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Monte Carlo Estimation of Production Function with Policy Relevance

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Abstract: *This study focuses on Monte Carlo estimation of production function with policy relevance. The ordinary least square (OLS) method is used to estimate the unknown parameters. The Monte Carlo simulation methods are used for the data generating process. In tables 1 to 3, the mean square error (MSE) of θ_1 are 0.007678, 0.001972 and 0.001253 respectively for sample sizes 20, 40 and 80. Our finding showed that the mean square error (MSE) value varies with the sum of the powers of the input variables, the smaller the mean square error the lesser the viability and the better the estimator. In addition, use of parameters estimated to guide policy formulation to producing firms or industries is treated.*

Keywords: *Cobb-Douglas model, Monte Carlo estimation, production function, returns to scale, Capital.*

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I. Introduction

For the past years, Production Function played an important role in modeling certain phenomena. This is due to increase computational powers and Monte Carlo Methodology. In economics, it is widely used in research works on production, cost function and demand. Econometric modeling requires the combination of an error term as well as the specification of its distribution. A production function is a quantitative link between production inputs and outputs. Johnson and Samuel (2012); said a production function establishes the functional relationship between the quantity of a specific product that can be produced within a time and a set of inputs used, given the existing technology in a socio-cultural environment.

The traditional theory of production function of a firm expresses output as a function of two inputs, capital (k) and labour (L) in the form of the Cobb-Douglas function. The Cobb-Douglas production function is the simplest production function widely used to represent the technological relationship between the amounts of two or more inputs, and the amount of output that can be produced by those inputs. It was used by Charles Cobb-Douglas and Paul Douglas in the study in which they modeled the growth of American Economy during the period (1899-1922). In spite of the important role played by the producing sectors or industries in Nigeria and other countries, they are faced with problems of estimation of parameters, finding the returns to scale in production function, evaluating policy issues on production to producing firms or industries.

1.1 Specification and Policy Issues

The specification of the production function is very relevant to this study. The production function may be wrongly specified and the form of mis-specification at micro level may not be exactly as it is at the macro level. For this and others reasons, policies of government on production to individual firms may not be useful for the whole economy. Policies of the authorities to industries or the economy should come after a rigorous and systematic aggregation in the surface of possible mis-specification. The Cobb-Douglas Production function (without the error term) is given by:

$$Y = \theta_0 K^{\theta_1} L^{\theta_2} \qquad \textbf{(1)}$$

The constant θ_0 represents efficiency. States (say), in an economy like Nigeria could have (for example yam production in Anambra and Benue states) therefore, they apply the same techniques but the state with higher value of θ_0 would produce more for every combination of Inputs, than the other state. So θ_0 expresses the relative efficiency of combining the given inputs. the parameter θ_0 in essence, can be used as an indicator of the general state of technology, policy of government in the agricultural sector of the economy should include advising states with lower θ_0 to consider as one of the priorities using judiciously some funds in improving the technology level in production.

We extend this reasoning to compare efficiencies of two countries for a given industry, for example, palm oil production in Nigeria and Malaysia. It is obvious at the moment judging from the present economic situation that Nigeria needs a higher level of technology in this case. The production function is also known to exhibit some returns to scale phenomena at some points under certain condition. The sum of the input variables K and L is interpreted as a measure of returns to scale. These returns to scale phenomena plays significant role in the policy of resource allocation and taxation of producing industries.

Marshall, as citied in Bhatia (1994) recommends that the output of an industry operating under diminishing returns should be taxed, while the output of an industry under increasing returns should be subsidized. An industry with constant returns to scale is neither to be taxed nor subsidized. Marshallian reasoning is not diminishing returns industry should be discouraged and increasing returns industry should be encouraged to produce. These suggestions lead to the conclusion that authorities may be able to wisely adopt a set of indirect taxes on a selective basis for obtaining a desired shift from the current allocation of production resources. This such as option is of great policy relevance in a county like Nigeria where investment in high priority, industries is to be encouraged. The production function shows the property of substitutability of inputs for one another. A local measure of such substitutability is the elasticity of substitution. It is usually denoted by σ. The elasticity of substitution σ is defined as the ratio of the proportionate change to the proportionate change in the ratio of Marginal products. It is given by:

$$\sigma = \frac{d[\text{In } K/L]}{d[\text{In } (MP_L / MP_k)]} \qquad \textbf{(2)}$$

Where MP_L and MP_k are respectively management product with respect to labour and capital. In the case of perfect competition and profit maximization, the ratio of the marginal production is the ratio of factor prices. That is:

$$\sigma = \frac{d[\text{In } K/L]}{d[\text{In } (W/r)]} \qquad \textbf{(3)}$$

The elasticity of substitution is thus a measure of how quickly factor proportions change for change in relative factor prices. The symbols W and r denote respectively labour and capital prices. The value of σ lies between 0 and α. The value of σ has policy relevance in economics. However, in the Cobb-Douglas case; since its value is unity , one may think that there is not much one can deduce from σ as far as policy is concerned with the right specification, the value of σ is unity. If the value of σ is different from units , this points to the fact that there can be a possible form of mis-specification. The Objectives of the study are:

i. Estimate Cobb-Douglas Production function

ii. Compute and Compare mean square errors for different returns to scale in Cobb –Douglas production model.

iii. Evaluate policy issues of Government on production to producing firms or industries.

Some works on mis-specification are cited in Essi (2000), Essi and Iyaniwuta (2007), Essi, Iyaniwuta and Ojekudo (2007), Essi (2009, 2010), Md, Tapati and Ajit (2013) and Maxwell and Essi (2019). These results, show that the consequence is more serious when a multiplicative error plagued data set if splinted with additive error based model than vice-verse, therefore in this circumstance, the values of other model parameters should not be relied upon heavily in policy marking and implementation. Work on correct specification cited in Essi (2011). Ashfag and Muhammad (2015) Essi, Olaomi and Iwabueze (2010) and Essi, onwuchekwa and Chikwendu (2010). Monte Carlo simulation and forecast are considered using correct specified nonlinear production function and are more reliable for use in policy formulation and implementation.

II. Materials and Methods

This study uses generated data.

2.1 Estimation Method

This study considers Cobb-Douglas production function with Multiplication error term. The model is given by:

$Y = \Theta_0 K^{\Theta_1} L^{\Theta_2} e^u$ **(4)**

Where Y = Production output

K = Capital invested in the production

L = Labour used in the production

Θ_0 = Positive constant or Technological constant.

Θ_1 and Θ_2 are positive parameters output elasticities of capital and Labour

U = Random or stochastic error

e = Base of natural logarithm

The model in (4) can be transformed to linear model by taking the natural logarithm of both sides of the equation to obtain a regression model of the form.

Lny $= \text{Ln}\, \Theta_0 + \Theta_1 \text{Ln K} + \Theta_2 \text{ Ln} + u$ **(5)**

The ordinary least square (OLS) estimation is used for the linear model to obtain the estimate $\Theta = (\Theta_0, \Theta_1, \Theta_2)$.

The choice of model parameters $(\Theta_0, \Theta_1, \Theta_2)$ is such that $\Theta_1 + \Theta_2 < 1$
$\Theta_1 + \Theta_2 = 1$ and $\Theta_1 + \Theta_2 > 1$, while the value of Θ_0 is arbitrary and kept constant at
$\text{Ln}\, \Theta_0 = 3$, $\Theta_0 = 20.09$
We use the following three sets of parameters:
$V_1 = (20.09, 0.35, 0.30)$, $V_2 = (20.09, 0.55, 0.45)$, $V_3 = (20.09, 0.75, 0.60)$
The input matrix is made of two variables K (Capital) and L (Labour) which are randomly generated and normally distributed independently.

2.2 Simulation

The Mont Carlo Study uses Sample size of 20, 40 and 80 with each experiment replicated 20 times under the following three conditions, varied one at a time while the others are kept: the sample size T and the parameters set $\Theta = (\Theta_0, \Theta_1, \Theta_2)$ used in the data generating process. Mean square error is the most important criterion used to evaluate the performance of an estimator.

2.3 Evaluation of Mean Square Error

The mean square error is the most important criterion used to evaluate the performance of an estimator. It is calculated as sum of the variance (V) of the estimator and the squared bias (B) of the estimator. The relationship is given by;

$$\text{MSE} = V + \beta^2 \qquad (6)$$

$$= E(\hat{\theta} - \theta)^2 \qquad (7)$$

III. Results and Discussion

We have estimated a total of 180 equations. $\theta = (20.09, 0.35, 0.30)$ with sample sizes 20,40 and 80, with 20 replications. In all the tables. N stands for the number of replication. The model (5) is a multiplicative error based model which is fitted to the data generated. The strata software package is used to analyze the data. Monte Carlo results showing estimates with their bias, variance, mean square error (MSE), Standard deviation, are summarized and presented in Tables 1to11.

Table 1: Monte Carlo Estimate for variables for sample size T
N = 20, $(\theta_0, \theta_1, \theta_2) = (20.09, 0.35, 0.30)$, T = 20
$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	**Bias**	**Variance(v)**	**MSE**	**σ**
θ_0	15.964248	4. 125752	45.104058	62.12588	6.715955
θ_1	0.368320	0.018320	0.007342	0.007678	0.085688
θ_2	0.333575	0.033575	0.005586	0.006713	0.074739
$\theta_1+\theta_2$	0.701867	0.051867	0.018881	0.021571	0.137408

Table 2: Monte Carlo Estimate for variables for sample size T
N = 20, $(\theta_0, \theta_1, \theta_2) = (20.09, 0.35, 0.30)$
T = 40
$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	**Bias**	**Variance(v)**	**MSE**	**σ**
θ_0	18.004218	2.085782	32.061799	36.412284	5.662314
θ_1	0.371508	0.021508	0.001510	0.001972	0.038855
θ_2	0.321911	0.021911	0.003444	0.003924	0.058683
$\theta_1+\theta_2$	0.697519	0.047519	0.004963	0.007221	0.070451

Table 3: Monte Carlo Estimate for variables for sample size T

$N = 20, (\theta_0, \theta_1, \theta_2) = (20.09, 0.35, 0.30)$

$T = 80$

$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	**Parameter Estimates**				
	Estimate	**|Bias|**	**Variance(v)**	**MSE**	**σ**
θ_0	21.108742	1.018742	40.699586	41.737421	6.379623
θ_1	0.345666	0.004334	0.001234	0.001253	0.035131
θ_2	0.298823	0.001177	0.002032	0.002034	0.045081
$\theta_1+\theta_2$	0.644486	0.005514	0.002765	0.002796	0.052586

Table 4: Monte Carlo Estimates for variables for sample size T

$N = 20, (\theta_0, \theta_1, \theta_2) = (20.09, 0.55, 0.45)$

$T = 20$

$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	**Parameter Estimates**				
	Estimate	**|Bias|**	**Variance(v)**	**MSE**	**σ**
θ_0	16.073577	4.016423	44.909952	61.041604	6.701489
θ_1	0.569329	0.019329	0.007371	0.007745	0.085855
θ_2	0.482515	0.032515	0.005562	0.006619	0.074576
$\theta_1+\theta_2$	1.051844	0.051844	0.018881	0.021569	0.137408

Table 5: Monte Carlo Estimates for variables for sample size T

$N = 20, (\theta_0, \theta_1, \theta_2) = (20.09, 0.55, 0.45)$

$T = 40$

$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	**Parameter Estimates**				
	Estimate	**|Bias|**	**Variance(v)**	**MSE**	**σ**
θ_0	17.954216	2.135784	32.249889	36.811462	5.678899
θ_1	0.571497	0.021497	0.001511	0.001973	0.038871
θ_2	0.476909	0.026909	0.002925	0.003649	0.054079
$\theta_1+\theta_2$	1.046966	0.046996	0.004665	0.006873	0.068298

Table 6: Monte Carlo Estimates for variables for sample size T

$N = 20, (\theta_0, \theta_1, \theta_2) = (20.09, 0.55, 0.45)$

$T = 80$

$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$ 9

Parameter	**Parameter Estimates**				
	Estimate	**|Bias|**	**Variance(v)**	**MSE**	**σ**
θ_0	21.108737	1.018737	40.699522	41.776603	6.379618
θ_1	0.548666	0.001334	0.001241	0.001243	0.035226
θ_2	0.448823	0.001177	0.002032	0.002034	0.045081
$\theta_1+\theta_2$	0.994489	0.005511	0.002766	0.002796	0.052592

Table 7: Monte Carlo Estimates for variables for sample size T

$N = 20, (\theta_0, \theta_1, \theta_2) = (20.09, 0.75, 0.60)$

$T = 20$

$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	**Parameter Estimates**				
	Estimate	|Bias|	Variance(v)	MSE	σ
θ_0	16.111710	3.978290	44.296391	60.123180	6.655553
θ_1	0.770279	0.020279	0.007360	0.007771	0.085790
θ_2	0.631881	0.031881	0.005410	0.006426	0.073554
$\theta_1+\theta_2$	1.402159	0.052159	0.018954	0.021675	0.137673

Table 8: Monte Carlo Estimates for variables for sample size T
$N = 20, (\theta_0, \theta_1, \theta_2) = (20.09, 0.75, 0.60)$
$T = 40$
$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(v)	MSE	σ
θ_0	17.953678	2.136372	32.254912	36.818999	5.679341
θ_1	0.771508	0.021508	0.001510	0.003384	0.038855
θ_2	0.626930	0.026930	0.002922	0.003647	0.054052
$\theta_1+\theta_2$	1.398438	0.048438	0.004931	0.007277	0.070220

Table 9: Monte Carlo Estimates for variables for sample size T
$N = 20, (\theta_0, \theta_1, \theta_2) = (20.09, 0.75, 0.60)$
$T = 80$
$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	Parameter Estimates				
	Estimate	Bias	Variance(v)	MSE	σ
θ_0	21.108740	1.018740	40.699587	41.737419	6.379623
θ_1	0.745666	0.004334	0.001234	0.001253	0.035131
θ_2	0.598823	0.001177	0.002032	0.002034	0.045081
$\theta_1+\theta_2$	1.344515	0.005485	0.002765	0.002795	0.052582

Table 10: Monte Carlo Values for variables.
Sum $\theta_1 + \theta_2$ with sample size T and fixed N = 20
$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

T	$\theta_1 + \theta_2 = 0.65$	$\theta_1 + \theta_2 = 1.00$	$\theta_1 + \theta_2 = 1.35$
20	0.021571	0.021569	0.021675
40	0.007221	0.006873	0.007277
80	0.002796	0.002796	0.002795

Table 11: Monte Carlo Values for variables .
Sample size T with sum $\theta_1 + \theta_2$ and fixed N = 20
$\ln y = \ln\theta_0 + \theta_1 \ln K + \theta_2 \ln L + u$

Parameter	T = 20	T = 40	T = 80
$\theta_1 + \theta_2 = 0.65$	0.021571	0.007221	0.002796
$\theta_1 + \theta_2 = 1.00$	0.021569	0.006873	0.002796
$\theta_1 + \theta_2 = 1.35$	0.021675	0.007277	0.002795

In Tables 1 to 3, the mean square error (MSE) of θ_1 are 0.007678, 0.001972 and 0.001253 respectively for sample sizes 20, 40 and 80. The mean square error (MSE) for θ_2 are 0.006713, 0.003924 and 0.002034 respectively for sample sizes 20, 40 and 80, These results show that the value of the mean square error for sample size 20 is greater than the mean square error (MSE) for sample size 40, and also the value of the mean square error for sample size 40 is greater than the value of the mean square error (MSE) for sample size 80 and vice-verse . From the results, we observed that as the value of the mean square error (MSE) decreases as the sample size increases and vise-versa. In the same vein, we observed the same trend in Tables 4 to 9.

In Table 10, shows Monte Carlo values for variables sum ($\theta_1 + \theta_2$) with sample size T and fixed N = 20. In this table, for instance, the mean square errors (MSE) for decrease return to scale (that is for $\theta_1 + \theta_2 = 0.65$) are 0.021571, 0.007221 and 0.002796 respectively, for sample sizes 20,40 and 80. These results indicate that the value of the mean square for the sample size 20 is greater than the value of the mean square error for sample size 40, and the value of the mean square error (MSE) for sample size 40 is greater than the value of the mean square error (MSE) for sample size 80.

In Table 10, we observe that for each return to scale, the value of the mean square error decreases as the sample size increases, and vice-versa. This observation is in line with the report of Maxwell and Essi (2019) who worked on power of Monte Carlo Test for Different Returns to Scale in Production Function.

IV. Conclusion

The study has revealed a good number of interesting results which are useful in empirical studies in terms of both methodologies and practical relevance. In economics, the sum of power of the input variables K and L is interpreted as a measure of returns to scale. The work has shown that the mean square errors (MSE) for θ_1 and θ_2 decrease as the sample size increases for various returns to scale. From the findings of the study, we can conclude that Cobb-Douglas Production function is useful and powerful tool for the analysis and evaluation of the governmental structural policies in the context of producing sector in Nigeria.

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