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Modeling the Dynamics of Measles and Control

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Abstract: *The attack of measles is endemic and dangerous to human existence; hence, this paper focused onmodeling of the transmission and dynamics with control of measles. Basic mathematical differential equation is used to model the rate of spread and possible control of the disease was discussed and computed. Data used was sourced secondarily. In this study, analysis revealed that the transmission of measles is significantly rising with about 5000 new individuals yearly. In addition, it was discovered that as time goes on (say t = 2025, I(t) = 70,582) the number of people that will be infected by measles if preventive or curative measure are not taken in metropolis will grow increasingly and will one day cover up the entire population.However, the study developed a model for the control and the result appear to significantly reduce the growth at which measles spread in the population given the parameters included in the model. It was recommended therefore that since the model shows that the spread of a disease largely depend on the number of people infected; therefore, the National Measles Control Programme should emphasize on the improvement in early detection of measles cases so that the disease transmission can be minimized.*

Keywords: *(Transmission, Dynamics, Control, Model and Epidemics)*

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I. Introduction

The attack Measles virus is endemic and dangerous to human existence; it is one of the public health issues challenging human population that demand attention. Measles is an infectious disease highly contagious through person-to-person transmission mode with greater than 95% secondary attack rates among susceptible persons. It is one of the worst cause of eruptive fever that can occur in many childhoods thereby leading to serious and fatal complications including: pneumonia, diarrhea and encephalitis. A number of infected children and adults subsequently suffer blindness, deafness and impaired vision. In fact, millions people were infected and died, hospitalized and developed chronic disability.However, measures to prevent outbreaks of measles diseases is a priority where poorly immunized populations are housed in overcrowded settings with limited water, sanitation and hygiene resources. The proportion of children vaccinated against measles was adopted as an indicator to measure progress towards curbing the epidemics but a rebound in measles deaths pose a substantial threat to achieving this goal. The main symptoms of measles are fever, runny nose, cough and a rash all over the body, it also produces characteristics-red rash and can lead to serious and fatal complications including pneumonia, diarrhea and encephalitis. Many infected children subsequently suffer blindness, deafness or impaired vision (Michel, 2011). Measles is a highly contagious virus that lives in the nose and throat mucus of an infected person which can spread to others through coughing and sneezing. In addition, measles virus can live for up to two hours in an airspace where the infected person coughed or sneezed. Measles is a disease of humans and not spread by any other animal species (Sharma, 2013).

There is no specific treatment for measles. People with measles need bed rest, fluids, and control of fever. The progress in measles control sometimes was based on implementation of recommended measles mortality reduction strategies including increasing routine immunization coverage, periodic Supplemental Immunization Activities. Also,Strategies adopted were strengthening routine vaccination, providing a second opportunity for measles vaccination through monitoring disease trends, and improving measles case management. There is a mass vaccination campaigns aimed at immunizing a good number of a predefined population within several days or weeks, laboratory-supported surveillance, and appropriate management of measles cases as by World Health organization. Also, global measles control and mortality reduction, and countries that have fully implemented and sustained these strategies have experienced reductions in measles incidences of greater than 90 percent (Otten, et al. 2005).

This endemic disease is really an issue in many parts of the world including developed and developing nations which continued to cause both economic and health challenges to large population globally. Hence, this paper is motivated to investigate the mathematical model for transmission and control dynamics of measles using ordinary differential equation model. The analysis of dynamics of measles can be used to predict measles outbreak before it occurs and plan vaccination programmes to prevent future measles outbreak.

Measles is caused by a virus which is transmitted on close contact via airborne propagules thereby leading to the development of a typical rash. The infectious period is in the order of a week, after which the hosts recover and develop lifelong immunity. Hosts are infected only once in their lifetime and, if the force of infection is sufficiently large, this happens at a young age, and hence measles is a childhood disease. In unvaccinated populations measles is a common disease, infection is not without danger. In developed countries, infection with measles leads to complications in one out of seven cases and is fatal in about one in 5,000 cases (Carabin, *et al.*2002). Measles is a highly infectious viral disease with an incubation period of 0-12 days. Transmission is through aerosol droplets or direct contact with the nasal and oral secretions of an infected person to susceptible individuals, especially children between the ages of 9 months to 15 years. This disease is considered endemic especially in the developing countries with a peak of transmission from October to March. Measles carries with it high morbidity and mortality especially when clinical cases are not properly managed (Wills, 2009). The case fatality rate of measles in the developing countries is around 3-5% and this could be as high as 10% during epidemics. Natural infection with the measles virus confers life-long immunity, those vaccinated with the vaccine would get up to 10 years of protection from the measles virus. However, it is interesting to note that about 85% of children vaccinated with a dose of the measles vaccine could also get life-long immunity. In spite of the global efforts to vaccinate all eligible children as early as 9 months of age as the maternal immunity wanes out around that age, it is disheartening to mention that measles remains one of the leading causes of death among the under-fives, especially in the Sub-Saharan African continent (Woods and Glibbs, 2013).

Rancho, (2001) stated that the laboratory confirms measles when a blood sample taken from a suspected case within 30 days of onset of rash shows measles antibodies. The caveat here with regards to antibodies from suspected blood sample is that measles vaccination as well as measles infection both results in raised antibodies; thus, presence of measles in those vaccinated with the measles antigen 30 days before the sample is collected does not imply disease but rather vaccination against measles. Rancho (2001) furthermore stated that an outbreak of measles is said to occur when there are at least 3 measles positive as confirmed by the laboratory in a health facility or district within 1-month. In spite of the increase in measles immunization coverage aimed at building population immunity against the disease, measles outbreaks still occur especially in the developing countries; this ranks measles top among the burden of vaccine-preventable diseases across the globe with worst picture seen in the developing countries that are not unconnected to malnutrition and overcrowding. Robbs (2000) stated that the sporadic measles outbreaks seen are often fatal with high morbidity and mortality in those under the age of 5 years. Interrupting transmission of this fatal disease requires an effort to achieve population immunity of at least 95%. This calls for the need to review the existing strategies for the control of this fatal but preventable disease.

In Nigeria, measles case-based surveillance started in response to measles catch-up supplementary immunization activities (SIAs) in the Northern part in the last quarter of 2005. However, the measles case-based surveillance was subsequently implemented across the country after the SIAs in the Southern states in late 2006. It is interesting to mention that 1,346 suspected measles cases were reported since January 2007 with 196 laboratory-confirmed by the laboratory and or epidemiological linkage and of these confirmed cases, 62% were 1-4 years and 23% aged 5-14 years. There is a need to emphasize that the pattern of measles in Nigeria is predominantly among the younger un-immunized population due to immunity gaps as a result of inadequate routine measles coverage among others (Adegoke, 2005).Onyiriuka (2011) in a secondary health center, in a Southern Nigerian city, it was observed that measles accounted for 3.1% of all pediatric admissions in the hospital; this figure is higher than the 2.3% reported in 1998 at a tertiary health center in the same city. Etuk et al. (2015) also noted a rise in the prevalence of measles in a tertiary health center, in a neighboring state. There are various studies on the trend of measles, causes of outbreaks, reasons for high vaccination dropout rates by mothers, and lack of awareness on the benefits of vaccination as a whole. In an attempt to look at the epidemiology of measles in South-West Nigeria.

Fatiregun, et al. (2014) analyzed measles case-based surveillance data from 2007 to 2012 used a descriptive analysis (persons, place, and time) of measles cases and which was confirmed through laboratory and epidemiological link. They predicted expected measles cases in 2015 using additive time series model. Furthermore, in a similar study on trends and patterns of under-fives vaccination in Nigeria, using four National Demographic and Health surveys datasets involving 44,071 (weighted) children from 1990 to 2008. The researchers examined child health information including the proportion of those who had some or completed their routine childhood vaccinations, the trends, as well as a pattern of vaccination over 18 years. The authors also selected certain factors and regressed them to obtain predictors of child vaccinations in Nigeria. Considering the importance of timeliness and completeness of reporting on all suspected infectious diseases, a retrospective review of surveillance records was conducted between January 1, 2007 and June 30, 2008. This was done by review of records of suspected cases of measles from the registers of 23 health facilities in Nigeria. Odega, et al. (2016) used a capture-recapture method to obtain an estimate of the total number of measles cases required for the study area within the period under review. Completeness of reporting was by calculating the ratio of a number of measles reported by hospitals to the number of estimated cases using the capture-recapture method. Although there are safe and effective vaccines against measles, measles remains a significant cause of childhood morbidity and mortality in Nigeria.

Muscat, et al. (2015) investigated measles using an epidemiological assessment tools where they engaged case-based data from 2006 to 2007 collected by the national surveillance institutions submitted by the 32 European countries. In addition, data were obtained for age group, confirmed diagnosis, vaccination status, hospital treatment, those presented with acute encephalitis as a complication, and those who died because of the disease. Cases were separated based on age as well as graded countries with indigenous measles incidence per 100,000 inhabitants per year. The data were analyzed based on clinical diagnosis, laboratory-confirmed cases, and epidemiologically linked cases in accordance with the requirements for national surveillance. The authors regarded indigenous case as those that not recorded as imported from another country and those with unknown importation status.Olaleye and Olayiwola (2010) also worked on the spread of measles with assumptions that P be the total number of people in the place, I(t) be the number of people infected with measles at time (t) and S as the susceptible that are at risk of contracting measles but are presently free from infection. They found that based on the evaluation of the two models they estimated to show conformity of differential equation on a real life situation. Also, the rate at which measles would be spreading if no control were introduced, is alarming and highly dangerous to the human population because it will be growing without limit. They found that the in the first ten years, the number of prevalence people infected with measles reduced to half, which shown that if the control parameter is used properly the number of people will reduce to zero.***Modeling the Dynamics of Measles***

In developing a model for the spread of measles, some assumptions need to be made these assumptions were as follows. The rate of spread of measles is proportional to the susceptible, number of people not infected with measles but liable to be infected at any time (t).

$$\frac{dI(t)}{dt} = \alpha S \quad (1)$$

α = rate of infection of measles.

$$S = P - I(t) \quad (2)$$

t = the time of infection in a given year.

P = the total population of people in a place

I(t) = the number of people infected with measles at a given time (t)

S = the susceptible, those that stand at risk of contracting measles but are presently free from the infection.

$$\frac{dI(t)}{P-I(t)} = \alpha dt \quad (3)$$

$$\frac{1}{P-I(t)} = Ae^{\alpha t}$$

$$A = e^{c}$$

$$I(t) = p(1 - e^{\alpha t}) \quad (4)$$

Modeling the Control of Spread of Measles

However, to control the rate of spread of measles, a threat to human population, attention is demanded to curb and manage the situation in order reduce the transmission to minimum level

Mathematically, it is given by the differential equation

$$\frac{dI(t)}{dt} = CI_c(t) \quad (5)$$

Rearranging and integrating the equation

$$I_c(t) = Ae^{-kt} \quad (6)$$

$$A = e^{k}$$

t = 0

$I_c(0) = A$

Substituting into equation (7)

$$7I_c(t) = I(0)e^{-ct} \quad (7)$$

I_c (t) = the controlled prevalence number of people infected with measles at time (t),

t = the time of infection.

Therefore, the rate of number of people infected with measles is inversely proportional to time (t), which imply that as time t increases, I_c (t) decreases.

I_c (0) = is the current number of people already infected with measles

C = the control parameter that decayed infection of measles (controlled rate of infection).

We assume that after the period of observation and warning about the spread of measles, the agencies involved would have worked out the prevention and then the control rate is assumed to be half of the rate of spread.

C = ½α

= ½ (0.0171)

= 0.00855

II. Data Presentation and Analysis

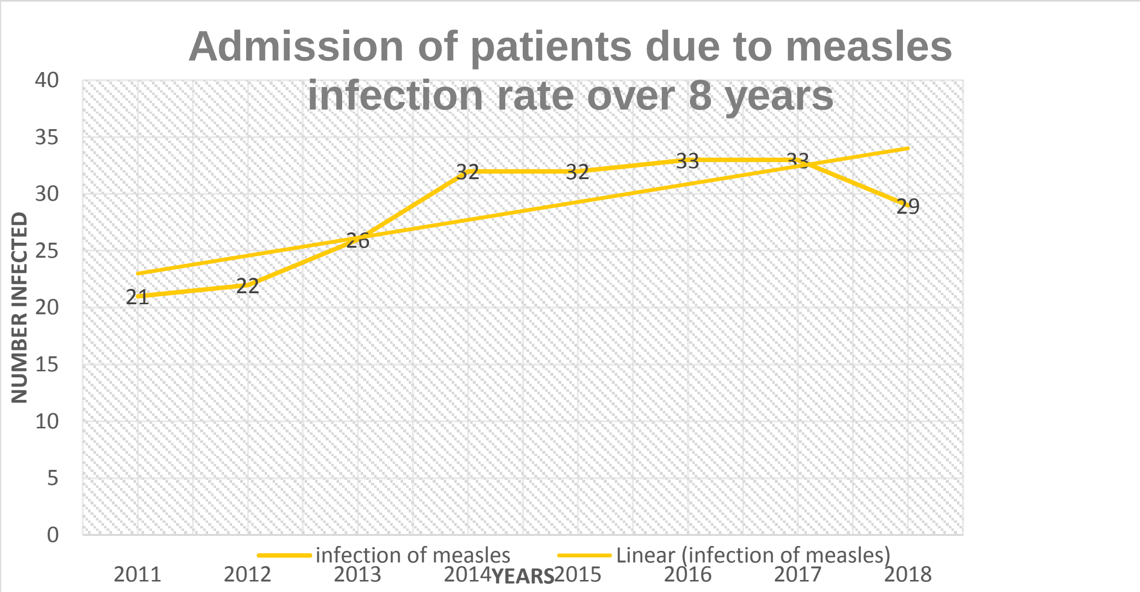
The data collected for this study is summarized in the table below for better description and analysis.

Table 1: (Admission of People Infected with Measles)

Years	Time	Infected (Admission Due to Measles Infection)	Susceptible (Exposed to Risk of being Infected)
2011	1	21	311949
2012	2	22	311948
2013	3	26	311944
2014	4	32	311938
2015	5	32	311938
2016	6	35	311935
2017	7	33	311937
2018	8	29	311941

Source:*(Central Hospital, Warri, 2019)*

Figure 1: (Graph showing the Trend of Admission due to Measles Infection)



Modeling the Spread and Computation

Given the national infection rate of measles as 0.0171.According to National Population census figure for 2006, the population Warri environs equals approximately 311970

Then applying the derived transmission model.

$$I(t) = p(1 - e^{\alpha t})$$

Where e is a numerical constant that is equal to 2.71828

$I(1) = 311970\left(1 - 2.71828^{-0.0171\,(1)}\right) = 311970(1 - 0.9830)$

$2011 = 311970 \times 0.0171 \ = 5334.69$

$I(2) = 311970\left(1 - 2.71828^{-0.0171\,(2)}\right) = 311970(1 - 0.0.9663)$

$2012 = 311970 \times 0.0337 = 10513.389$

$I(3) = 311970\left(1 - 2.71828^{-0.0171\,(3)}\right) = 311970(1 - 0.9499) = 311970 \times 0.0501$

$2013 = 15629.697$

$I(4) = 311970\left(1 - 2.71828^{-0.0171\,(4)}\right) = 311970(1 - 0.9339)$

$2014 = 311970 \times 0.066 = 20625.32$

$I(5) = 311970\left(1 - 2.71828^{-0.0171\,(5)}\right) = 311970(1 - 0.9181)$

$2015 = 311970 \times 0.0819 = 25{,}550.34$

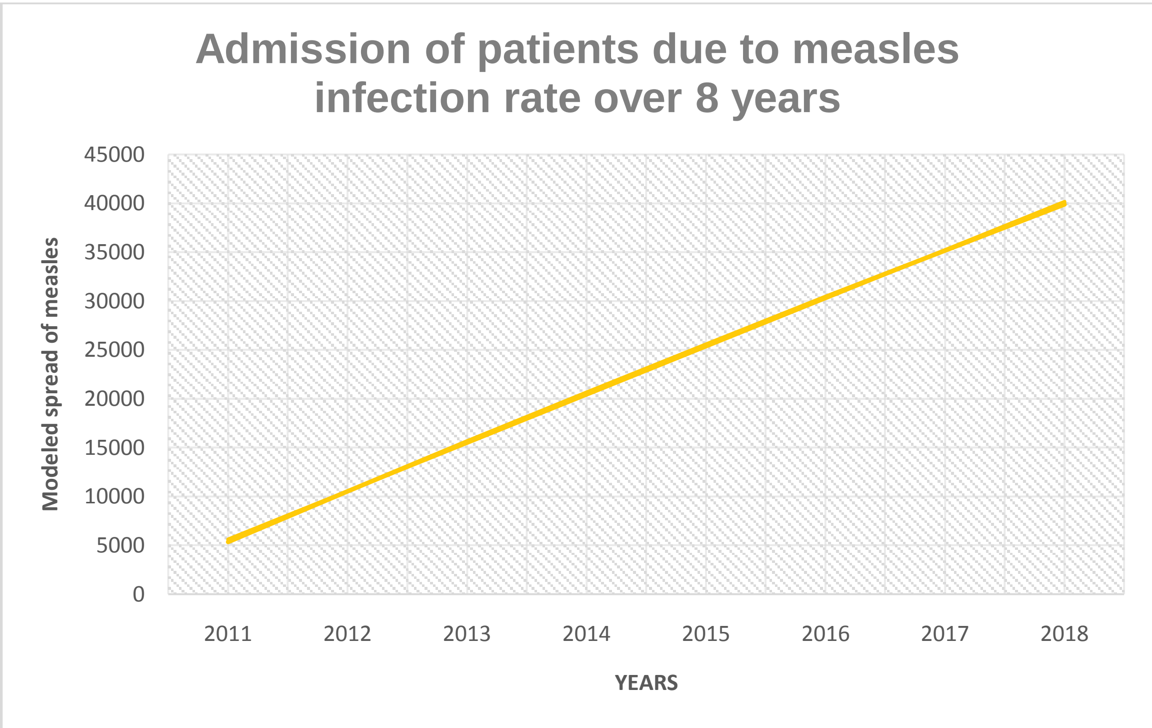
$I(6) = 311970(1 - 2.71828^{-0.0171(6)}) = 311970(1 - 0.9025)$
$2016 = 311970 \times 0.0975 = 30{,}417.075$
$I(7) = 311970(1 - 2.71828^{-0.0171(7)}) = 311970(1 - 0.8872)$
$2017 = 311970 \times 0.1128 = 35{,}190.22$
$I(8) = 311970(1 - 2.71828^{-0.0171(8)}) = 311970(1 - 0.8721)$
$2018 = 311970 \times 0.1279 = 39{,}900.96$

Table 2: (Transmission of Measles Based in Model Estimation)

Years	*Time*	*Transmission of measles within the population of 311970*
2011	1	5335
2012	2	10513
2013	3	15630
2014	4	20625
2015	5	25550
2016	6	30417
2017	7	35190
2018	8	39901

Source: Central Hospital, Warri(2019)

Figure 2: Graph showing the transmission of measles in Warri



Modeling the Control and Computation

After the period of observation and the warning against the spread of measles, the health agencies involved would have worked on how to prevent the occurrence as the rate of control of the disease is assumed to be half of the rate of spread.

C = ½α = ½ (0.0171) = 0.0086

$$I_c(t) = I(0)e^{-ct}$$

2011 $I_c(1) = 311949(1 - 2.71828^{-0.0086(1)}) = 311949(1 - 0.9914)$
$= 311949 \times 0.0086 = 2682.942$
2012 $I_c(2) = 311948(1 - 2.71828^{-0.0086(2)}) = 311948(1 - 0.9829)$
$= 311948 \times 0.017 = 5334.3108$
2013 $I_c(3) = 311944(1 - 2.71828^{-0.0086(3)}) = 311944(1 - 0.9745)$
$= 311944 \times 0.0255 = 7954.572$

2014 $I_c(4) = 311938\left(1 - 2.71828^{-0.0086(4)}\right) = 311938(1 - 0.9662)$
$= 311938 \times 0.0338 = 10543.5044$
2015 $I_c(5) = 311938\left(1 - 2.71828^{-0.0086(5)}\right) = 311938(1 - 0.9579)$
$= 311938 \times 0.0421 = 13132.5898$
2016 $I_c(6) = 311937\left(1 - 2.71828^{-0.0086(6)}\right) = 311937(1 - 0.9497)$
$= 311937 \times 0.0503 = 15690.3305$
2017 $I_c(7) = 311937\left(1 - 2.71828^{-0.0086(7)}\right) = 311937(1 - 0.9416)$
$= 311937 \times 0.0584 = 18217.1208$
2018 $I_c(8) = 311941\left(1 - 2.71828^{-0.0086(8)}\right) = 311941(1 - 0.09335)$
$= 311941 \times 0.0665 = 20744.0765$

Table 3:(Control of Measles based on Model Estimation)

Years	Time	Control of Measles within the Population of 311970
2011	1	2683
2012	2	5334
2013	3	7955
2014	4	10544
2015	5	13133
2016	6	15690
2017	7	18217
2018	8	20744

Figure 3:(Graph showing the Control of Measles)

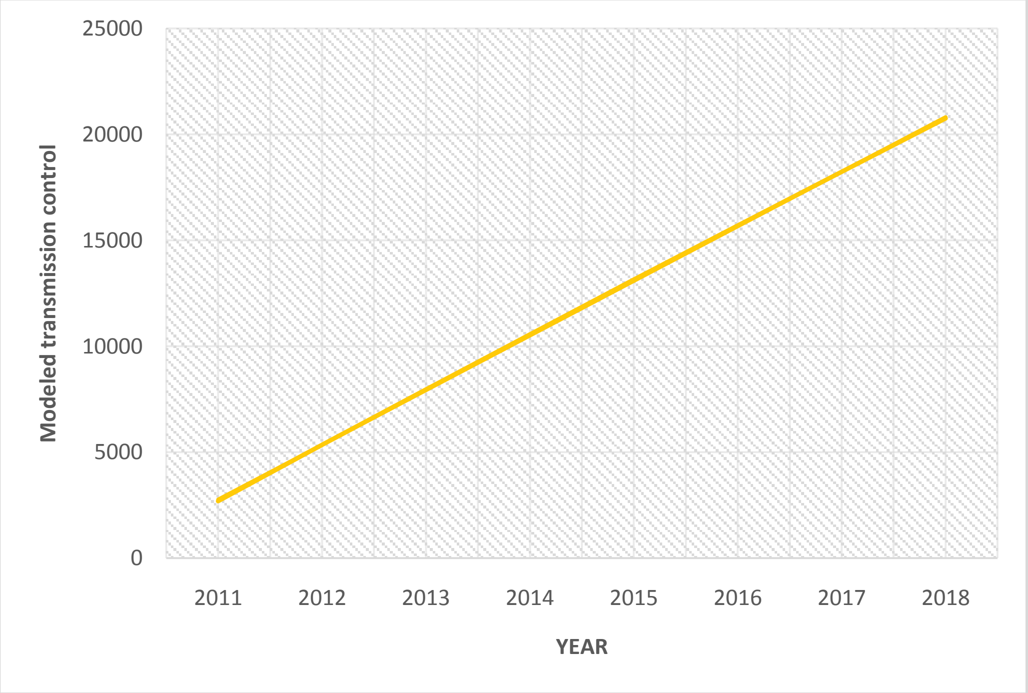
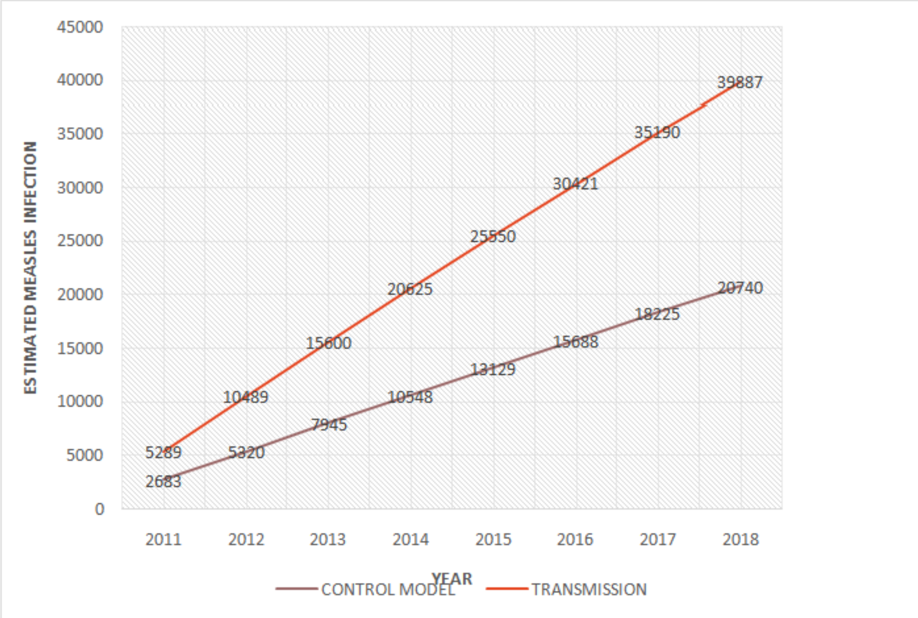


Figure 4: *(Graph showing a comparison between the spread and Control of measles)*



III. Discussion of Results

In figure (1) result depicts that the reported cases of measles infection in Warri is on the increase and it indicates a level of morbidity in the population. From the model specification and available data presented in this study, the analysis of the transmission dynamics is presented. Based on the model evaluation, the result indicated that with the current status, if the precautions are not taken, the widespread of measles will eat too deep into the population. Model evaluation supported the description of people admitted due to measles infection as in indicated in figure 1 above.In table (2), it is clear that transmission of measles is significantly rising with about 5000 new individuals yearly which implies that as time tend to infinity (say at year 2025, $t = 15$, $I(t) = 70{,}582$) the number of people that will be infected by measles if preventive or curative measure are not taken in the metropolis will grow increasingly and will one day cover up the entire population. Also, figure (2) indicates a very smooth upward trending of the transmission of measles disease in the population. However, on the control model, it was found that if the warnings are yielded to, the spread of measles appears to depreciate gradually as compared to the spread rate.

IV. Summary

Measles disease has continued causing both economic and health problems to large population worldwide mostly affecting children. This paper developed a mathematical representation for the transmission dynamics and control of measles. The study focused on the model of Susceptible, Infected and Recovered (SIR) differential equation for the transmission dynamics and control of the disease. The results on the spread of the disease reported from the model estimation summary is clear that the transmission of measles is significantly rising with about 5000 new individuals yearly. This implies that as time tend to infinity (say $t = 15$, $I(t) = 70{,}582$) the number of people that will be infected by measles if preventive or curative measure are not taken in Warri metropolis will grow increasingly and will one day cover up the entire population.

V. Conclusion

The study concluded on its findings that measles transmission is still rapid in the population and if not carefully managed, it will cause an unimaginable outbreak which will result to low economic grading in terms of health by the World Health Organization. The diseases can be controlled through provision of adequate health facilities and education of public prevention.

VI. Recommendation

Eradication of contagious diseases such as measles has remained one of the biggest challenge facing developing counties like Nigeria. Therefore, there is an urgent need for Health Ministry to come up with some new control strategies and more efficient ones to fight the spread of the disease in the country. Therefore; from the outcome of the results shows that the spread of a disease largely depends on the number of people infected, therefore, the National Measles Control Programme should emphasize on the improvement in early detection of measles cases so that the disease transmission can be minimized. To attain high level immunity for the disease, mass vaccination exercise should be encouraged to cover the majority of the population to prevent outbreak of the disease in developing country. Any researcher may use this model as a foundation to perform a case study at a specific region and obtain practical results. Measles infected individuals should be treated early in order to limit its transmission

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