying capacity and the maximum rate of growth. He suggested the population growth to be limited. This means the rate of population growth may change depending on relationship between the initial population and carrying capacity. The logistic model is an extension of exponential model that includes the ideal conditions that somewhere excluded in the model as follows

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right), Where\ r, K > 0 \tag{1.4}$$

Where K is the maximum sustainable population (carrying capacity) and r is the growth rate, r and r/K are vital constants. For small population $P \leq K$ then $P^2 \longrightarrow 0$ and the logistic model reduces to exponential growth signifying that as P is greater K then the rate of growth becomes negative and population decreases.

From equation (1.4) we can find the solution of the non-linear differential equation as follows;

$$\frac{dP}{P\left(\frac{K-P}{K}\right)} = rdt$$

$$\int_{P_0}^{P(t)} \frac{K}{P(K-P)} dP = \int_0^t rdt$$

$$\int_{P_0}^{P(t)} \left[\frac{1}{P} + \frac{1}{K-P}\right] dP = \int_0^t rdt$$

It follows,

$$e^{rt} = \frac{P(t)(K-P)}{P_0(K-P(t))}$$

By rearrangement gives

$$P(t) = \frac{K}{1+\left(\frac{K}{P_0}-1\right)e^{-rt}} \tag{1.5}$$

$$P_{max} = \lim_{t\longrightarrow\infty} P(t) = \lim_{t\longrightarrow\infty}\left(\frac{K}{1+\left(\frac{K}{P_0}-1\right)e^{-rt}}\right) = K(Carring\ Capacity)$$

Determination of the parameters r and K by estimation by using equation (1.5)

Let P_0: Population at $t = t_0 = 0$, P_T: Population at $t = T$ and P_{2T}: Population at $t = 2T$, then

$$\frac{1}{K}[1 - e^{-rT}] = \frac{1}{P_T} - \frac{e^{-rt}}{P_0} \tag{1.6}$$

$$\frac{1}{K}[1 - e^{-2rT}] = \frac{1}{P_{2T}} - \frac{e^{-2rt}}{P_0} \tag{1.7}$$

By dividing equation (1.7) to (1.6) gives

$$\frac{[1 - e^{-rT}]}{[1 - e^{-2rT}]} = \frac{\frac{1}{P_T} - \frac{e^{-rt}}{P_0}}{\frac{1}{P_{2T}} - \frac{e^{-2rt}}{P_0}}$$

So that $e^{-rt} = \frac{P_0(P_{2T}-P_T)}{P_{2T}(P_T-P_0)}$
This implies that, If$0 < \frac{P_0(P_{2T}-P_T)}{P_{2T}(P_T-P_0)} < 1$

$$r = \frac{1}{T} ln \left(\frac{P_0(P_{2T} - P_T)}{P_{2T}(P_T - P_0)} \right)$$

By direct substitution into equation (1.6) yields

$$K = \frac{P_T(P_0P_T - 2P_0P_{2T} + P_TP_{2T})}{(P_T)^2 - P_0P_{2T}} \tag{1.8}$$

Both exponential and logistic models originated from observations of biological re- production process. However, human population believed to be dynamic, and then the growth rate cannot be constant as stipulated in the two models and one method by the constants. As per our study, we must show the implementation of this model based on constant growth rate and explain in mathematical terms however, the reality remains in controversy.

c. Method of least square

A French mathematician Adrien-Marie Legendre (1805), the method of least squares is an ideal algorithm in regression analysis that has most important application in data fitting. This algorithm involves minimization of sum of squared residuals for the sake of maximizing objective function of the model. In various areas of experimental sciences, maximization theory is associated with accuracy and precision of the predicted output attained by error reduction. The essence and necessity of using this method is to fit the exponential function that has similar properties with the previous models.

$$P = Ae^{at}$$

Where A and α are constants which are unknown parameters to be estimated for the model. For many observed data point, the method of least square is reasonably most systematic procedure to fit the unique curve. Suppose we have set of observation, $\big((t_0, P_0), (t_1, P_1), \ldots, (t_{n-1}, P_{n-1})\big)$.We transform the exponential model ($P = Ae^{at}$) into linear model by taking natural logarithm (ln) both and the linear model can be written as

$$lnP = lnAe^{at} = lnA + at$$

Assume that $y = lnP$, and $\gamma = lnA$, Generally we have

$$y_i = at_i + \gamma, i = 1,\ 2, 3, \ldots, 37 \tag{1.9}$$

We define the error associated in the set of data with the equation (1.9) by

$$E(a, \gamma) = \sum_{i=1}^{n} \big(y_i - (at_i + \gamma)\big)^2$$

For (n) times the variance of the data set $\{(y_1 - (at_1 + \gamma), \ldots, y_n - (at_n + \gamma))\ \}$
The main target to determine the value of a and γ is to minimize the error. On the other hand, getting the optimal values of a and γ so as to minimize the residual ($E(a, \gamma)$) that can be done by using the concept of Calculus,

$$\frac{\partial E}{\partial a} = \frac{\partial E}{\partial \gamma} = 0$$

By using this condition there is no need to about boundary even if$|a|$ and $|\gamma|$ became very large. Differentiating $E(a, \gamma)$ with respect to α and γ and solve for α and γ respectively.

$$\frac{\partial E}{\partial a} = 2\sum_{i=1}^{n} \big(y_i - (at_i + \gamma)\big)(-t_i) = 0$$

$$\frac{\partial E}{\partial \gamma} = 2\sum_{i=1}^{n} \big(y_i - (at_i + \gamma)\big) = 0$$

After some rearrangement, we can get the following normal equation of the partial derivative of the error with respect to the parameters ($\alpha \& \gamma$).

$$\begin{cases} \left(\sum_{i=1}^{n} (t_i)^2\right)\gamma + \left(\sum_{i=1}^{n} t_i\right)a = \left(\sum_{i=1}^{n} y_i t_i\right) \\ \left(\sum_{i=1}^{n} t_i\right)\gamma + n * a = \left(\sum_{i=1}^{n} y_i\right) \end{cases} \tag{1.10}$$

It is invertible matrix; therefore it is easy to determine the value of γ and α even using the equations the simultaneously.

VI. Result

The unknown values for the models that they described in the above were estimated using untied republic of Tanzania population data and the selection of the models also based on this data.

A. For Exponential model (equation 1.2): The model has one parameter (growth rate) of the population for the country and its value was estimated using a deterministic way using points $(t_0, P_0) = (0, 18683157)$ and $(t_1, P_1) = (1, 19277108)$:

$$\beta = \frac{\ln(P(t)) - \ln(P_0)}{t} = \frac{\ln(19277108) - \ln(18683157)}{1} = 0.0313$$

Therefore, the general exponential model of the population growth is

$$P(t) = 18683157 * e^{0.0313*t};$$
$$0 \le t \le 36$$

B. For Logistic growth model: By using equation (1.8), the Carrying Capacity of United republic of Tanzania population can be estimated at $(P_0, P_T, P_{2T}) = (18683157, 32451713, 55572201)$. The better choice of P_T and P_{2T} lead to better approximation of K.

$$K = \frac{P_T(P_0P_T - 2P_0P_{2T} + P_TP_{2T})}{(P_T)^2 - P_0P_{2T}}$$

By substitute the values on the equation, the carry capacity of United republic of Tanzania population was estimated as K= 728133426.

This implies that the maximum carry capacity of the country is 728133426. This can have written as,

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty}\left(\frac{K}{1+\left(\frac{K}{P_0}-1\right)e^{-rt}}\right) = 728133426.$$

Then after, the growth rate (r) of the logistic model was used the following expression to estimate it

$$P(t) = \frac{K}{1 + (\frac{K}{P_0} - 1)e^{-rt}}$$

By considering the points $(t_0, P_0) = (0, 18683157)$ and $(t_1, P_1) = (1, 19277108)$ as well as the carrying capacity K= 728133426. Finally, substitute the values and solve for the growth rate of the model.

$$19277108 = \frac{728133426}{1 + (\frac{728133426}{18683157} - 1)e^{-r}}$$

After some rearrangement, we can have estimated the growth rate as $r = 0.032$ and hence the logistic growth models is written as follows

$$P(t) = \frac{728133426}{1 + 37.97 * e^{-0.032*t}}$$

C. Estimating the Exponential model using method of least square. We were estimating α and γ after transforming the exponential model into linear model. We used R software, estimate the parameters by using least square model (using function lm()). The results of the estimates given in table 1 below. As indicated below, the estimates were statistically significant at 1%.

D.

***Table 1**: summary of the estimates of the parameters*

Coefficients	Estimates	Std. Error	t value	Pr(>|t|)
γ	1.675e+01	2.330e-03	7188.7	<2e-16***
α	2.989e-02	1.113e-04	268.5	<2e-16***

*** significant at 0.001

Then we transformed the linear model into exponential model $\boldsymbol{\gamma = 16.75}$ and $\boldsymbol{\alpha = 0.03}$ such that $\boldsymbol{ln(A) = \gamma,\ A = exp(\gamma) = exp(16.75)}$ = 18811896.The general equation is given by $\boldsymbol{P(t) = A * exp(\alpha t)}$ = 18811896$*\ \boldsymbol{exp(0.03 * t)}$.

These models were candidate to model the population of united republic of Tanzania. However, we need to select the best model to predict the population of Tanzania.

To select the best model, Numerical and graphical comparison were very important to compare the models to predict the population. The following table indicates the numerical diagnosis of the models using SEE, MAD and R^2, which are essential to select the best models to predict the population. The model with the smallest SEE, MAD and the model having the highest R^2 were the best models to predict the population of Tanzania. As a result, the exponential model, that parameters estimated by using method of least square, has the least SEE, MAD and the higher R^2 as compared to the other modes. Hence, $P(t) = Ae^{at}$ (linear form) is the best model to predict the population of Tanzania.

Table 2: *The diagnosis for the models*

Model	SEE	MAD	R^2
$P(t) = P_0 e^{\beta t}$	1,094,581	758,653	0.99952
$P(t) = Ae^{at}$	241,806	188,413	0.99953
$P(t) = \frac{K}{1+(\frac{K}{P_0}-1)e^{-rt}}$	505,997	369,409	0.99952

On the other hand, figure 1 confirmed that $P(t) = Ae^{at}$ (linear form) is the best model to predict the population of Tanzania. The plot indicated by green color approaches to the actual population, which indicated by red color in figure 1, presented below. Due to this, an exponential model, which estimated by method of least square technique, is the best model to predict the population of Tanzania.

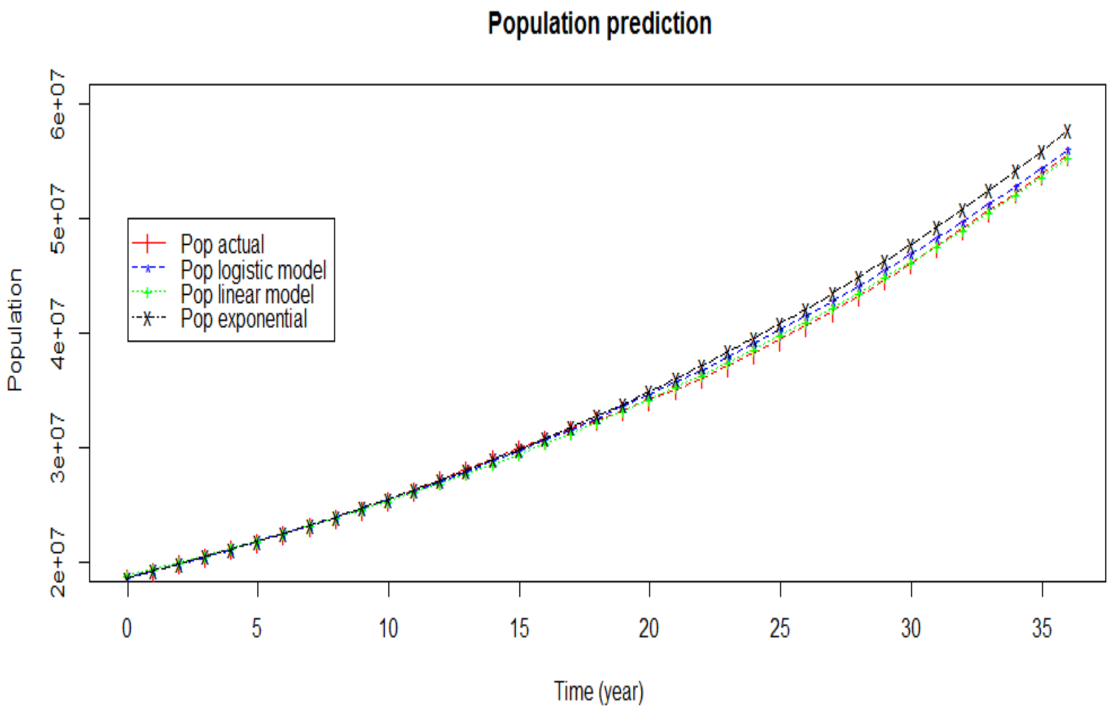

Figure 1: *Prediction of the population by models which is compared with the actual population*

VII. Discussion

Figure 1 in the above shows that from 1980 to 2016 years, Tanzania has a monotonic increase of population. The actual populations and predicted populations were almost the same for all models such as exponential and logistic model for the first 15 years. This indicates that the variation is very small and the drawback of the models is that they did not use all data points to estimate the parameters. Mussa A. et al (2019) explained thatmethod of least the square was not reliable method for the population of Tanzania. However, in this study, the method of least square was the closest to the actual population because the parameter estimation for the model was depend on the data points. As time increases, the population also increases and prediction of the population using method of least square relatively approaches to the actual population with R^2=99.953% with least SEE and MAD as compared to the other models. This result also supported by the plot in figure 1 that indicates the closeness of the prediction of the population and actual population. This result contrasted with the result Mussa A. et al (2019).

Table 2 Shows that the method of least square model is more reliable and effective model in Human population estimation with the smallest SEE and MAD and relatively the highest R^2. The 2nd best model for the population prediction is logistic growth model ($\boldsymbol{P(t)} = \frac{\boldsymbol{K*e^{-rt}}}{\boldsymbol{e^{-rt}+(\frac{K}{P_0}-1)}}$) with the 2nd smallest SEE and MAD as well as the 2nd highest R^2. So that this model will not be the best model to predict, the population that is contradicted with the result Mussa A. et al (2019).

The 3rd model for the prediction of population of Tanzania is exponential model ($\boldsymbol{P(t) = P_0 e^{\beta t}}$) with the highest SEE and MAD and relatively the smallest R^2.

Using various statistical methods to decide the best model that is acceptable to predict the population and we proposed exponential model ($\boldsymbol{P(t) = A * e^{\beta t}}$), that the parameters were estimated using least square methods, is the best model to predict the future population of Tanzania.

VIII. Conclusion

The study has reviewed to revise the paper ofMussa A. et al (2019) which followed the same procedure with misunderstanding of the result. We have proved that method of least square, logistic and exponential models were the best models to predict the population of Tanzania. We used Exponential, Logistic and method of Least Squares to determine the parameters of the model that triggered the best prediction of the future population of Tanzanian. Finally, it observed that the population parameters from exponential model using least square estimation technique for the parameters. This model is the best for the prediction of population Tanzania because it was the best model numerically and graphically.

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Appendix

A. Method of least square

```
Call:lm(formula = log(pop) ~ t, data = t)

Residuals:
      Min        1Q    Median        3Q       Max
-0.009448 -0.005628 -0.001303  0.004339  0.015346

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.675e+01  2.330e-03  7188.7   <2e-16 ***
t          2.989e-02  1.113e-04   268.5   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.007231 on 35 degrees of freedom
Multiple R-squared:  0.9995,        Adjusted R-squared:  0.9995
F-statistic: 7.208e+04 on 1 and 35 DF,  p-value: < 2.2e-16
```

B. R code for the result

```
setwd("C:/Users/Admin/Desktop/Data")
t<-read.csv("pop.csv")
head(t) # see help("mtcars")
dim(t)
        time<-t$t
        yt<-t$pop
        p0<-18683157
        y1<-p0*exp(3.024e-02*time)
        ylog<-728133426/(1+((728133426/18683157)-1)*exp(-3.152e-02*time))
        yp<-exp(predict(mls))
        Ermls<-yt-yp
        Erlog<-yt-ylogl
        bets<-data.frame(time,y1,yt,ylog,yp)
        plot ( yt ~ time , data = bets ,
        xlab =" Time (year)", ylab =" Population",
            main =" Population prediction", type ="n",ylim=c(2e+7,6e+7),xlim=c(0,36))
        lines ( yt ~ time , data = bets, lty =2,col="red")
        points(time, yt, col="red", pch="l")
        lines ( ylog ~ time , data = bets, lty =2,col="blue")
        points(time, ylog, col="blue", pch="*")
        lines (yp ~ time , data = bets, lty =3,col="green")
        points(time, yp, col="green", pch="+")
        lines ( y1 ~ time , data = bets, lty =5)
        points(time, y1, col="black",pch="x")
        legend(0,5e+7,legend=c("Pop actual","Pop logistic model","Pop linear model","Pop exponential"),
col=c("red","blue","green","black"),   pch=c("l","*","+","x"),lty=c(1,2,3,4,5), ncol=1)
```

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