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Optimal Pricing and Ordering Policy for Non-Instantaneous Deteriorating Items with Price and Stock Dependent Demand and Partial Backlogging

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Abstract: *In this paper we deals non-instantaneous deteriorating items with multivariable demand which depend on selling price and available stock level. The rate of deterioration is constant which start after a certain time because items are non-instantaneous. Shortages are permitted and partially backlogged with fixed rate. The aim to develop this model is to find the optimal ordering quantity and optimal selling price. To illustrate the proposed model, a numerical example is carried out.*
Keywords: *Inventory, Deterioration, Multivariable demand, Partial Backlogging*

I. Introduction

In this existing modern literature, numerous researcher giving their work by formulating different inventory models for deteriorating items by using different parameters to influence the model. Every existing product can't last life with the same utility value, that's why deterioration is the most important parameter for any inventory model. As we all are aware of the fact the many of items doesn't vanish immediately, usually there is a certain time period of this process of deterioration. By keeping this fact in mind, in the modern environment, non-instantaneous deterioration is taken as a key parameter for the research. Deterioration concept was first given by Ghare and Schrader (1963) in their study. Many researchers like Covert and Philip (1973), Dave and Patel (1981), Goyal & Giri (2001), etc. contribute significantly in this direction. Later, the non-instantaneous deterioration rate becomes the choice of researchers. Wu etal (2006) defined their inventory model for non-instantaneous deteriorating items. Dye (2013) used preservation technology on a non-instantaneous deteriorating stock model. Vipin et al. (2013) developed a two-warehouse inventory model for deteriorating items with ramp type demand. Tayal et al. (2014) illustrated an inventory model for deteriorating items with seasonal products. Jaggi et al. (2015) discussed credit financing in economic ordering policies for non-instantaneous decaying goods with price dependent demand under permissible delay in payments. Sanjay Sharma (2015) A Generalized EOQ Model for Time Dependent Deteriorating Items under Inflation .Vipin Kumar et al. (2015) developed a deterministicinventory model for Weibull deteriorating items with selling price dependent demand. Anupama Sharma et al. (2018) discussed an optimal inventory policy for deteriorating items with stock level and selling price dependent demand under trade credit. As in many situations, there can a possibility of stock out for the supplier, in such condition the customer can wait for the consignment to refill or can move to some other supplier for the same. In case the customer can wait for some time for their items to get delivered is termed as backlogging as it always for a span of time so it is called partial backlogging. In many inventory systems, the waiting time for the next replenishment would determine whether the backlogging would be accepted or not. Therefore, the normally backlogging rate is kept as variable and dependent on the time of waiting for the next replenishment. Wee (1993) gave economic production lot size model for deteriorating items with partial back-ordering. Teng (2004), Chaudhary (2010) gave their economic quality model with the concept of partial backlogging. Abad(1996) defined Optimal pricing and lot-sizing under conditions of perishability and partial back-ordering. Different optimal pricing and ordering policies were considered by many researchers in order to maximize the profit. Sheen (2008) developed dynamic pricing, promotion, and replenishment policies for deteriorating items with trade credit facilities. Later, Wu et. al (2009) and Goyal (2010) developed their model for Optimal Pricing and Ordering Policy for Non-Instantaneous Deteriorating Items.

In this paper we deals non-instantaneous deteriorating items with multivariable demand which depend on selling price and available stock level. The rate of deterioration is constant which start after a certain time because items are non-instantaneous. Shortages are permitted and partially backlogged with fixed rate. The aim to develop this model is to find the optimal ordering quantity and optimal selling price. To illustrate the proposed model, a numerical example is carried out.

The remaining part of the paper is organized after introduction part as follows: "Assumptions and Notations"section presents the assumptions and notations for the development of the model. "Mathematical modeling" section describes the mathematical formulation of themodel. Cost analysis and solution procedure is provided in IV and V section respectively. To illustrate the Developedmodel, a numerical example "Numerical Example" Finally, in "Conclusion" section concluding remarks and the future direction ofresearch are presented.

II. Assumptions and Notations

Assumptions

The following assumptions are used throughout the modelling of the problem:

1. The time horizon is infinite.
2. The replenishment rate is infinite.
3. $D\left(p,t,I(t)\right) = \dfrac{a+\alpha I(t)}{p^{\beta}}$ is the demand rate in no shortage phase,

   $D\left(p,t\right) = \dfrac{a}{p^{\beta}}$ is the demand rate in shortage phase, where ($a > 0$, $\alpha > 0$, $\beta > 0$)
4. The shortages are partially backordered with constant rate.
5. The rate of deterioration is constant per unit time.
6. There is no repair or replacement of the deteriorated inventory.

Notations

A : The ordering cost

C_1 Deteriorating cost per unit

C_2 : The holding cost per unit time per unit item

C_3 : The backorder cost per unit per unit time

C_4 : The cost of lost sales per unit

C_5 : The cost of purchasing per unit

p : selling price per unit

t_d : The length of the time in which the product exhibits no deterioration

t_1 : The length of the time in which there is no inventory shortage p>c

T : The duration of the replenishment cycle

θ : The parameter of deterioration rate of the stock

Q :The optimal ordering quantity

S : The maximum inventory level

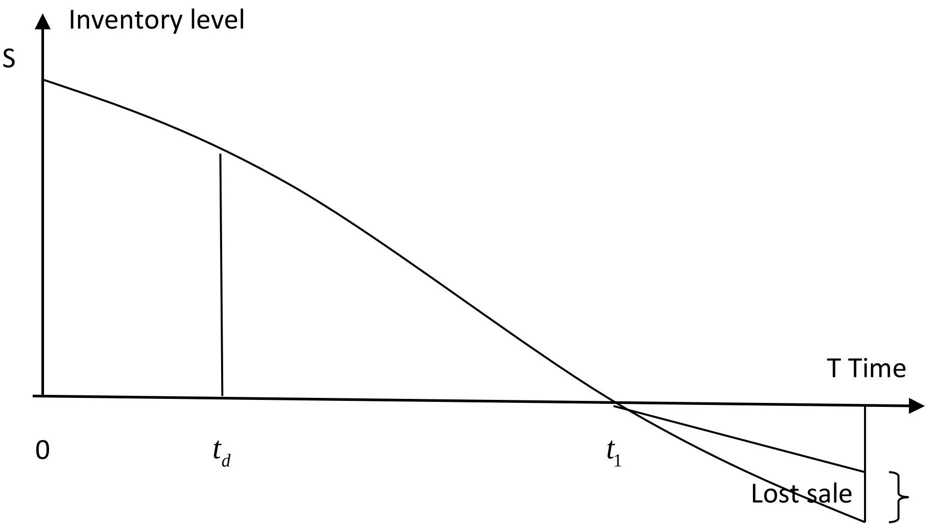
Q_2 : The maximum amount of demand backlogged

$I_1(t)$: The inventory level at time $t \in [0, t_d]$

$I_2(t)$: The inventory level at time $t \in [t_d, t_1]$

$I_3(t)$: The inventory level at time $t \in [t_1, T]$

III. Mathematical Modeling



During the time interval $\left[0,t_d\right]$, the differential equation representing the inventory status is given by

$$\frac{dI_1(t)}{dt}=-\frac{a+\alpha I_1(t)}{p^{\beta}} \qquad 0\le t\le t_d \quad (1)$$

With the initial condition $I(0)=S$, the solution of (1) is

$$I_1(t)=\frac{a}{\alpha}\left(e^{-p^{-\beta}t\alpha}-1\right)+Se^{-p^{-\beta}t\alpha} \quad 0\le t\le t_d \ (2)$$

In the second interval $\left[t_1,t_d\right]$, the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:

$$\frac{dI_2(t)}{dt}=-\theta I_2(t)-\frac{a+\alpha I_2(t)}{p^{\beta}} \qquad t_d\le t\le t_1 \quad (3)$$

With the condition $I_2(t_1)=0$, (3) yields

$$I_2(t)=\frac{a}{\alpha+p^{\beta}\theta}\left(e^{-p^{-\beta}(t-t_1)\left(\alpha+p^{\beta}\theta\right)}-1\right) \ t_d\le t\le t_1 \ (4)$$

During the third interval $\left[t_1,T\right]$, shortage occurred and the demand is partially backlogged according to the fraction β(T−t). That is, the inventory level at time t is governed by the following differential equation:

$$\frac{dI_3(t)}{dt}=-k\frac{a}{p^{\beta}} \qquad t_1\le t\le T \quad (5)$$

With boundary conditions $I_3(t_1)=0$,(5) yields

$$I_3(t)=-akp^{-\beta}(t-t_1) \ t_1\le t\le T \quad (6)$$

It is clear from Fig. 1 that $I_1(t_d)=I_2(t_d)$; therefore the maximum inventory level I0 can be obtained:

$$S=e^{p^{-\beta}t_d\alpha}\left(\frac{a}{\alpha}\left(1-e^{-p^{-\beta}t_d\alpha}\right)-\frac{a}{\alpha+p^{\beta}\theta}\left(e^{-p^{-\beta}(t_d-t_1)\left(\alpha+p^{\beta}\theta\right)}-1\right)\right) \quad (7)$$

Substituting (7) in into (2), we get

$$I_1(t)=\frac{a}{\alpha}\left(e^{-p^{-\beta}t\alpha}-1\right)+e^{p^{-\beta}(t_d-t)\alpha}\left(\frac{a}{\alpha}\left(1-e^{-p^{-\beta}t_d\alpha}\right)-\frac{a}{\alpha+p^{\beta}\theta}\left(e^{-p^{-\beta}(t_d-t_1)\left(\alpha+p^{\beta}\theta\right)}-1\right)\right) (8)$$

If we put t=T into (6), the maximum amount of demand backlogging will be obtained as follows:

$$Q_2 = -I_3(T) = akp^{-\beta}(T - t_1) \quad (9)$$

The order quantity per cycle (Q) is the total of S and Q , i.e.

$$Q = S + Q_2$$

$$Q = akp^{-\beta}(T - t_1) - a\mathrm{e}^{p^{-\beta}t_d\alpha}\left(\frac{\mathrm{e}^{-p^{-\beta}t_d\alpha}}{\alpha} - \frac{1}{\alpha} + \frac{1}{\alpha + p^{\beta}\theta}\left(\mathrm{e}^{-p^{-\beta}(t_d - t_1)(\alpha + p^{\beta}\theta)} - 1\right)\right) \quad (10)$$

Now, we can obtain inventory costs and sales revenue per cycle, which consists of the following elements:

IV. Cost Analysis

(a) **Ordering Cost**

$$OC = A \quad (11)$$

(b) Deteriorating cost

$$DC = C_1\int_{t_d}^{t_1}\theta I_2(t)\,dt$$

$$DC = C_1\frac{ap^{\beta}\theta}{(\alpha + p^{\beta}\theta)^2}\left(\mathrm{e}^{-p^{-\beta}(t_d - t_1)(\alpha + p^{\beta}\theta)} - 1\right) \quad (12)$$

(c) The Inventory Holding Cost

$$HC = C_2\left[\int_0^{t_d} I_1(t)\,dt + \int_{t_d}^{t_1} I_2(t)\,dt\right]$$

$$HC = C_2\frac{a}{(\alpha + p^{\beta}\theta)^2}\left(p^{\beta}\left(2\mathrm{e}^{-p^{-\beta}(t_d - t_1)(\alpha + p^{\beta}\theta)} - \mathrm{e}^{t_1(p^{-\beta}\alpha + \theta)} - 1 + \theta(2t_d - t_1)\right) + (2t_d - t_1)\alpha\right) \quad (13)$$

(d) Shortage Cost

$$SC = C_3\int_{t_1}^{T}\left[-I_1(t)\right]dt$$

$$SC = \frac{1}{2}C_3akp^{-\beta}(T - t_1)^2 \quad (14)$$

(e) The Opportunity Cost Due to Lost Sales

$$LSC = C_4\int_{t_1}^{T} D(p,t)(1-k)\,dt$$

$$LSC = C_4a(1-k)p^{-\beta}(T - t_1) \quad (15)$$

(f) The Purchasing Cost

$$PC = C_5 Q$$

$$PC = C_5\left\{akp^{-\beta}(T - t_1) - a\mathrm{e}^{p^{-\beta}t_d\alpha}\left(\frac{\mathrm{e}^{-p^{-\beta}t_d\alpha}}{\alpha} - \frac{1}{\alpha} + \frac{1}{\alpha + p^{\beta}\theta}\left(-1 + \mathrm{e}^{-p^{-\beta}(t_d - t_1)(\alpha + p^{\beta}\theta)}\right)\right)\right\} \quad (16)$$

Total average cost of the system is

$$TAC = \frac{1}{T}\left[OC + DC + HC + SC + LSC + PC\right] \quad (17)$$

Next, we drive the objectives function for the above inventory system. The problem could be formulated as follows,

Min $TAC(p, t_1)$

Subject to $p > C_5,\ 0 < t_1 < T$ (18)

According to Eq. (17), if p and t_1 are the real numbers, then the total average cost is a function of thetwo variables p and t_1

V. Solution Procedure

Since the T.A.C. is the function of two variables p and t_1 with the constraint $p > C_5$, and $0 < t_1 < T$.
Accordingly, we could deduce the optimal solution as to the following:

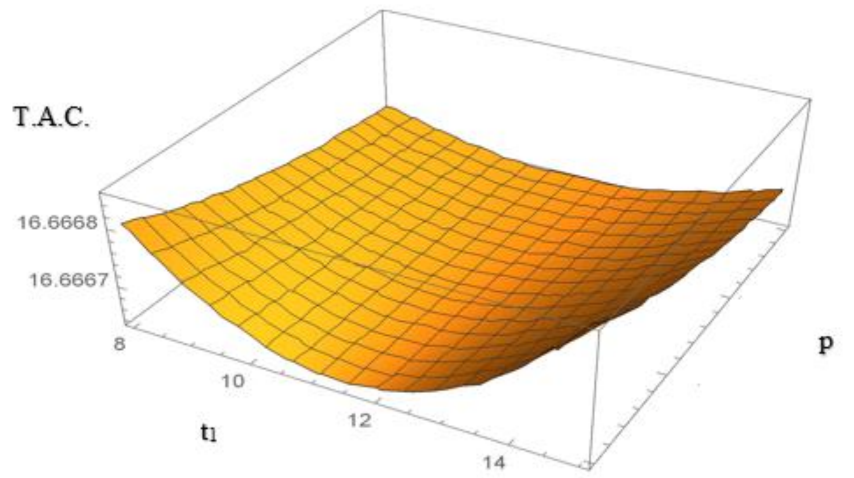
$$\frac{\partial TAC(p,t_1)}{\partial p} \quad (19)$$

$$\frac{\partial TAC(p,t_1)}{\partial t_1} \quad (20)$$

VI. Numerical Example

In practice, the related parameters can be used for historical transaction data. The inputs values are $A \to 250$, $a \to 25$, $\alpha \to 0.02$, $\beta \to 2.5$, $C_1 \to 18$, $C_2 \to 25$ Rs. /unit, $C_3 \to 25$ Rs. /unit, $C_4 \to 25$ Rs. /unit, $C_5 \to 20$ Rs. /unit, $k \to 0.8$, $t_d \to 3$ $\theta \to 0.08$, $T \to 15$ days

For these given parameters, the optimal value of p and t_1 are Rs. 32.1178 and 12.2325 days, respectively and corresponding to these the optimal value of T.A.C. and ordering quantity are Rs. 16.0203 and 107.9 units, respectively.



VII. Conclusion

In this paper, we developed an EOQ model for non-instantaneous deteriorating items with selling price and stock dependent demand which is deterministic to find out the optimal selling price and optimal ordering quantity. The rate of deterioration is fixed. The shortages are allowed, and it is assumed that the occurring shortages are partially backlogged. We considered here, has incorporated more realistic feature of inventory control. Numerical example and graph shown in the model have indicated that the model was practical and satisfactory. For future, the model can be extended in several ways like, permissible delay in payment, time dependent rate of deterioration, stochastic demand.

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