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Finding Optimal Solution of Transportation Problem Using Different Solving Techniques

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Abstract: *In this paper, a fuzzy transportation problem is taken in such a way that the transportation cost, demand ,supply all are in interval numbers and solved in two stages. In the first stage the interval numbers are fuzzified into hexagonal numbers. In the second stage by using ranking technique hexagonal numbers are converted into crisp numbers. Then by applying different methods optimum solution is obtained and the results are compared. This is illustrated by means of numerical examples.*

Keywords: *Interval numbers, Hexagonal fuzzy numbers, ranking technique*.

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I. Introduction

The transportation problem is a special class of the Linear Programming Problems in which the objective is to transport various quantities that are sources (origins) to different destinations in such a way that the total transportation cost will be minimized. To achieve the objective the amount of available supplies and the quantities demanded should be known. Transportation models have wide applications in logistics and supply chain for reducing cost efficiency. The origin of the transportation methods dates back to 1941 by F.L.Hitchcock. Several algorithms have been developed for solving the transportation problem when the cost coefficients of the supply and demand quantities are known exactly.

A fuzzy transportation problem is a transportation problem in which the transportation costs, supply, and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the optimum schedule that minimizes the total fuzzy transportation cost. Many authors discussed the solutions of fuzzy transportation problem using various techniques. Chanas et al [3] developed a method for solving transportation problems by the parametric programming technique. Chanes and Kuchta [4] introduced a new method by transforming the problem to a bicriteria transportation problem which provides only a crisp solution to the given transportation problem. Zero-point method was introduced by Pandian and Natarajan [11] to find the fuzzy optimal solution of fuzzy transportation problem.

In this paper the interval numbers are converted into hexagonal numbers. In order to solve the transportation problem with hexagonal numbers it is converted into a crisp problem by using the ranking method.Now different solving techniques are applied to find the optimum solution to the transportation problem. This is illustrated by means of an example and comparative study is made.

II. Preliminaries

Definition:2.1 (Fuzzy number)
A fuzzy set Ã, defined on the set of real numbers R is said to be fuzzy number if it has the following characteristics i). Ã is normal ii). Ã is convex set iii). The support of Ã is closed and bounded.

Definition:**2.2**
An interval number A is defined as A= [a, b] $=\{x \,/\, a \le x \le b \,, x \in \Re\}$. Here a, $b \in \Re$ are the lower and upper bound of the intervals.

Definition: 2.3 (Hexagonal fuzzy number)
A fuzzy number $\overline{A_H}$ is a hexagonal fuzzy number denoted by $\overline{A_H}$ = (a_1,a_2,a_3,a_4,a_5,a_6) where a_1,a_2,a_3,a_4,a_5,a_6 are real numbers and its membership function $\mu_{\overline{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \le x \le a_3 \\ 1 & a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ 0 & x > a_6 \end{cases}$$

Definition: 2.4 (Ranking Function)
We define a ranking function R:F(R) →R which maps each fuzzy numbers to real line F(R) represent the set of all hexagonal fuzzy numbers. If R be any linear ranking functions. Then

$$R(\overline{A}_H) = \frac{a_1+a_2+a_3+a_4+a_5+a_6}{6}$$

Definition: 2.5
Given a fuzzy set 'A' defined on 'X' and any number α ∈ [0,1], the α –cut is denoted by A(α) and is defined by the crisp set $A(\alpha) = \{x: A(x) \ge \alpha\}$. i.e. A(α)={x: μ(x) ≥ α , α ∈ [0,1]}

Definition: 2.6 (Fuzzification method)
A new approach is used to fuzzify the given interval data into a hexagonal fuzzy number. Consider an interval number [L, U] Thedifference of this interval is d = $\frac{U-L}{5}$.The required hexagonal fuzzy number will be in arithmetic progression.

III. Numerical example

Consider the following transportation problem with interval fuzzy numbers:

	D1	D2	D3	D4	supply
S1	[5,30]	[-5,70]	[4,39]	[6,36]	**[1,6]**
S2	[-4,61]	[1,31]	[6,21]	[6,46]	**[7,12]**
S3	[4,24]	[5,30]	[1,31]	[-5,70]	**[3,18]**
demand	**[7,12]**	**[1,6]**	**[2,12]**	**[3,18]**	

Solution:
The problem is not balanced

	D1	D2	D3	D4	supply
S1	[5,30]	[-5,70]	[4,39]	[6,36]	**[1,6]**
S2	[-4,6]	[1,31]	[6,21]	[6,46]	**[7,12]**
S3	[4,2]	[5,30]	[1,31]	[-5,70]	**[3,18]**
S4	[0,0]	[0,0]	[0,0]	[0,0]	**{2,12]**
demand	**[7,12]**	**[1,6]**	**[2,12]**	**[3,18]**	**[13,48]**

Using the fuzzification method, the given transportation table reduced to hexagonal numbers.

	D1	D2	D3	D4	supply
S1	(5,10,15,20,25,30)	(-5,10,25,40,55,70)	(4,11,18,25,32,39)	(6,12,18,24,30,36)	**(1,2,3,4,5,6)**
S2	(-4,9,22,35,48,61)	(1,7,13,19,25,31)	(6,9,12,15,18,21)	(6,14,22,30,38,46)	**(7,8,9,10,11,12)**
S3	(4,8,12,16,20,24)	(5,10,15,20,25,30)	(1,7,13,19,25,31)	(-5,10,25,40,55,70)	**(3,6,9,12,15,18)**
S4	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	(0,0,0,0,0,0)	**(2,4,6,8,10,12)**
demand	**(7,8,9,10,11,12)**	**(1,2,3,4,5,6)**	**(2,4,6,8,10,12)**	**(3,6,9,12,15,18)**	

Crisp values are obtained by using the ranking technique.

	D1	D2	D3	D4	supply
S1	17.5	32.5	21.5	21	**3.5**
S2	28.5	16	13.5	26	**9.5**
S3	14	17.5	16	32.5	**10.5**

S4	0	0	0	0	**7**
demand	**9.5**	**3.5**	**7**	**10.5**	**30.5**

Applying different solving techniques, to find optimum solution.

Method 1:
VAM

	D1	**D2**	**D3**	**D4**	**supply**
S1	**3.5** 17.5	32.5	21.5	21	**3.5**
S2	28.5	**2.5** 16	**7** 13.5	26	**9.5**
S3	**6** 14	**1** 17.5	16	**3.5** 32.5	**10.5**
S4	0	0	0	**7** 0	**7**
demand	**9.5**	**3.5**	**7**	**10.5**	**30.5**

IBFS is = (3.5x17.5) + (2.5x16)+(7x13.5)+(6x14)+(1x17.5)+(3.5x32.5)+(7x0) = 411

MODI method:

	D1	**D2**	**D3**	**D4**	**supply**
S1	17.5	32.5	21.5	**3.5** 21	**3.5**
S2	28.5	**2.5** 16	**7** 13.5	$\in$ 26	**9.5**
S3	**9.5** 14	**1** 17.5	16	32.5	**10.5**
S4	0	0	0	**7** 0	**7**
demand	**9.5**	**3.5**	**7**	**10.5**	**30.5**

Total transportation cost = (3.5x21) + (2.5x16)+(7x13.5)+($\in$x26)+(9.5x14)+(1x17.5)+(7x0) = 358.5

Method 2:
Alternate method:[8]

	D1	**D2**	**D3**	**D4**	**supply**
S1	17.5	32.5	21.5	**3.5** 21	**3.5**
S2	28.5	**2.5** 16	**7** 13.5	26	**9.5**
S3	**9.5** 14	**1** 17.5	16	32.5	**10.5**
S4	0	0	0	**7** 0	**7**
demand	**9.5**	**3.5**	**7**	**10.5**	**30.5**

Total transportation cost = (3.5x21) + (2.5x16)+(7x13.5)+(9.5x14)+(1x17.5)+(7x0) = 358.5

Method 3:
Fuzzy New method: [3]

	D1	**D2**	**D3**	**D4**	**supply**
S1	17.5	32.5	21.5	**3.5** 21	**3.5**
S2	28.5	**3.5** 16	**6** 13.5	26	**9.5**
S3	**9.5** 14	17.5	**1** 16	32.5	**10.5**
S4	0	0	0	**7** 0	**7**
demand	**9.5**	**3.5**	**7**	**10.5**	**30.5**

Fuzzy modified new method:

	D1	**D2**	**D3**	**D4**	**supply**
S1	17.5	32.5	21.5	**3.5** 21	**3.5**
S2	28.5	**2.5** 16	**7** 13.5	$\in$ 26	**9.5**
S3	**9.5** 14	**1** 17.5	16	32.5	**10.5**
S4	0	0	0	**7** 0	**7**
demand	**9.5**	**3.5**	**7**	**10.5**	**30.5**

Total transportation cost = (3.5x21) + (2.5x16)+(7x13.5)+($\in$x26)+(9.5x14)+(1x17.5)+(7x0) = 358.5

Method 4:
Zero suffix method:[9]

	D1	**D2**	**D3**	**D4**	**supply**
S1	**3.5** 17.5	32.5	21.5	21	**3.5**
S2	28.5	**3.5** 16	**6** 13.5	26	**9.5**
S3	**6** 14	**1** 17.5	16	**3.5** 32.5	**10.5**
S4	0	0	0	**7** 0	**7**
demand	**9.5**	**3.5**	**7**	**10.5**	**30.5**

Total transportation cost = (3.5x17.5)+(3.5x16) +(6x13.5)+(6 x14) +(1x17.5)+(3.5x32.5)+(7x0) = 412

Method 5:

Direct method algorithm: [1]

	D1		**D2**		**D3**		**D4**		**supply**
S1	**3.5**	17.5		32.5		21.5		21	**3.5**
S2		28.5	**2.5**	16	**7**	13.5		26	**9.5**
S3	**6**	14	**1**	17.5		16	**3.5**	32.5	**10.5**
S4		0		0		0	**7**	0	**7**
demand	**9.5**		**3.5**		**7**		**10.5**		**30.5**

Total transportation cost = (3.5x17.5)+(2.5x16) +(7x13.5)+(6 x14) +(1x17.5)+(3.5x32.5)+(7x0) = 411

Method 6:

Minimum supply demand method:[14]

	D1		**D2**		**D3**		**D4**		**supply**
S1	**3.5**	17.5		32.5		21.5		21	**3.5**
S2		28.5		16	**3.5**	13.5	**6**	26	**9.5**
S3	**6**	14		17.5		16	**4.5**	32.5	**10.5**
S4		0	**3.5**	0	**3.5**	0		0	**7**
demand	**9.5**		**3.5**		**7**		**10.5**		**30.5**

Total transportation cost = (3.5x17.5)+(6x14) +(3.5x13.5)+(6x26)+(4.5x32.5)+(3.5x0)+(3.5x0) = 554.75

Comparison table

S.No	**Methods**	**Optimum cost**
1	MODI method using VAM	358.5
2	Alternate method :	358.5
3	Fuzzy modified new method	358.5
4	Zero suffix method :	412
5	Direct method algorithm	411
6	Minimum supply demand method	554.75

From the table, we see that MODI method, Alternate method & Fuzzy modified new method gives lesser transportation cost when compared to remaining methods. Also the three methods number of iterations are very less when compared to remaining methods. The steps are very easier and calculating time is also very less.

IV. Conclusion

Transportation problems with hexagonal numbers are considered. Various techniques are applied to find the optimum solution of the transportation problem. A comparison is made between each method and the results are discussed for the numerical problem.

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