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Numerical Investigation of Transport in a Couette Flow with Unsteady Suction

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Abstract
This study investigates the fluid flow and transport in a vertical channel with an exponentially decaying suction and mobile wall. The governing equations are derived based on the assumptions of incompressible flow with buoyancy forces and viscous dissipation. A finite-difference scheme is formulated and implemented. The numerical results are presented graphically. The results show that the increase in Brinkman number, Suction Parameter and Prandtl number increase the temperature distribution, while the increase in Thermal Grashof number, Mass Grashof number and Suction parameter increase the flow velocity.

Keywords: Finite difference scheme, Heat and Mass Transport, Couette flow, Unsteady Suction and Mobile wall, Viscous Dissipation

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I. Introduction:

Transport in Porous media flow has applications in geothermal extraction, agricultural water distribution, dispersion in Aquifer. It is also important to curb or control both industrial and natural pollutions like (oil spillage and water bodies), due to the natural actions of man and the survival of other living things in the atmosphere. This has pulled much enthusiasm to the modeling and simulation of such flows.

A lot of research has been done on the effects of pollutant and thermal energy in porous media. Amhalhel *et al.* [1] studied boundary layer effects of heat and mass transfer in porous media. Amos *et al.* [2] investigated the effect of heat and mass transfer on free convection boundary layer flow of a rotating MHD fluid past a vertical porous plate with thermal radiation. The solutions to the governing equation are solved analytically using the perturbation technique by neglecting viscous dissipation and pressure term. It was observed that increase in magnetic field parameter and Schmidt number decrease the velocity of flow while the increase in temperature increases the velocity of the flow system.

Attia *et al.* [3] studied the effect of porosity and magnetic field on heat transfer between two parallel porous plates for couette flow under pressure gradient and hall current. Chein [4] investigated the effects of turbulent shear stress and the rate of dissipation near a solid wall temperature using the Taylor series expansion. Da Silva and Bejan [5] the maximization of heat transfer density using a new design concept for generating multi-scale structures in natural convection was investigated. Nwaigwe [6] investigated the sequential implicit numerical scheme for pollutant and heat transport in a plane-poisecuille flow with variable viscosity and variable diffusion. Das etal [7] investigated the radiative effects on free convection MHD couette flow with variable wall temperature in the presence of heat generation. The analytical solutions are solved using the Laplace transform technique. Israel-Cookey *et al*. [8] Investigated significant effects of radiation and magnetic field on MHD oscillatory couette flow of a viscous fluid in a porous medium with periodic wall temperature. The increase in magnetic field, Grashof number and porosity parameter enhance the fluid velocity while the increase in radiation parameter reduces the velocity of flow in the system. The governing equation did not consider the concentration equation which means that the effect of diffusion was outside the scope of this study.

Levintal *et al*. [9] The problem of modeling flow and heat transfer in porous media adopting the two mechanisms that can transfer gas at high rates across the earth-atmosphere interface in dry porous media was investigated. Malviya and Dwivedi [10] investigated the effect of heat transfer in porous media and describing how the governing equations are formed by the law of thermodynamics with constant wall temperature. The pressure of the fluid flow is reduced by the increase in porous layer and decrease in mass transfer decreases the velocity of the flow. Nield and Bejan [11] studied the effect of radiation in a porous media by concentrating on the equation that viscous dissipation and neglecting the work done by the pressure. The result shows that increase in temperature decreases the porosity of the system. Ogulu and Amos [12] studied the problem of suction/injection on free convective flow of a non- Newtonian fluid past a vertical porous plate with viscous dissipation.

Reddy *et al.* [13] studied the effect of MHD boundary layer flow, heat and mass transfer analysis over a rotating disk through a porous medium saturated by Cu-water and Ag-water nanofluid chemical reaction. In [2, 13] the pressure term and viscous dissipation were not incorporated in the model for momentum and energy equation. Tamayol *et al*. [14] Studied the similarity solution for boundary layer flow through a porous medium over stretching wall temperature or heat flux in which the result indicates that the heat transfer is increased by the Prandtl number and suction to the surface.

This paper investigates the effects of pollutant and heat transport in a porous media couette flow which extends the work of Amos *et al.* [2] and Nwaigwe [6] by incorporating the pressure and viscous dissipation terms on a well-developed model of momentum and temperature equations with appropriate initial conditions. The results are solved using a finite difference scheme.

II. Mathematical Formulation

The problem under investigation involves a vertical channel with two parallel walls separated by a distance r_0 apart. The channel is filled with a porous solid which is fully saturated with an incompressible Newtonian fluid with temperature variations. Further, we assume the presence of some pollutant which is continuously injected into the fluid. By assuming a time-dependent suction, viscous dissipation and Brinkman flow, the mathematical equations governing the velocity $u^{'}$, temperature $T^{'}$ and concentration $C^{'}$ are given as Amos *etal* [2] and Nwaigwe *et al.* [15].

$$\frac{\partial u^{'}}{\partial t^{'}} + W^{'}\frac{\partial u^{'}}{\partial y^{'}} = g\beta_T(T^{'} - T_0) + g\beta_c(C^{'} - C_0) + \mu\frac{\partial^2 u^{'}}{\partial y^{'2}} - \frac{\sigma B_0^2 u^{'}}{\rho} - \mu\frac{u^{'}}{K_p^{'}} - \frac{1}{\rho}\frac{\partial p^{'}}{\partial x^{'}}, \quad (1)$$

$$\frac{\partial C^{'}}{\partial t^{'}} + W^{'}\frac{\partial u^{'}}{\partial y^{'}} = D\frac{\partial^2 C^{'}}{\partial y^{'2}} - S_0^{'}e^{-a(C^{'}-C_0)}, \quad (2)$$

$$\frac{\partial T^{'}}{\partial t^{'}} + W^{'}\frac{\partial T^{'}}{\partial y^{'}} = \frac{K}{\rho C_p}\frac{\partial^2 T^{'}}{\partial y^{'2}} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^{'}} + \mu\left(\frac{\partial u^{'}}{\partial y^{'}}\right)^2\frac{1}{\rho C_p}, \quad (3)$$

where $t^{'}$ is the time variable, $y^{'}$ is the space variable (coordinate) along the width of the channel and $x^{'}$ is the space variable along the channel length.

The above problem is complimented with the conditions:

$$u^{'} = 0, T^{'} = 0, C^{'} = 0 \text{ at } y^{'} = 0,$$
$$u^{'} = u_0, T^{'} = T_0, C^{'} = C_0 \text{ at } y^{'} = r_0, \quad (4)$$
$$u^{'} = \begin{cases} w_0, \wedge\, y = r_0 \\ 0, \wedge\, else \end{cases}, T^{'} = \quad (5)$$

where $W^{'} = -w_0\gamma e^{-1000t^{'}\left(\frac{w_0^2}{\mu}\rho\right)}$. (6)

where gamma is a non-dimensional parameter.

We introduce the Roseland approximation:

$$q_r = \frac{-4\sigma}{3K}\frac{\partial T^{'4}}{\partial Z^{'}}\ , \quad (7)$$

where σ is Stefan–Boltzmann constant, K^{*} is the mean absorption coefficient. Further assume that the temperature difference within the flow are sufficiently small and therefore T'^4 can be expressed as a linear function of temperature about the free stream T_∞ using Taylor's series expansion in which the higher other terms were neglected,

$$T'^4 \cong 4T_\infty^{'3}T^{'} - 3T_\infty^{'4}, \quad (8)$$

$$q_r = \frac{-4\sigma}{3K}\frac{\partial\left[4T^{'}(T_\infty^{'3} - 3T_\infty^{'4})\right]}{\partial y^{'}},$$

Substituting equation (8) into (9)

$$q_r = \frac{-16\sigma \quad T_\infty^{'3}}{3K}\frac{\partial T^{'}}{\partial y^{'}}, \quad (9)$$

Putting equation (9) into (2), we have

$$\frac{\partial T^{'}}{\partial t^{'}} + W^{'}\frac{\partial T^{'}}{\partial y^{'}} = \frac{K}{\rho C_p}\frac{\partial^2 T^{'}}{\partial y^{'2}} + \frac{1}{\rho C_p}\frac{16\sigma \quad T_\infty^{'3}}{3K}\frac{\partial^2 T^{'}}{\partial y^{'2}} + \mu\left(\frac{\partial u^{'}}{\partial y^{'}}\right)^2\frac{1}{\rho C_p}, \quad (10)$$

Non-Dimensional Variables and Parameters

Defining the following non-dimensional quantities:

$$u = \frac{u^{'}}{w_0}, t = \frac{t^{'}w_0^2\rho}{\mu}, Pr = \frac{\mu C_p}{K}, \ \phi = \frac{C^{'} - C_0^{'}}{C_w - C_0}, \ Sc = \frac{\mu}{\rho D}, \quad Gc = \frac{g\beta_c\mu(C_w - C_0)}{w_0^3\rho}, \quad Gr = \frac{g\beta_T\mu(T_w - T_0)}{w_0^3\rho}, \ y = \frac{y^{'}}{r_0},$$

$$K_p = \frac{w_0^2\rho^2 K_p^{'}}{\mu^2}, \ \theta = \frac{T^{'} - T_0^{'}}{T_w - T_0}, Rd = \frac{16\sigma \quad T_\infty^{'3}}{3K \quad K}, x = \frac{x^{'}}{h}, \ M = \frac{\sigma B_0^2\mu}{\rho^2 w_0^2}, P_x = \frac{\mu}{hw_0^2\rho^2}\left(\frac{\partial p^{'}}{\partial x}\right),$$

$$a = \frac{\alpha}{C_w - C_0}, S_0 = \frac{\mu S_0^{'}}{w_0^2 \rho (C_w - C_0)}, N = -P_x, Br = \frac{w_0^2 \mu}{K(T_w - T_0)}, y^{'} = r_0 \quad (11)$$

Substituting the non-dimensional variables on equation (1)- (6) and (10), we obtain the following dimensionless equations for velocity, temperature and concentration.

$$\frac{\partial u}{\partial t} - \gamma e^{-1000t} \frac{\partial u}{\partial y} = Gr\theta + Gc\phi + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K_p}\right) u + N \quad (12)$$

$$\frac{\partial \phi}{\partial t} - \gamma e^{-1000t} \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - S_0 e^{-\alpha \phi} \quad (13)$$

$$\frac{\partial \theta}{\partial t} - \gamma e^{-1000t} \frac{\partial \theta}{\partial y} = \frac{1}{Pr}(1 + Rd) \frac{\partial^2 \theta}{\partial y^2} - \frac{Br}{Pr}\left(\frac{\partial u}{\partial y}\right)^2 \quad (14)$$

subject to,

$u = 0, \theta = 0$, and $\phi = 0$ at $y = 0$.

$u = 1, \theta = 1, \phi = 1$ at $y = 1$ (15)

with initial conditions.

$$u = \theta = \phi = \begin{cases} 1, y = 1, \\ 0, else. \end{cases} \quad (16)$$

where Gr, Gc, K_p, Sc, Pr and Br are Thermal Grashof number, Mass Grashof number, the permeability of the porous medium, the Schmidt number, the Prantl number and the Brinckman number respectively.

III. Numerical Formulation

The numerical scheme for the non-dimensionalized problem above is presented

Let $\Omega_h = \{z_i : z_i = ih, h = \frac{1}{Q}, Q = Z^{+}\}$. Define $\partial_z^{\pm} u_i = u_{i+1} - u_i, \forall u_i \in \{u_1, \ldots, u_{Q-1}\}$, $\partial_t^{+v^n = v^{n+1} - v^n}$ (17)

Following Nwaigwe [6] and Nwaigwe *etal* [15], we propose the following scheme:

$$h^2 \partial_t^{+u_i^{n+1} - h\Delta t \gamma (g^{n+1}) \partial_z^{+u_i^{n+1} = \Delta t}}, \quad (18)$$

$$h^2 \partial_t^{+\theta_i^{n+1} - h\Delta t \gamma (g^{n+1}) \partial_z^{+\theta_i^{n+1} = \frac{\Delta t}{Pr}}} \quad (19)$$

$$h^2 \partial_t^{+\phi_i^{n+1} - h\Delta t \gamma (g^{n+1}) \partial_z^{+\phi_i^{n+1} = \frac{\Delta t}{Sc}}} \quad (20)$$

and, $$\left.\begin{aligned} \varphi_0^{n+1} &= \varphi(0, t^{n+1}) = \varphi(1, t^{n+1}), \forall n \\ \varphi_i^0 &= \varphi(z_i, 0) \forall n z_i \in \Omega_h \end{aligned}\right\} \forall \varphi \in \{u, \theta, \phi\}, \quad (21)$$

In the above formulation, $\gamma^n = e^{-1000t^n}$

IV. Results and Discussion

The governing equation (12)-(14) given by the boundary conditions in equation (15), subject to the initial conditions in equation (16) are solved using the numerical scheme in (17)-(21). Figures 1, 2 and 3 show the time evolution of velocity, concentration and temperature respectively. The increase in time enhances the velocity of flow fields, pollutant concentration and the temperature of the plate. Hence, has a smooth convergence in the flow.

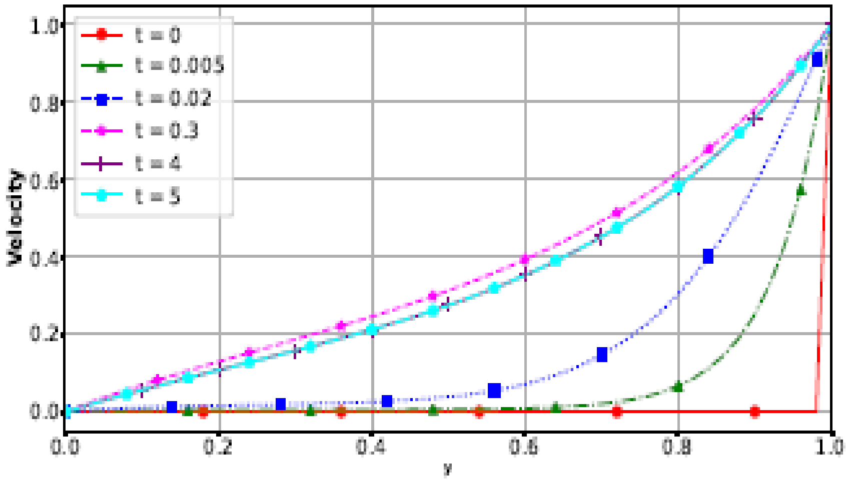


Figure 1: Time Evolution of Velocity Distribution.

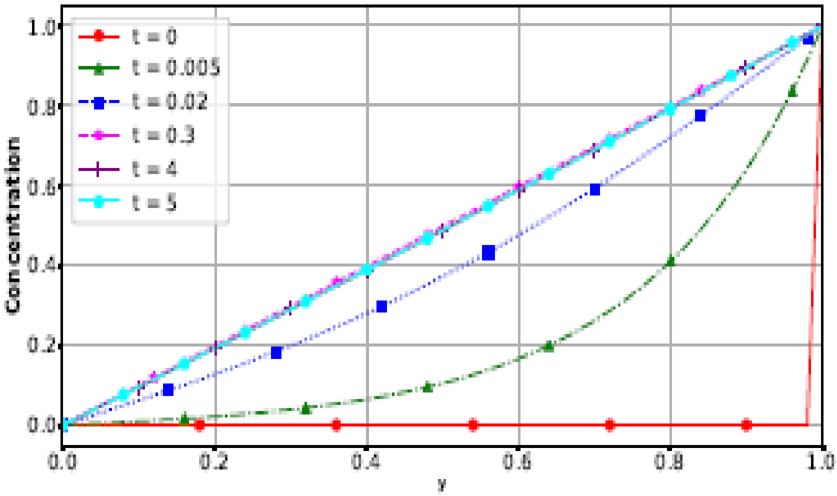


Figure 2: Time Evolution of Concentration Distribution

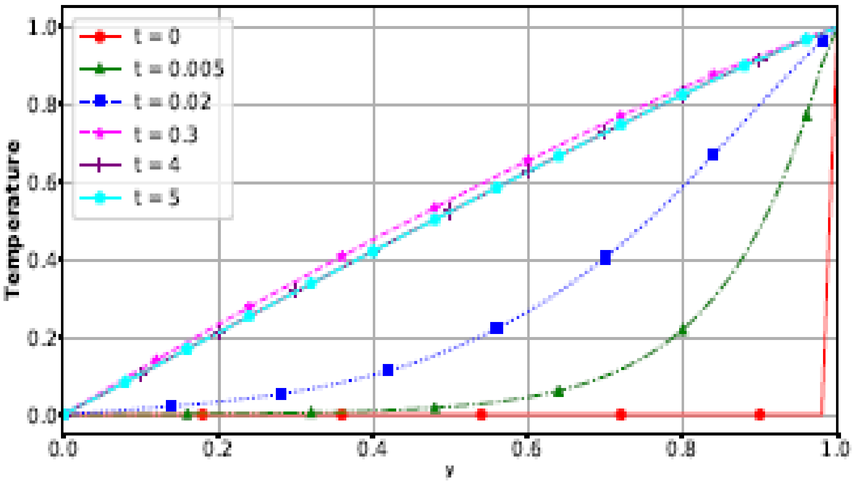


Figure 3: Time Evolution of Temperature Distribution.

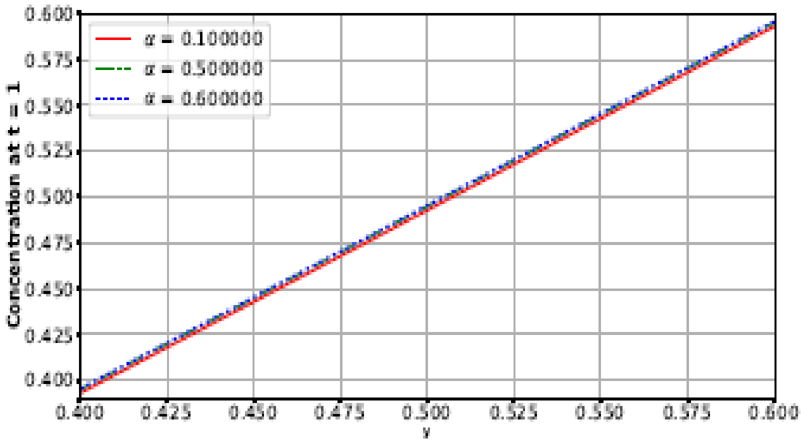


Figure 4: Effect of Viscosity Parameter α on the pollutant Concentration

Figure 4 indicate the influence of viscosity variation on the pollutant concentration which is affected by the type of concentration causing resistance to the flow but has a slight increase in the fluid velocity. The viscosity increases exponentially and reduces the temperature of the moving plate.

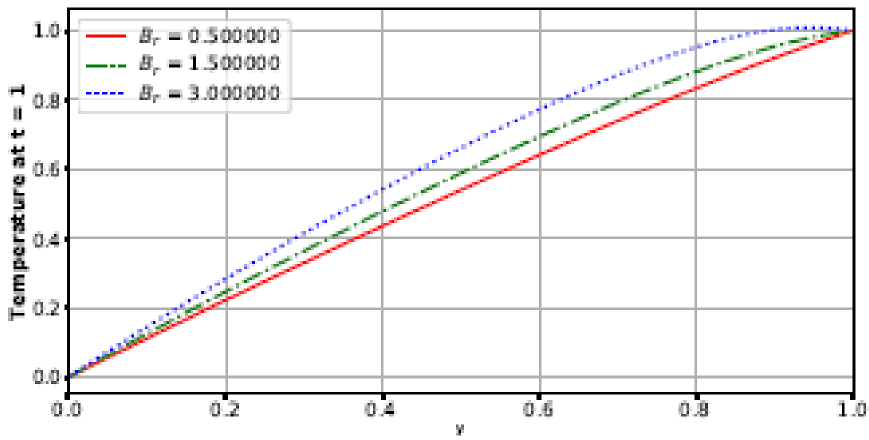
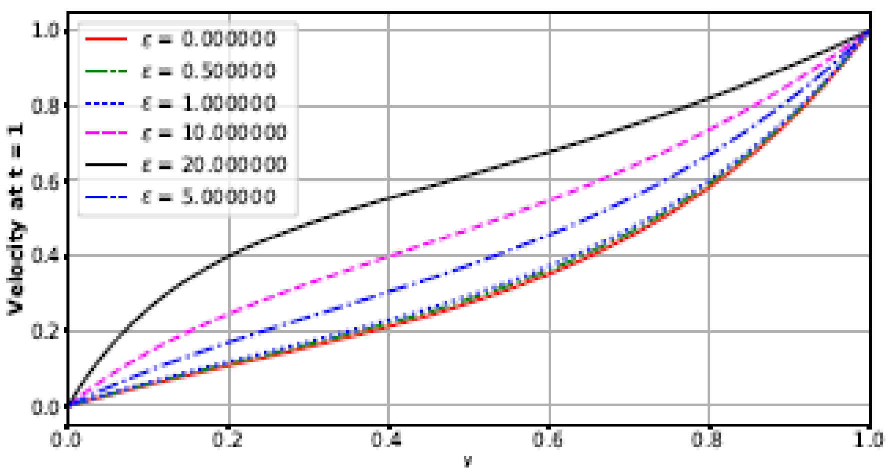


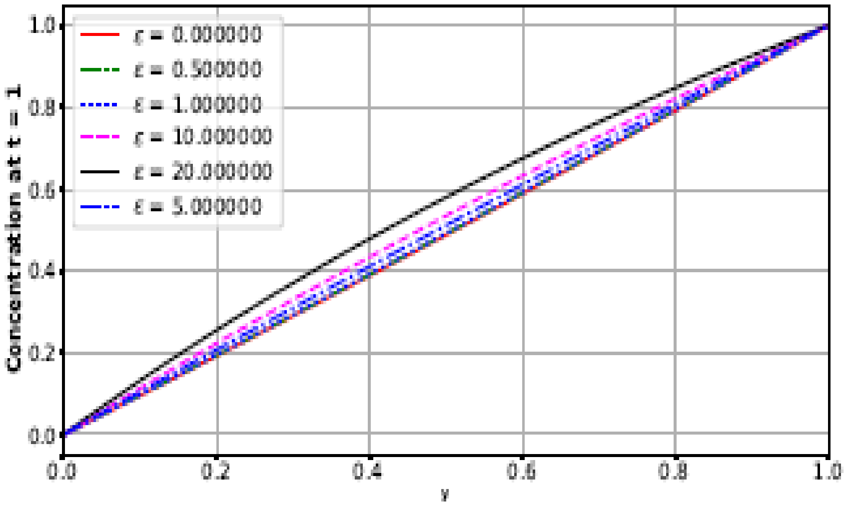
Figure 5: Effect of Brinkman number B_r on the Fluid Temperature.

It can be seen that the fluid temperature in figure 5 increases with increasing value of Brinkman number due to the heat generated by viscous dissipation. The temperature increase causes an increase in buoyancy force on the moving fluid.

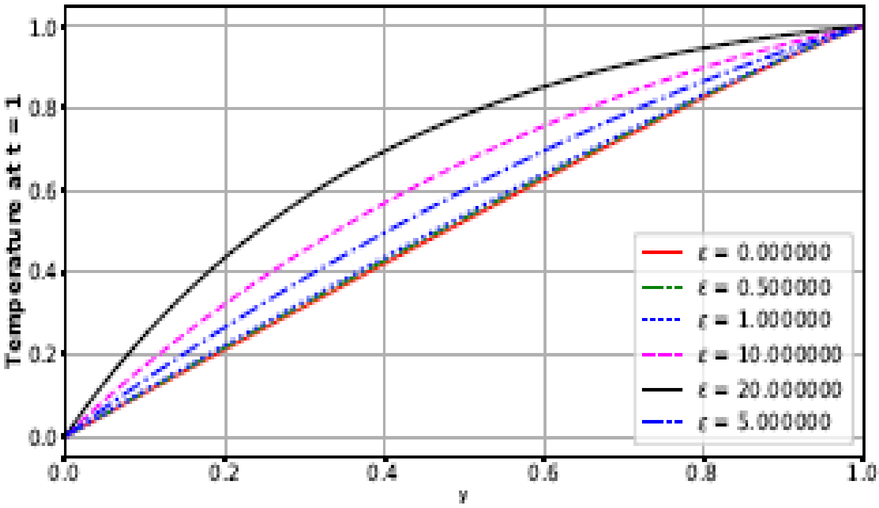


(a) Velocity Profile.

Figure (6a) show the effect of suction parameter on velocity profile. It's observed that the velocity increases with the increase in suction parameter ε. The increase in suction parameter value leads to slight increase on the concentration profile as shown in figure (6b) and the increase in Schmidt number causes decrease in suction parameter. While figure (6c) display the impact of suction parameter ε on the temperature profile, showing that the temperature of the plate increases with increasing suction ε in the flow. It's seen that the increase in suction enhances the velocity, concentration and temperature profiles respectively and hence, cause a drop on the flow.

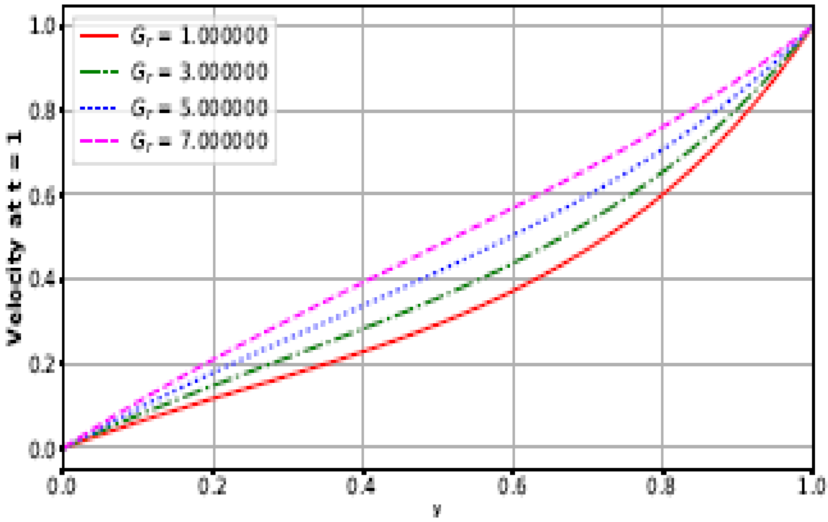


(b) Concentration Profile.

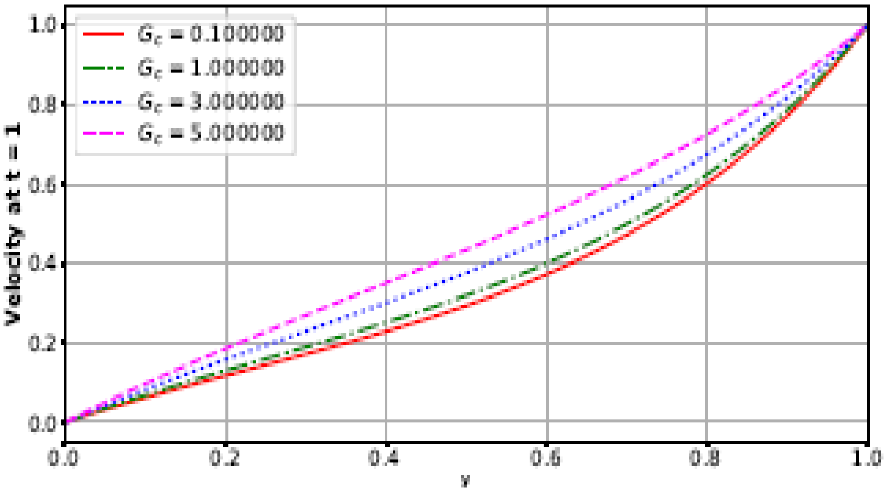


(c) Temperature Profile.

Figure 6: Effect of Suction Parameter εon the Flow Fields.



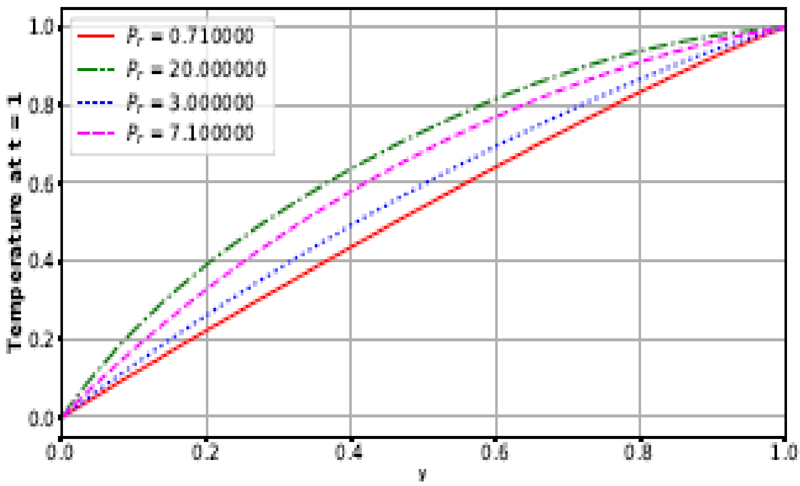
(a) Effect of G_r on the velocity.



(b) Effect of G_c on the Velocity.

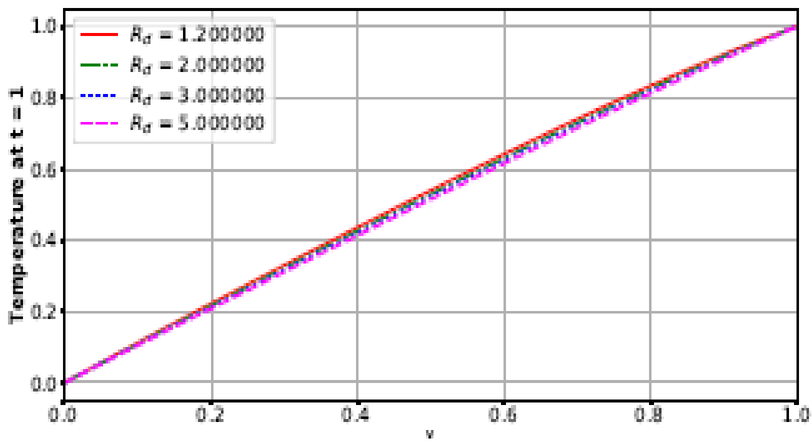
Figure 7: Effect of G_r and G_c on the Velocity.

The velocity increases with the increase in Thermal Grashof number at t=1 in Figure (7a) and the increase in Mass Grashof number leads to increase in velocity of flow as shown in Figure (7b), the Thermal Grashof number and the Mass number do not appear on temperature and concentration fields, hence have no effect on the fields. The increase in viscous force decreases the Grashof number and decrease with the decreasing of buoyant force.



(a) Effect of Pr on the Temperature Field.

Figure (8a) show that the temperature increases with the increasing value of Prandtl number on temperature fields due to the viscosity and thermal conductivity on the plates of moving couette flow. In Figure (8b), it can be seen that the increasing value of the Thermal radiation parameter lead to a slight increase in the temperature with the influence of specific heat capacity.



(b) Effect of Rd on the Temperature Field.

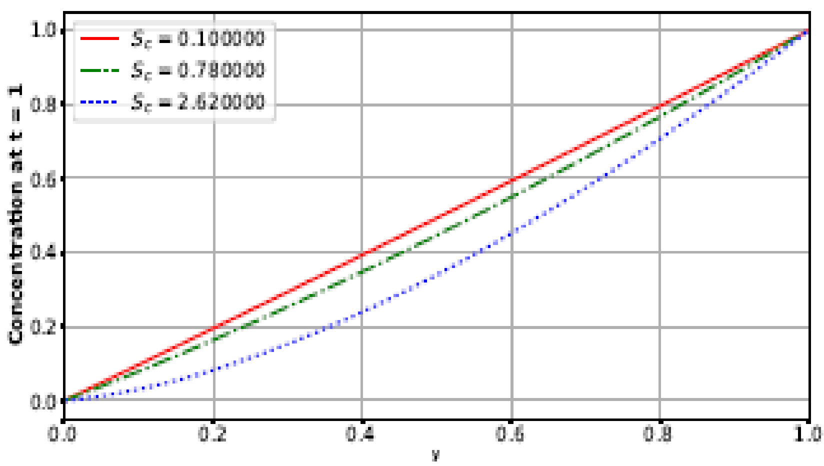
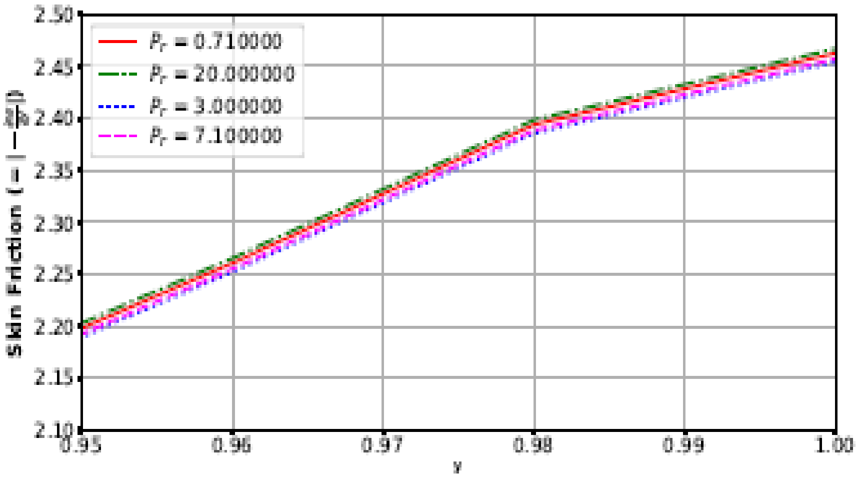


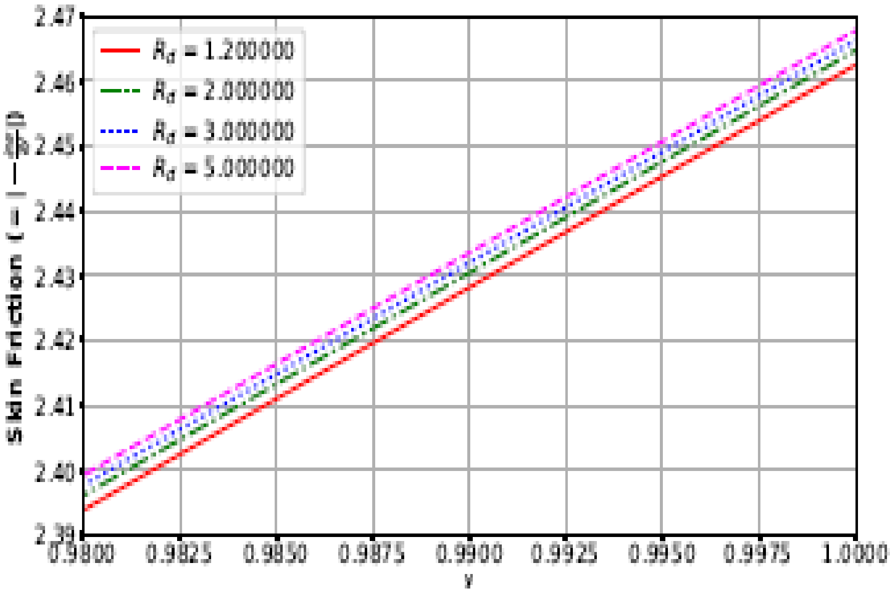
Figure 9: Effect of Schmidt Number Scon the Pollutant Concentration.

Figure 9 show that the pollutant concentration increases with the increasing value of Schmidt number due the impact of diffusion coefficient and the increase in density reduces the concentration.

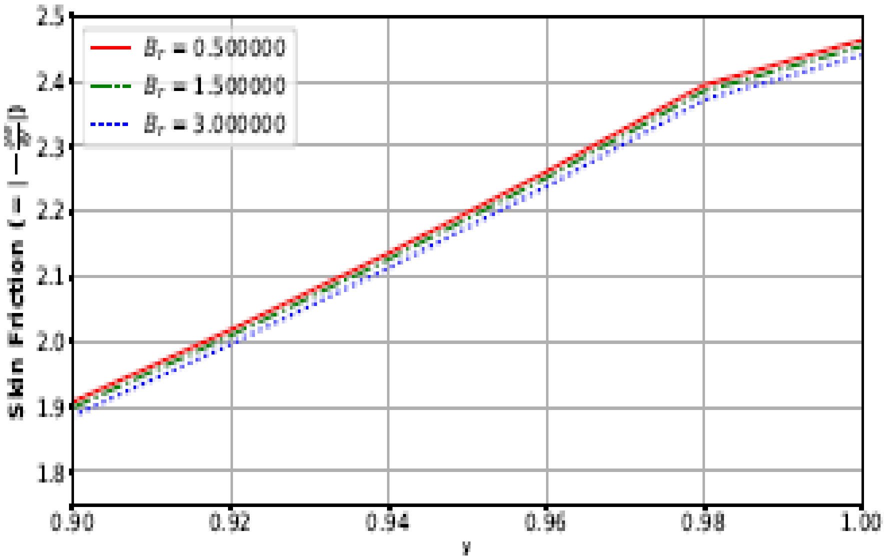


(a) Effect of Prandtl Number on the Skin Friction.

The Skin Friction in Figure (10a) increases with the increase in Prandtl number and have a slight drop in the flow. This is due to the frictional force exerted on the velocity field. Figure (10b) is indicating the influence of Skin Friction on different variation number of thermal radiation. It can be seen that the increase in thermal radiation enhances the Skin Friction and decreases with the increasing value of Brinkman number in the moving fluid as shown in figure (10c).



(b) Effect of Radiation on the Skin Friction.



(c)Effect of Brinkman Number on the Skin Friction.

V. Conclusion

The significant findings of this study are as follows;

1. The increase in time evolution increase the Velocity field, pollutant Concentration and Temperature field of the flow.
2. The increase in Brinkman number, Suction Parameter and Prandtl number increase the Temperature distribution of the flow field.
3. The increase in Thermal Grashof number, Mass Grashof number and Suction parameter increase the velocity of the flow system.
4. The increase in Radiation parameter increases the Skin Friction while the increase in Brinkman number reduces the Skin Friction.

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