IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 6 Ser. III (Nov – Dec 2019), PP 09-25
www.iosrjournals.org

Impacts of Priority Parameters on the Traffic Performance at a Road Intersection

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Abstract: *In this paper, the evolution of traffic flow on the road intersection of a single lane three legs roundabout is analyzed from a macroscopic point of view following Lighthill–Whitham–Richards model. The single lane three legs roundabout is modeled as a sequence of 1×2 and 2×1 junctions. The priority parameter is introduced for 2×1 junctions to analyze the traffic evolution on the road network of the roundabout. Also, analyzed is the performance of the roundabout with and without priority parameter to evaluate the traffic evolution on the road network. Thereafter, the evolution of density and flux versus priority parameter at different time steps through numerical simulation using Godunov scheme is illustrated.*

Keywords: *Traffic flow, Priority Parameter, Roundabout, Traffic evolution, Numerical simulation.*

Date of Submission: 01-11-2019 Date of Acceptance: 16-11-2019

I. Introduction

Modern civilization would not be possible without extensive, reliable transportation systems. Technology is poised to transform transportation and impact society and the environment in ways we cannot fully predict but must be prepared to manage. Traffic flow models are used to describe and predict traffic on roads. Besides transports have a positive impact on economic development, they attract all the negative effects such as pollution of the environment, traffic congestion, accidents, etc. [19]. The study of the basic traffic flow characteristics like traffic density, speed and flux are the pre-requisites for the effective planning, design, operation and management of roadway systems at road intersections.

Macroscopic traffic models were introduced during the 1950s by Lighthill, Whitham [15] and independently Richards [17]. They were the first to propose a hydrodynamics model for traffic flow using a nonlinear scalar hyperbolic Partial Differential Equation (PDE). The PDE equipped with an initial data is commonly referred to as the LWR model. This model was later on extended to work on networks. In fact, over the years, several authors proposed models on networks that are able to describe the dynamics at intersections, see for example [8, 20, 21] and reference therein. Each of these models considers different types of solutions for different types of junctions, according to the different number of lanes, incoming and outgoing links. In this article, we focus on a Riemann problem for roundabouts. This roundabout can be seen as concatenation of 2×1 (merging) and 1×2 (diverging) junctions, but the approach can be generalized to a more general network. The density and flux evolutions on the entrance roads, exits roads and on the overall portion of roundabout are described by a scalar hyperbolic conservation law. Roundabouts are junctions with a one-way circulatory carriageway around a central island. Vehicles on the circulatory carriageway have priority over those approaching the roundabout [11, 16]. At each junction, the Riemann problem is uniquely solved using right-of-way and traffic distribution parameters.

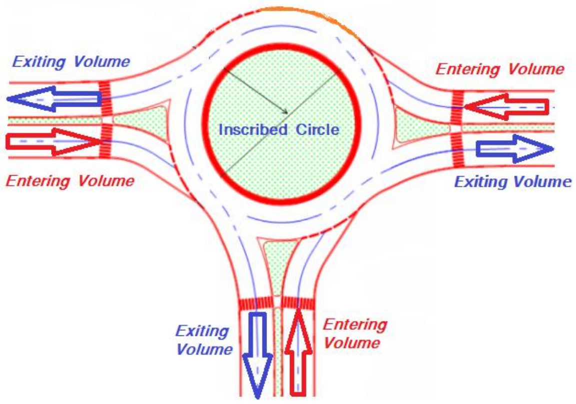
The goal of this paper is to analyze the performance of the roundabout through numerical simulation on its road networks. The fundamental reason for using simulation technique in traffic flow studies is that traffic flow is a highly complex phenomenon and is difficult to understand and analyze by simple mathematical techniques. A simulation of model is the imitation of the operation of a real world process or system over time. It facilitates the system planner to study and evaluate the performance of transport-network systems at various possible operating conditions [22].

The article is organized as follows: In Section 2 we introduce formally the mathematical model by describing the network and the mathematical description of the traffic evolution on each link and on each junction. In Section 3 we introduce the Riemann Solver at junctions. We first introduce some necessary notations and we describe step by step the construction of the Riemann Solver for the different types of junctions. In Section 4 we describe the numerical scheme used to find the numerical solution of the problem on the road network and illustrate the results using the Godunov scheme. Lastly, in Section 5 we give some conclusions about this article.

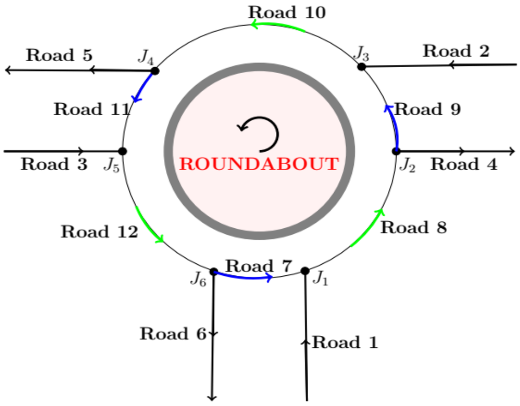
II. Mathematical Model

Roundabouts are special forms of road network having short links and connected to external incoming and outgoing roads. It is a self-controlled traffic flow regulating devices. A roundabout in its design may have three, four, five or more incoming and outgoing flow directions on which traffic can flow. These flow directions are commonly called legs or arms of the roundabout. A roundabout may be a single-lane, double-lane, three-lane or multi-lane. In this work three legs single-lane roundabout have been considered. In a single-lane roundabout only a single vehicle can enter to the roundabout at a time; others wait at the yield line one after the other.

Traffic evolutions on the roundabout differ from other conventional road networks due to priorities given for traffic circulating on the main road of the roundabout. Due to this fact, it is considered as an alternative traffic control device that can improve safety and operational efficiency at intersection when compared to other conventional intersection control. In this work we deal with a mathematical point of view to describe the details of the traffic road network illustrated in Figure 1(a), more clearly with links and junction types as shown in Figure 1(b).



(a)



(b)

Figure 1: (a) Represents the 3-incoming and 3-outgoing roads roundabout modeled in the paper and (b) represents different links and junction types of the figure a).

The road network of the Roundabout shown in Figure 1 (b) consists of 12 roads in all. Of them Roads 1, 2, and 3 are called Entrance roads, Roads 4, 5, and 6 are called Exit roads while the remaining Roads 7, 8, 9, 10, 11, and 12 are called Main roads. Also, it consists of 6 Junctions and they are denoted respectively by $J_1, J_2, J_3, J_4, J_5, J_6$. Of these junctions J_1, J_3, J_5 are called Merging Junctions while the remaining J_2, J_4, J_6 are called Diverging Junctions.

On each road we consider the LWR model for traffic and at junctions we consider boundary condition with Riemann solver satisfying the conservation of cars [8]. When there is more traffic demand on the incoming roads we introduce a right of way parameter that describes how many cars can drive through the junction from the incoming roads. The evolution of the traffic on the networks of a roundabout is governed by

$$[\partial \rho_i/\partial t] + [\partial f_i(\rho_i)/\partial x] = 0, \ (t, \ x) \in R^+ \times Road\ i, \quad i = 1, 2, \ldots 12 \qquad (2.1)$$

Here $\rho_i = \rho_i(t, \ x) \in [0, \ \rho_{max,i}]$ is the mean traffic density and $\rho_{max,i}$ the maximal density on each single road. Pipes-Munjal was an early researcher, who proposed the speed-density relationship in [18] and expressed in terms of an n[th] degree polynomial

$$v_i(\rho) = v_{max,i} \left[1 - \left(\rho_i/\rho_{max,i}\right)\right]^n, \quad n \geq 1 \qquad (2.2)$$

Here $v_{max,i}$ is the maximal speed on each link and $v_i: [0, \ \rho_{max,i}] \to \mathbb{R}^+$ is a smoothdecreasing function denoting the mean traffic speed. The flux functions $f_i: [0, \ \rho_{max,i}] \to \mathbb{R}^+$ defined by

$$f_i(\rho_i) = \rho_i\ v(\rho_i)$$

$$f_i(\rho_i) = v_{max,i}\ \rho_i\left[1 - \left(\rho_i/\rho_{max,i}\right)\right], \quad for\ n = 1 \qquad (2.3)$$

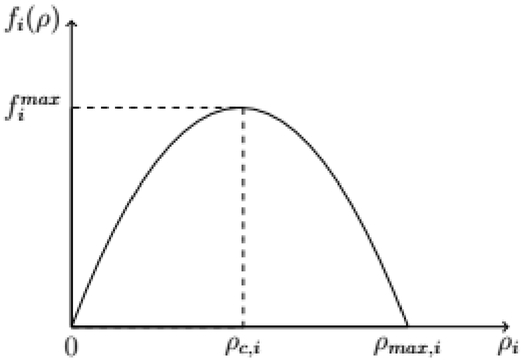


Figure 2: Fundamental diagram considered.

This fundamental diagram is illustrated in Figure 3. In this paper we use the normalized form of the vehicle density $\rho(t, \quad x)$ to be $0 \leq \rho \leq 1$ and we assume the following:

(i) $\rho_{max,i} = 1$
(ii) The speed v_i depends only on the density ρ_i
(iii) The flux f_i is a strictly concave C^2 function
(iv) $f_i(0) = f_i(1) = 0$

Assumptions (iii) and (iv) give that f_i has a unique point of maximum $\rho_{c,i} \in (0, \quad 1)$.

In the roundabout that we are modeling, there are 2 types of junctions: merge junction (2 incoming and 1 outgoing roads) and diverge junction (1 incoming and 2 outgoing links) see Figure 1 for the different locations of the junctions and Figure 3 for a more detailed representation of the different types of junctions used in this study.

(a) Merge junction (b) Diverge junction

Figure 3 Different types of junctions modeled

Definition 2.2: Let $f_i: [0, \quad 1] \to \mathbb{R}$ be a continuous, strictly concave function such that $f_i(0) = f_i(1) = 0$.Then there exist $\rho_{c,i} \in (0, \quad 1)$ such that f_i is smooth on $[0, \quad \rho_{c,i}]$ and $(\rho_{c,i}, \quad 1]$ with $0 < |f^{'}(x_i)| < \infty$ for each $x_i \in [0, \quad \rho_{c,i}) \cup (\rho_{c,i}, \quad 1]$.

Definition 2.3: Let $\tau : \quad [0, \quad 1] \to [0, \quad 1]$ be the map such that $f(\tau(\rho)) = f(\rho)$ for every $\rho \in [0, \quad 1]$ and $\tau(\rho) \neq \rho$ for every $\rho \in [0, \quad 1] \setminus \{\rho_c\}$. (For further properties see [8, 13])

III. Riemann problems at the junction

In this section we describe the construction of the Riemann solver at a junction. Let us first set some notations. In the following of the paper the subscripts $_{inc}$ indicates that quantities belonging to the incoming links on a junction while $_{out}$ indicates the outgoing ones.

Definition 3.1: Let us define that following quantities

1. For every $l \in \{inc\}$ define $$\gamma_{inc}^{max}(\rho_i) = \begin{Bmatrix} f(\rho_l) & if\ 0 \leq \rho_l \leq \rho_{cr} \\ f_{max} & if\ \rho_{cr} \leq \rho_l \leq \rho_l^{max} \end{Bmatrix}$$

2. For $j \in \{out\}$ define $$\gamma_{out}^{max}(\rho_j) = \begin{Bmatrix} f_{max} & if\ 0 \leq \rho_j \leq \rho_{cr} \\ f(\rho_j) & if\ \rho_{cr} \leq \rho_j \leq \rho_j^{max} \end{Bmatrix}$$

Moreover, let us fix a matrix A belonging to the set of matrices:

$$\mathcal{A} := \left\{ A = a_{1,j} \quad j \in \{out\} : 0 \le a_{1,j} \le 1, \sum_{j \in \{out\}} a_{1,j} = 1 \right\}$$

And a priority vector $p = (p_1, \; p_2) \in \mathbb{R}^2$ with $p_l > 0$, $\sum_{l=1}^{2} p_l = 1$ indicating priorities among incoming roads. Moreover, we define a function τ as follows. For further properties see [8] and [13].

Definition 3.2: Let $\tau : [0, \; 1] \to [0, \; 1]$ be the map such that

(i) $f(\tau(\rho)) = f(\rho)$ for every $\rho \in [0, \; \rho^{max}]$

(ii) $\tau(\rho) \neq \rho$ for every $\rho \in [0, \; 1] \setminus \{\rho_{cr}\}$

We are now ready to describe the construction of the Riemann Solver for different types of junctions. Fix $\rho_{1,0}, \; \rho_{2,0}, \; \cdots \; \rho_{12,0} \in [0, \; \rho_i^{max}]$. Consider a Riemann problem at a junction J_i

$$\partial_t \rho_i + \partial_x f(\rho_i) = 0, \;\; \rho_i(0,.) = \rho_{i,0} \quad i \in 1, \ldots, 12.$$

A solution to the Riemann problem at J_i is defined as follows:

3.1 Merge junctions: Let us consider first a merging junction, i.e. a junction with two incoming and one outgoing road, see Figure 3, left. Let us fix constants $\rho_{1,0}, \; \rho_{2,0}, \; \rho_{3,0} \in [0, \; \rho_i^{max}]$ for $i = 1, \; 2, \; 3$, and a priority parameter p. The Riemann solver $\mathcal{RS}(\rho_{1,0}, \; \rho_{2,0}, \; \rho_{3,0}) = (\hat{\rho}_1, \; \hat{\rho}_2, \; \hat{\rho}_3)$ at the junction is constructed in the following way.

1. Compute:

$$\gamma_1^{max} = \gamma_{inc}^{max}(\rho_1)$$
$$\gamma_2^{max} = \gamma_{inc}^{max}(\rho_2)$$
$$\gamma_3^{max} = \gamma_{out}^{max}(\rho_3)$$

2. Fix:

$$\hat{\gamma}_3 = \min(\gamma_1^{max} + \gamma_2^{max}, \quad \gamma_1^{max})$$
$$\hat{\gamma}_1 = \min(\gamma_1^{max}, \quad \max(\hat{\gamma}_3 - \gamma_2^{max}, \quad p\hat{\gamma}_3))$$
$$\hat{\gamma}_2 = \hat{\gamma}_3 - \hat{\gamma}_1$$

3. Set $\hat{\gamma}_{inc} = (\hat{\gamma}_1, \; \hat{\gamma}_2)$ and $\hat{\gamma}_{out} = (\hat{\gamma}_3)$

3.2 Diverging junctions: We consider a diverging junction, i.e. a junction with once income and two outgoing links, see Figure 3, center. Let us fix constants $\rho_{1,0}, \; \rho_{2,0}, \; \rho_{3,0} \in [0, \; \rho_i^{max}]$ for $i = 1, \; 2, \; 3$, and a distribution matrix $A = [\alpha, \; 1 - \alpha]$. The Riemann solver $\mathcal{RS}(\rho_{1,0}, \; \rho_{2,0}, \; \rho_{3,0}) = (\hat{\rho}_1, \; \hat{\rho}_2, \; \hat{\rho}_3)$ at the junction is constructed in the following way.

1. Compute:

$$\gamma_1^{max} = \gamma_{inc}^{max}(\rho_1)$$
$$\gamma_2^{max} = \gamma_{out}^{max}(\rho_2)$$
$$\gamma_3^{max} = \gamma_{out}^{max}(\rho_3)$$

2. Then

$$\hat{\gamma}_1 = \min\left(\gamma_1^{max}, \quad \frac{\gamma_2^{max}}{\alpha}, \quad \frac{\gamma_1^{max}}{1 - \alpha}\right)$$
$$\hat{\gamma}_2 = \alpha\hat{\gamma}_1$$
$$\hat{\gamma}_3 = (1 - \alpha)\hat{\gamma}_1$$

3. Set $\hat{\gamma}_{inc} = (\hat{\gamma}_1)$ and $\hat{\gamma}_{out} = (\hat{\gamma}_2, \; \hat{\gamma}_3)$

IV. Numerical Simulations

In this section, we describe the numerical scheme used to solve problem (2.1), and simulation results for the roundabout represented by 2×1 and 1×2 junction type.

4.1 Numerical Scheme

We de ne a numerical grid in $(0, \; T) \times \mathbb{R}$ using the following notation: Δx is the fixed grid space, Δt is the time step given by the CFL condition and $(t^n, \; x_j) = (n\Delta t, \; j\Delta x)$ for $n \in \mathbb{N}$ and $j \in \mathbb{Z}$ are the grid points.

Each road is divided in $N + 1$ cells numbered from 0 to N. The first and last cells of an edge are always a junction and we assume that these cells are ghost cells. The scheme used for solving equation (2.1) is the Godunov scheme as introduced in [9, 13] and it is based on exact solutions to the Riemann problem. The main idea of this method is to approximate the initial datum by a piecewise constant function, then the corresponding Riemann problems are solved exactly and a global solution is simply obtained by piecing them together. Finally, one takes the mean on the cell and proceeds by induction. Under the CFL (Courant-Friedrichs-Lewy) condition, [5] it holds:

$$\Delta t \, max_{j \in Z} \left| \lambda^n_{j+\frac{1}{2}} \right| \leq \Delta x \quad (3.2)$$

Here in (3.2), $\lambda^n_{j+\frac{1}{2}}$is the speed of the wave of the Riemann problem solution at the interface $x_{j+\frac{1}{2}}$at the timet^n, the numerical scheme can be written as

$$\rho_j^{n+1} = \rho_j^n - \frac{\Delta t^n}{\Delta x}[F(\rho_j^n, \quad \rho_{j+1}^n) - F(\rho_{j-1}^n, \quad \rho_j^n)]$$

Where the numerical flux F for a concave flux function is given by

$$F(u, \quad v) = \begin{cases} \min f(u, \quad v), & if\ u \leq v \\ f(u), & if\ v < u < \rho_c \\ f^{max}, & if\ v < \rho_c < u \\ f(v), & if\ \rho_c < v < u \end{cases}$$

4.2 Simulation results

In this section, we analyze the results of the simulations of the model presented for road network. For illustration, we choose a concave fundamental diagram as introduced in equation (2.3) with the following values for the parameters$v_{max,i} = 1,\ \rho_{max,i} = 1,\ L = 50,\ \rho_{c,i} = 0.5,\ T = 20,\ \Delta x = 0.0196,\ \Delta t = 0.0196.$

For the initial condition on the roads of the network, we assume that at initial time $t = 0$ all the roads are empty and influx at boundary of incoming edges is equal to 0.9. In the case of low traffic, we do not need priority rule, the traffic evolution is only governed by conservation law and the splitting rate to describe how traffic coming from the incoming roads chooses to distributed to their corresponding intermediate (main) roads and the external exiting roads. Thus,I n this case rarefaction wave fill the portion of the roads of the roundabout. But, in the case of high traffic, congestion can occur at merging junctions and shock wave propagating back.

Hence, the performance of roundabout is reduced in controlling traffic flow problem. In order to show the different state of traffic evolution on the network, we assume that the roundabout with and without priority parameter at merging junctions for the simulation purpose.

The results obtained are shown in Figures * to **. As example we show the evolution of the density and flux on an entrance road, an exit road, a roads between merging and diverging junctions, and a roads between diverging and merging junctions versus space Discretization as shown in Figure ***. The other Figures **** shows evolution of traffic with priority parameter. In all of them we can see the evolution of the density and flux during the simulation time.

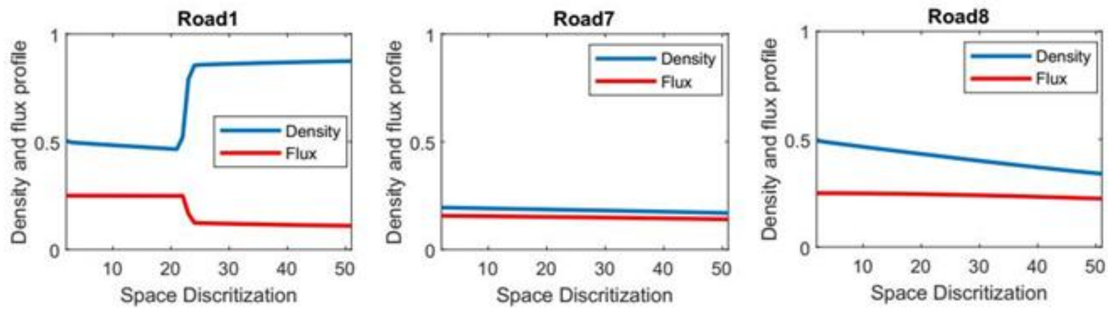


Figure 5: The merging junction at T = 6 without priority parameter

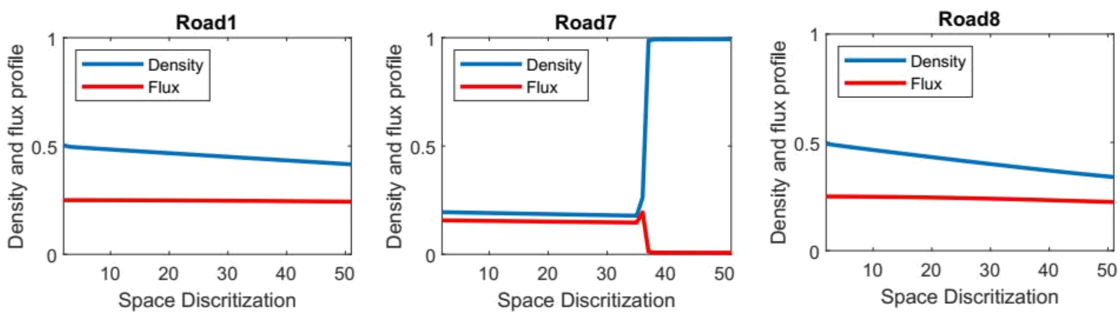


Figure 6: The merging junction at T = 6 with priority parameter

Here it is to be recalled that whenever traffic is low, priority is not required to introduce. However, if the traffic is high then introduction of priority is one way of resolving the problem of congestion and paving a way for smooth traffic flow. However, the Priority will not completely remove the problem of congestion but the congestion is shifted from important road to unimportant road. These facts have been illustrated in the Figures 5 and 6.

Figure 5 shows simulation study of traffic flow at the merging JunctionJ_1 . At this junction 1 and 7 are incoming roads and 8 is the outgoing road. Here priority is not considered. It can be observed that the traffic flow is smooth on Road 1 but congested on Road 7. However, the resultant traffic on Road 8 is also smooth. But, the congestion in traffic flow on Road 7is not desirable, as it is a part of roundabout and as a result this congestion will propagate back.

To avoid congestion on Road 7 priority is introduced to the traffic on this road and the results of the simulation study are presented in Figure 6. That is, more priority is given to the traffic on Road 7 than to that on Road 1. As a result the congestion on Road 7 could be reduced and the traffic on the whole roundabout is now smooth. However, congestion now takes place on Road 1 which does not do much damage for the overall traffic flow.

The traffic congestion on the main road or round about is more problematic than that on the external incoming roads. This problem is seen in Figure 5 and resolved in Figure 6.

The simulation study at the merging JunctionJ_1given in Figures 5 and 6 can be extended to the remaining two merging Junctions J_3andJ_5. However, the results of these simulation studies will just be similar to the present one and hence they are not presented here.

However, the Priority will not completely remove the problem of congestion but the congestion is shifted from important road to unimportant road. This fact is visualized in Figures 5 and 6. As the result of introduction of the concept of Priority the traffic congestion is shifted from Road 7 to Road 1.

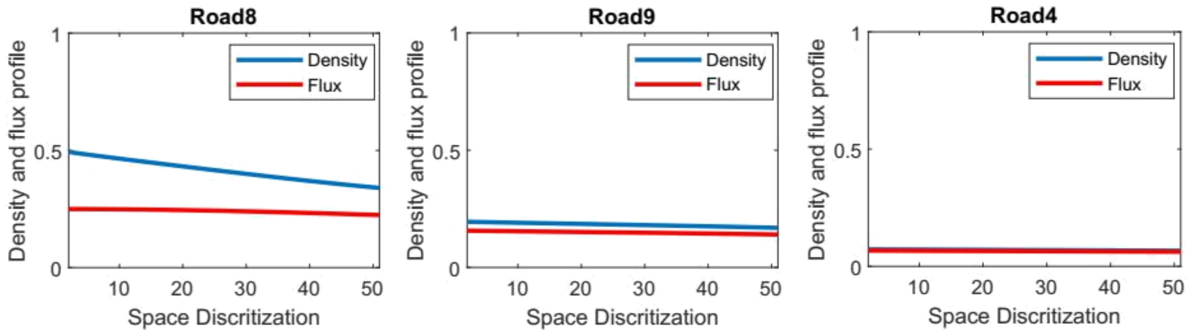


Figure 7: The diverging junction at T = 6 without priority parameter

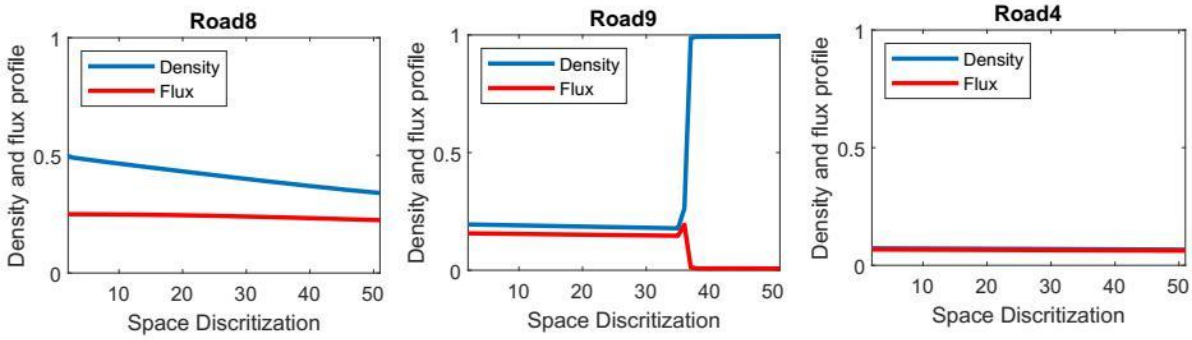
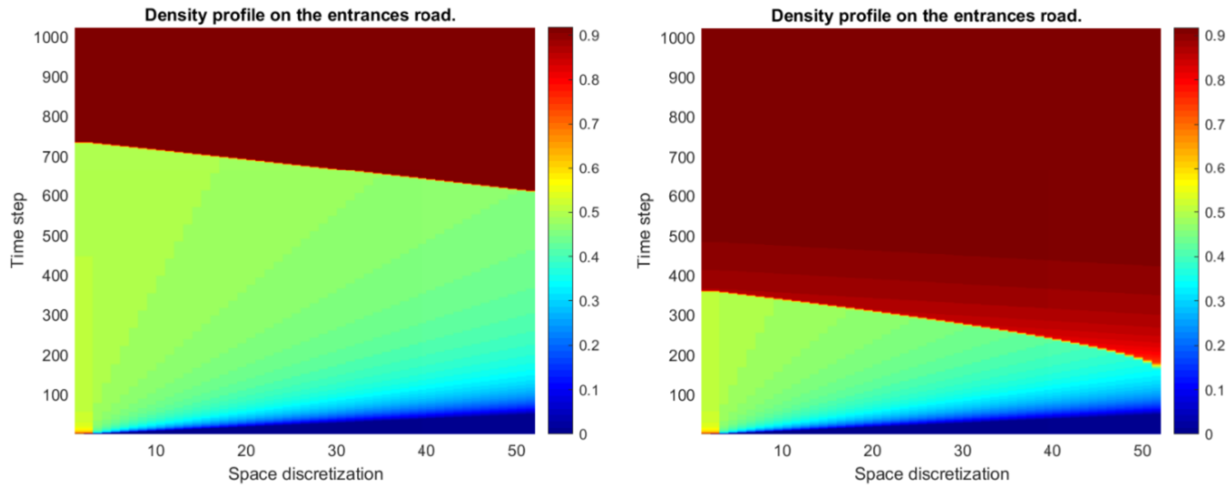


Figure 8: The diverging junction at T = 6 with priority parameter

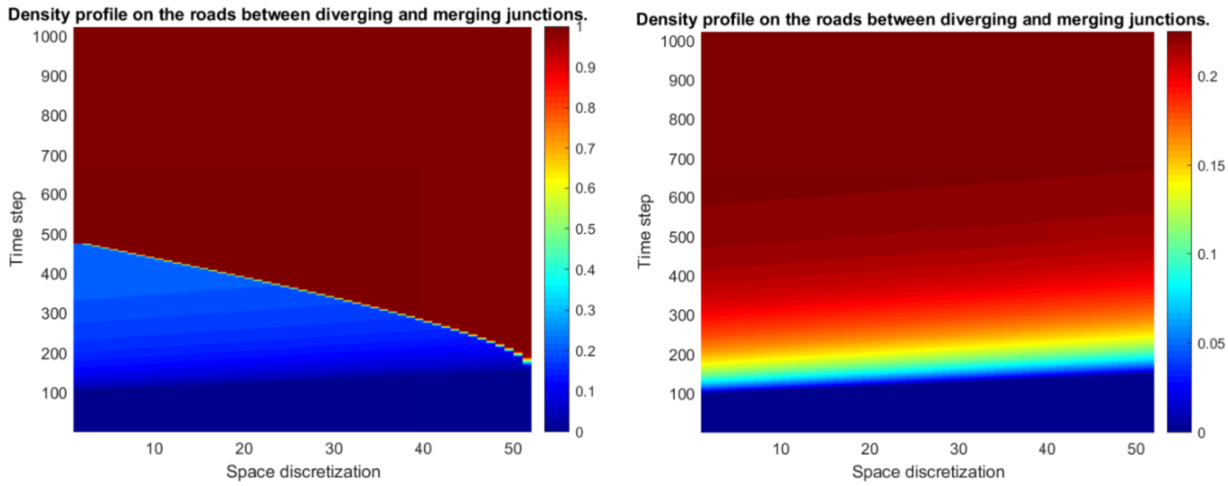
Figures 7 and 8 show the traffic situation at Junction J_2before and after a priority is introduced at Junction J_1 respectively. It can be observed that the traffic on the Road 9 is congested and propagated back before the priority but the same is resolved as a result of introducing priority at J_1.



(a) Without priority parameter (b) With priority parameter

Figure 9: Density profiles on the entrances Roads 1, 2, and 3

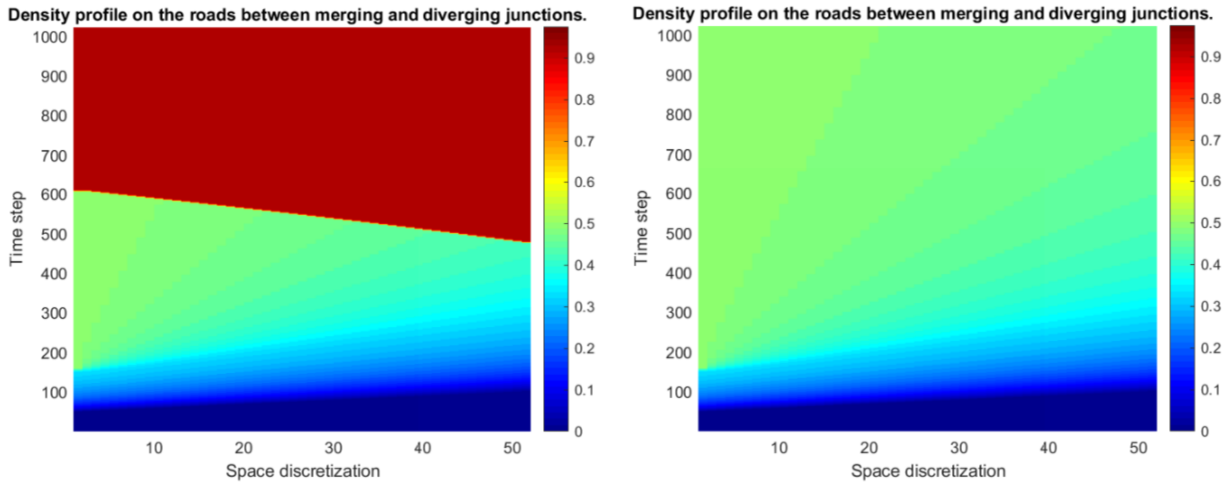
To reduce the traffic congestion on the Main roads the concept of priority is introduced and as a result the congestion is shifted from Main roads to the Entrance roads viz. 1, 2, and 3. This fact is illustrated in the Figure 9. The simulation study shows that on Entrance roads 1, 2, and 3 the traffic congestion is lesser before applying priority at the Merging junctions J_1, J_3, J_5 but higher after the priority.



(a) Without priority parameter (b) With priority parameter

Figure 10: Density profiles between diverge and merge junctions

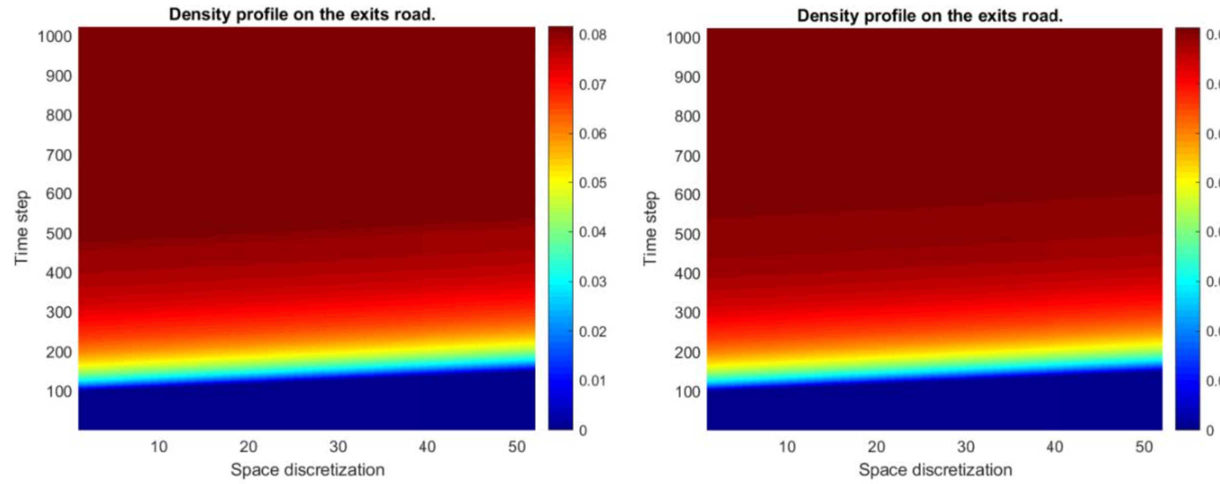
To reduce the traffic congestion on the Main roads the concept of priority is introduced and as a result the congestion is shifted from Main roads to the Entrance roads viz. 1, 2, and 3. This fact is illustrated in the Figure 10. The simulation study shows that on Main roads viz. 7, 9 and 11, leading to merging junctionsJ_1, J_3, J_5, the traffic congestion is more before applying priority but lesser after applying. Thus, the congestion on main roads is reduced due to the application of priority.



(a) Without priority parameter (b) With priority parameter

Figure 11: Density pro les between merge and diverge junctions

To reduce the traffic congestion on the Main roads the concept of priority is introduced at the merging junctions $J_1,\ J_3,\ J_5$and as a result the congestion is shifted from Main roads to the Entrance roads viz. 1, 2, and 3. This fact is illustrated in the Figure 11. The simulation study shows that on Main roads viz. 8, 10 and 12, leading to diverging junctions$J_2,\ J_4,\ J_6$, the traffic congestion is more before applying priority but lesser after applying. Thus, the congestion on main roads leading to diverging junctions is reduced due to the application of priority.

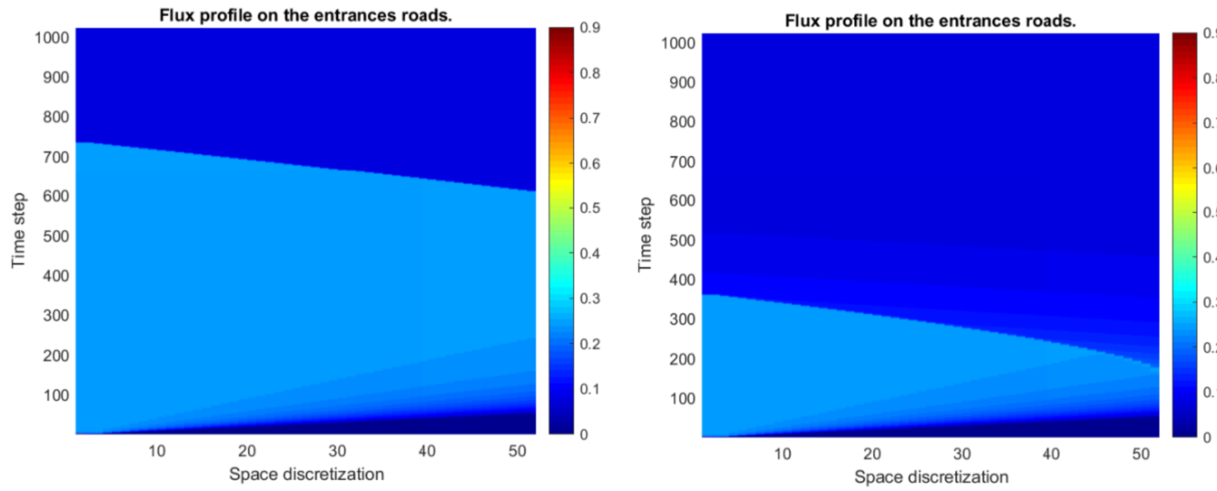


(a) Without priority parameter (b) With priority parameter

Figure 12: Density pro les on the exits road

To reduce the traffic congestion on the Main roads the concept of priority is introduced at the merging junctions J_1, J_3, $J_5$and as a result the congestion is shifted from Main roads to the Entrance roads viz. 1, 2, and 3.

However, the concept of prioritization shows no effect on the traffic flow on the Exit roads 4, 5, and 6 at the diverging junctionsJ_2, J_4, J_6. That is, the traffic flow on these roads is just remains the same both before and after the implementation of prioritization. This fact is illustrated in the Figure 12.

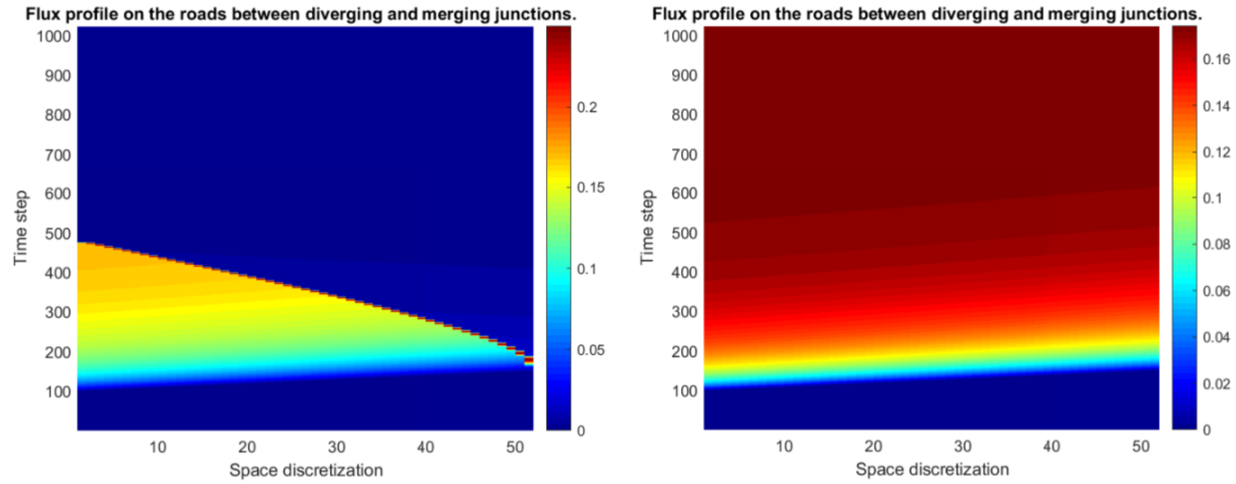


(a) Without priority parameter (b) With priority parameter

Figure 13: Flux pro les on the entrance roads

To increase the traffic flux on the Main roads the concept of priority is introduced at the merging junctions J_1, J_3, $J_5$and as a result the flux is improved on the Main roads but reduced on the Entrance roads viz. 1, 2, and 3.

This fact is illustrated in the Figure 13. The flux of traffic is reduced on the Entrance roads 1, 2, and 3 after the application of priority at the merging junctions J_1, J_3, J_5

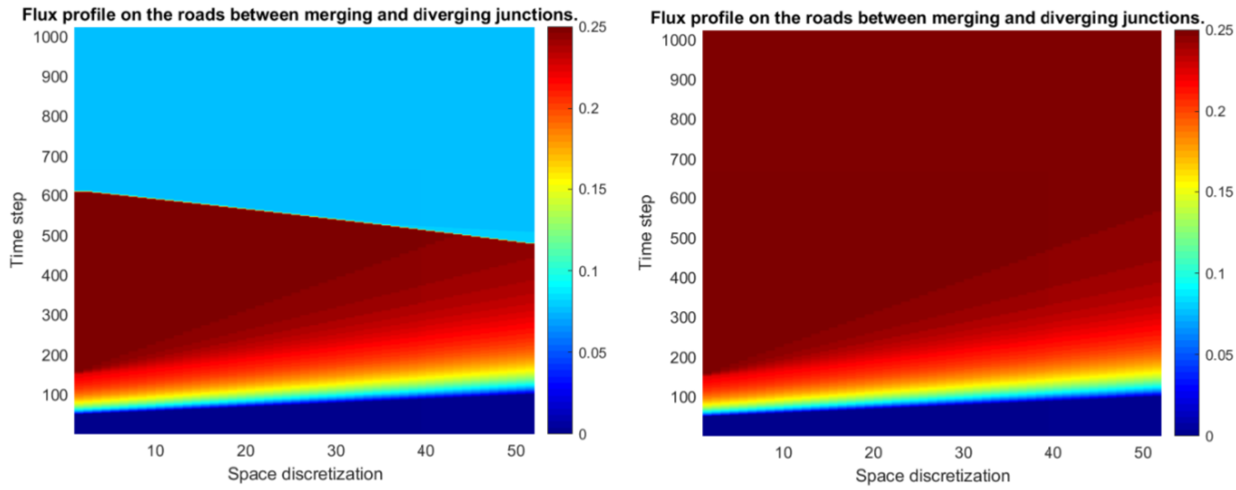


(a) Without priority parameter (b) With priority parameter

Figure 14: Flux pro les between diverge and merge junctions

To increase the traffic flux on the Main roads the concept of priority is introduced at the merging junctions J_1, J_3, $J_5$and as a result the flux is improved on the Main roads but reduced on the Entrance roads viz. 1, 2, and 3.

This fact is illustrated in the Figure 14. The flux of traffic is increased on the main roads 7, 9, and 11 leading to Merging junctions after the application of priority at the merging junctions J_1, J_3, J_5

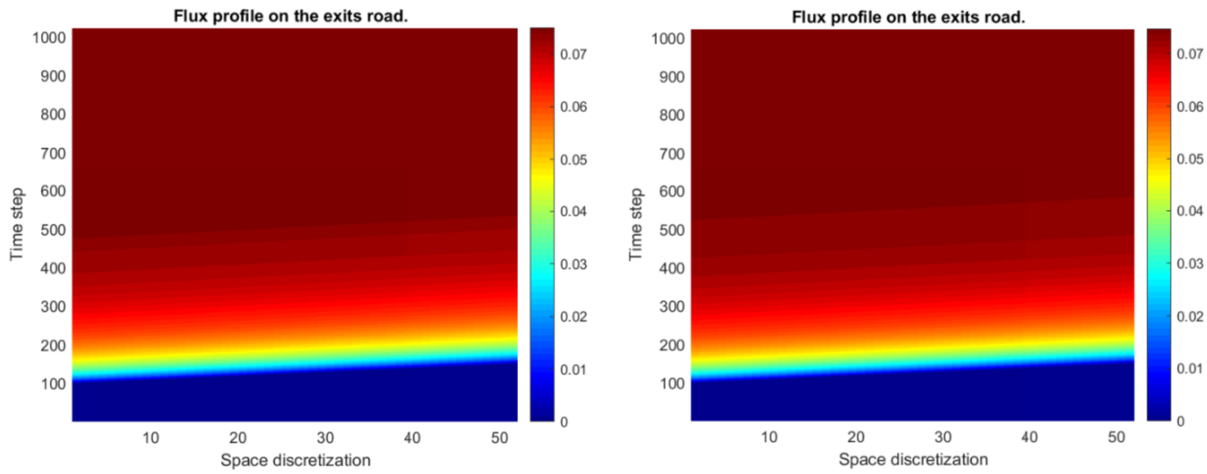


(a) Without priority parameter (b) With priority parameter

Figure 15: Flux pro les between merge and diverge junctions

To increase the traffic flux on the Main roads the concept of priority is introduced at the merging junctions J_1, J_3, $J_5$and as a result the flux is improved on the Main roads but reduced on the Entrance roads viz. 1, 2, and 3.

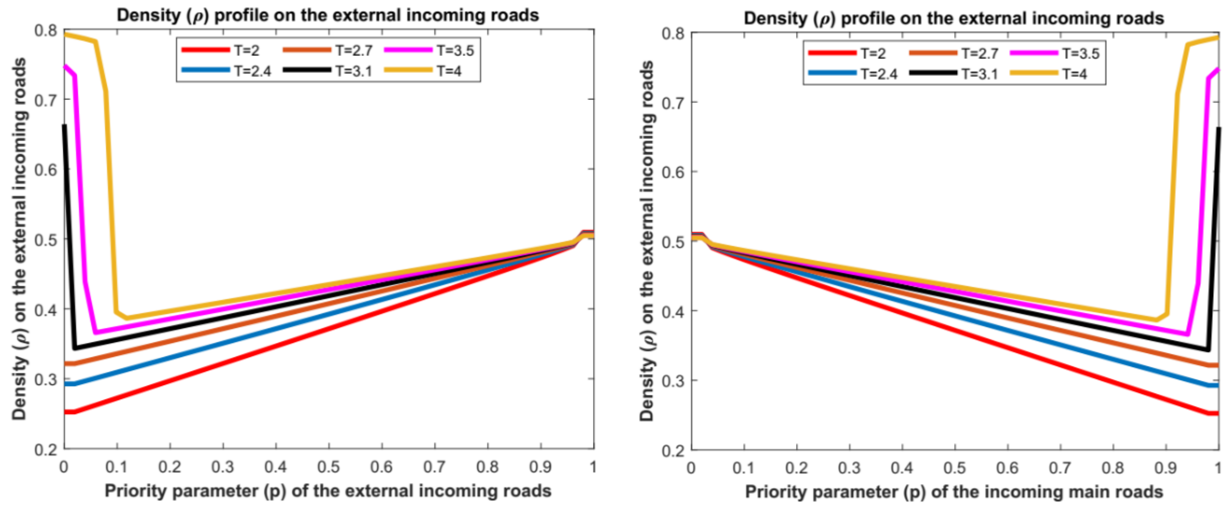
This fact is illustrated in the Figure 15. The flux of traffic is increased on the main roads 8, 10, and 12 leading to Diverging junctions after the application of priority at the merging junctions J_1, J_3, J_5.



(a) Without priority parameter (b) With priority parameter

Figure 16: Flux pro les on the exits road

To increase the traffic flux on the Main roads the concept of priority is introduced at the merging junctions J_1, J_3, $J_5$and as a result the flux is improved on the Main roads but reduced on the Entrance roads viz. 1, 2, and 3. However, the concept of priority shows no influence on the flux on the Exit roads 4, 5, and 6. This fact is illustrated in the Figure 15. The flux of traffic remains the same on the Exit roads 4, 5, and 6 both before and after the application of priority at the merging junctions J_1, J_3, J_5.

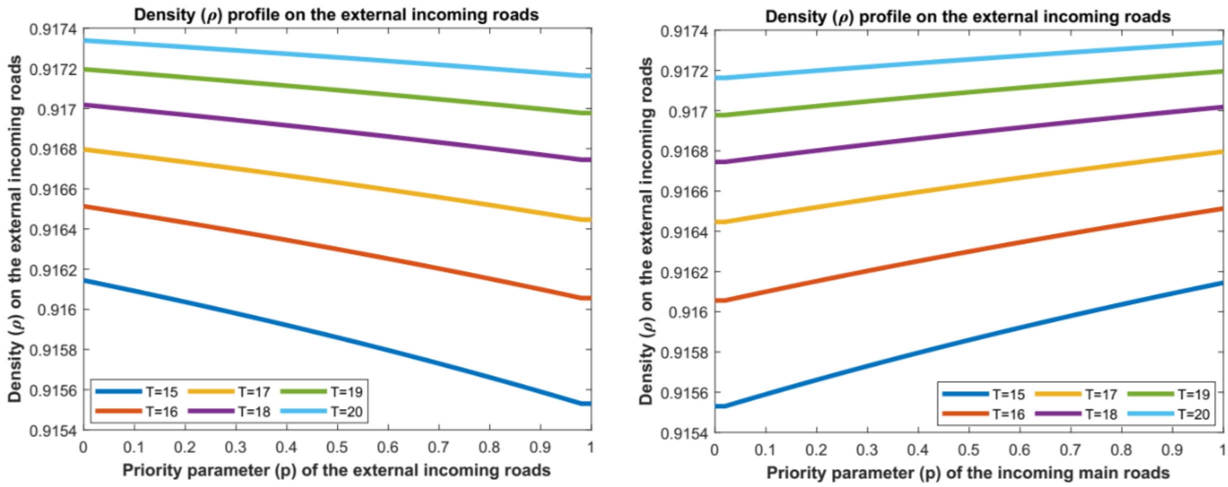


(a) Density on the entrances road (b) Density on the incoming main roads

Figure 17: The density of entrances and incoming main roads decrease as the priority given to each road increase

Figure 17 (a) shows the density profile of the traffic flow on the Entrance roads against priority parameter applied on these roads. It is observed that on these roads the density falls down initially and then rises to a converging point as the priority parameter grows from 0 to 1.

Figure 17 (b) shows the density profile of the traffic flow on the Entrance roads against priority parameter applied on Main roads. It is observed that on the entrance roads the density falls down initially from a fixed point to certain level and rises to the corresponding maximum values, as the priority parameter grows from 0 to 1 on the main roads.

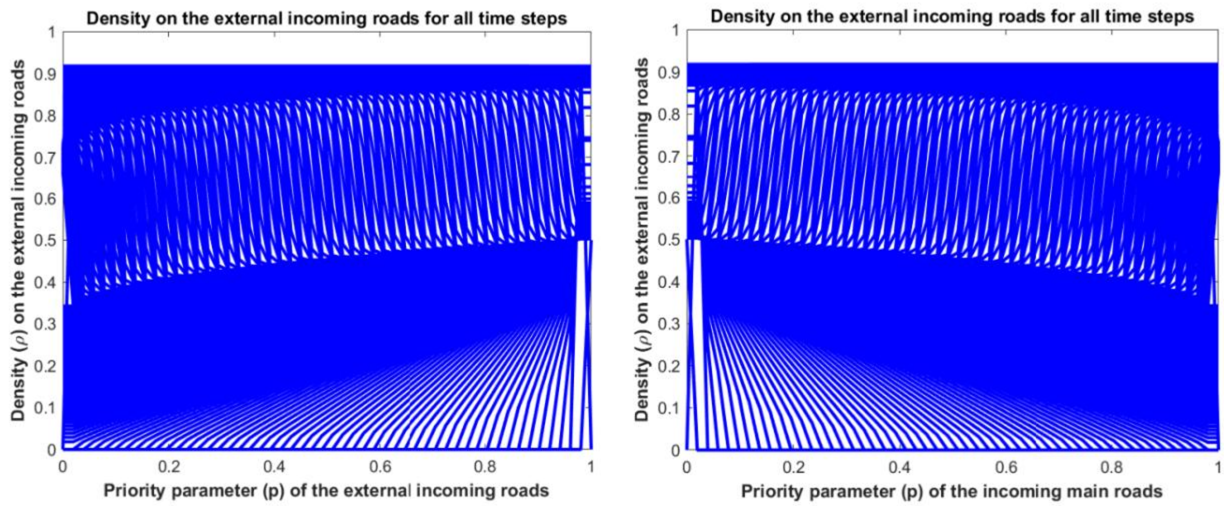


(a) Density on the entrances road (b) Density on the incoming main roads

Figure 18: The density of entrances and incoming main roads decrease as the priority given to each road increase

Figure 18 (a) shows the density profile of the traffic flow on the Entrance roads against priority parameter applied on these roads. It is observed that on these roads the density falls down to a point as the priority parameter grows from 0 to 1. Density is decreasing since the traffic flows smoothly as the priority value increases.

Figure 18 (b) shows the density profile of the traffic flow on the Entrance roads against priority parameter applied on Main roads. It is observed that on the entrance roads the density increases, as the priority parameter grows from 0 to 1 on the main roads. That is, the flow in the entrance road is slowed down and hence the density is increasing.

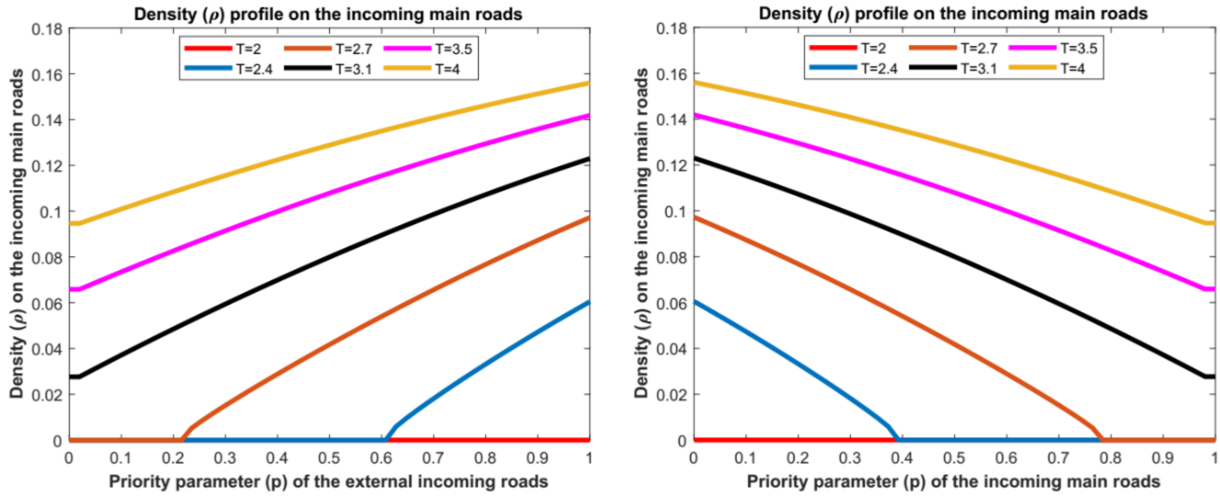


(a) Density on the entrances road(b) Density on the incoming main roads

Figure 19: The density of entrances and incoming main roads increase as the priority given to each road increase

Figure 19 (a) shows the density profile of the traffic flow on the Entrance roads against priority parameter applied on these roads for all simulation time from $0 \leq T \leq 20$. It is observed that on these roads the density falls down initially and then rises to a converging point as the priority parameter grows from 0 to 1 and propagating back.

Figure 19 (b) shows the density profile of the traffic flow on the Entrance roads against priority parameter applied on Main roads for all simulation time from $0 \leq T \leq 20$. It is observed that on the entrance roads the density falls down initially from a fixed point to certain level and rises to the corresponding maximum values, as the priority parameter grows from 0 to 1 on the main roads and propagating back.

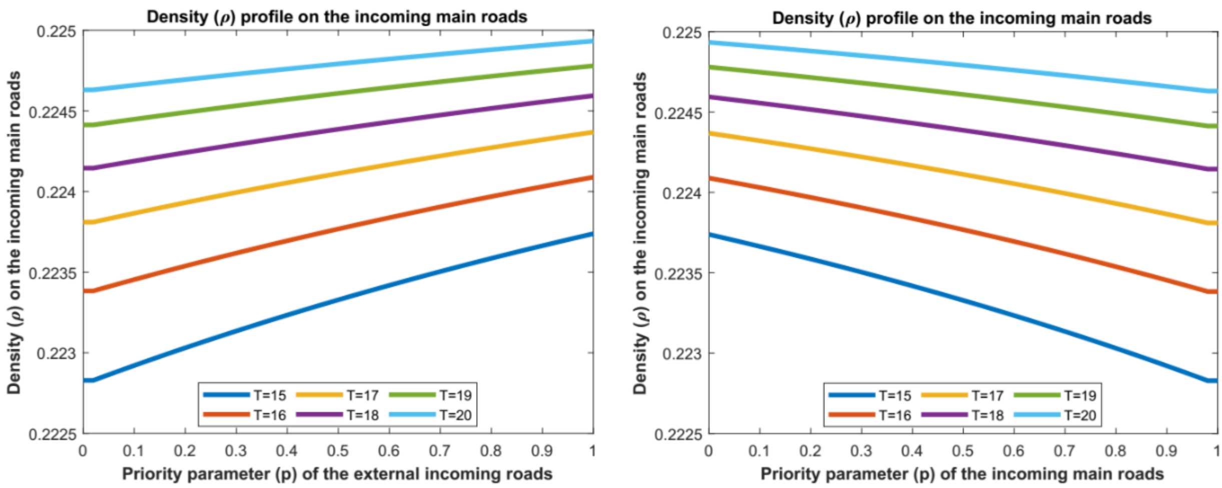


(a) Density on the entrances road (b) Density on the incoming main roads

Figure 20: The density of entrances and incoming main roads decrease as the priority given to each road increase

Figure 20 (a) shows the density profile of the traffic flow on the Main roads against priority parameter applied on Entrance roads. It is observed that on Main roads the density increases as the priority parameter grows from 0 to 1 on the Entrance roads. It can be concluded that the density on the main roads is increasing since the priority value is increasing on the Entrance roads.

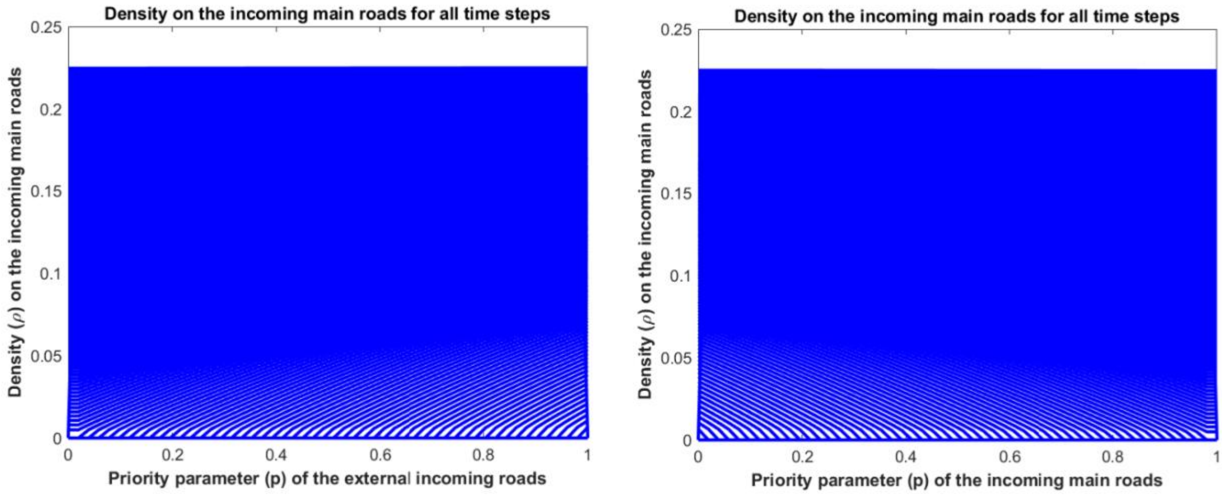
Figure 20 (b) shows the density profile of the traffic flow on the Main roads against priority parameter applied on these roads. It is observed that on these roads the density falls down as the priority parameter grows from 0 to 1. Density is decreasing since the traffic flows smoothly as the priority value increases.



(a) Density on the entrances road (b) Density on the incoming main roads

Figure 21: The density of entrances and incoming main roads decrease as the priority given to each road increase

The Figures 20 and 21 are just similar to each other with just a difference in time. In these figures the observations, interpretations and the conclusions are the same.

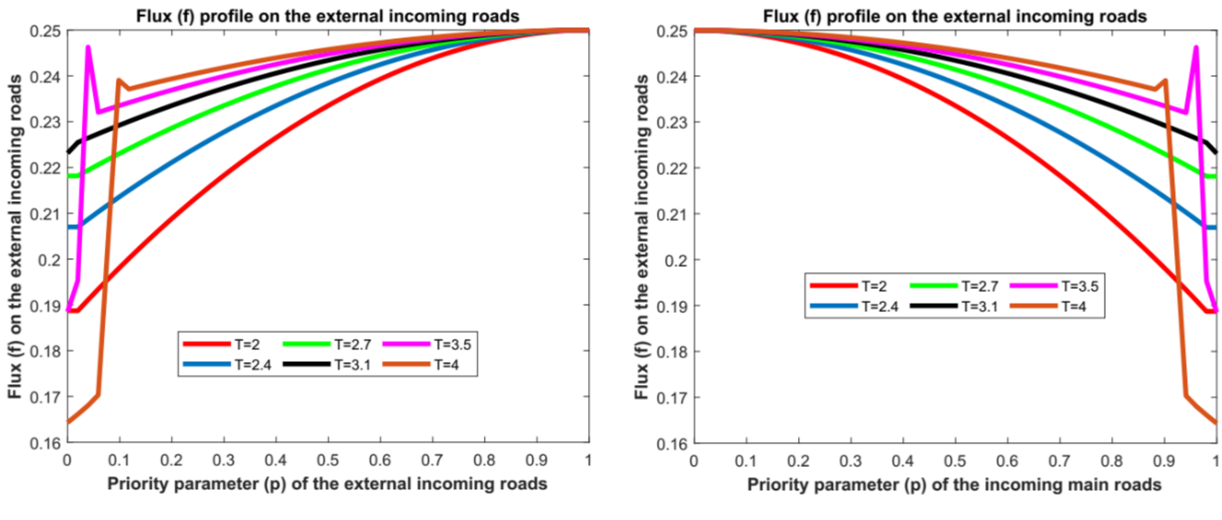


(a) Density on the entrances road (b) Density on the incoming main roads

Figure 22: The density of entrances and incoming main roads decrease as the priority given to each road increase

Figure 22 (a) shows the density profile of the traffic flow on the Main roads against priority parameter applied on Entrance roads for all simulation time from $0 \leq T \leq 20$. It is observed that on Main roads the density increases as the priority parameter grows from 0 to 1 on the Entrance roads. It can be concluded that the density on the main roads is increasing since the priority value is increasing on the Entrance roads.

Figure 22 (b) shows the density profile of the traffic flow on the Main roads against priority parameter applied on these roads for all simulation time from $0 \leq T \leq 20$. It is observed that on these roads the density falls down as the priority parameter grows from 0 to 1. Density is decreasing since the traffic flows smoothly as the priority value increases.

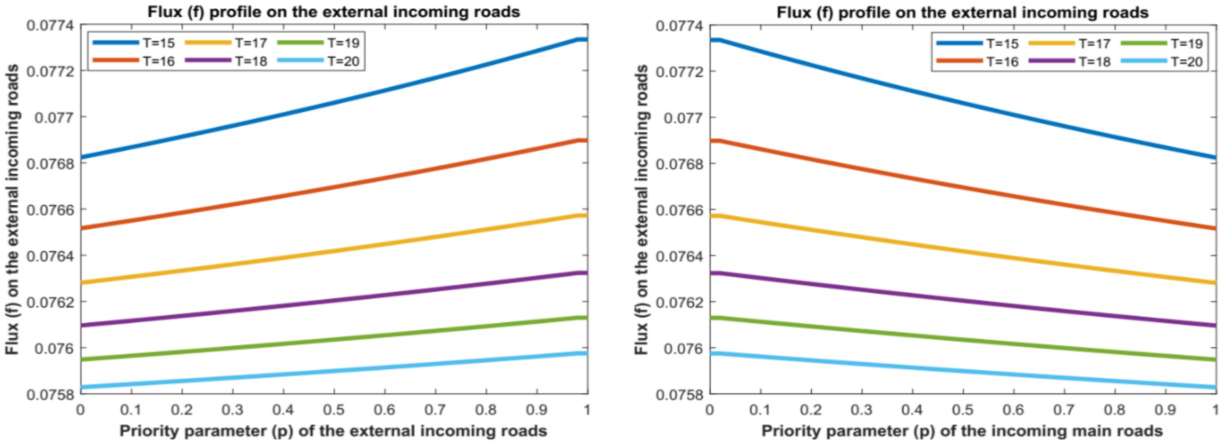


(a) Priority for the entrances roads (b) Priority for the main roads

Figure 23: The Flux on entrance roads

Figure 23 (a) shows the flux profile of the traffic flow on the Entrance roads against priority parameter applied on these roads. It is observed that on these roads the flux rises to a converging point as the priority parameter grows from 0 to 1.

Figure 23 (b) shows the flux profile of the traffic flow on the Entrance roads against priority parameter applied on Main roads. It is observed that on the entrance roads the flux falls down to certain level and start propagating back, as the priority parameter grows from 0 to 1 on the main roads.

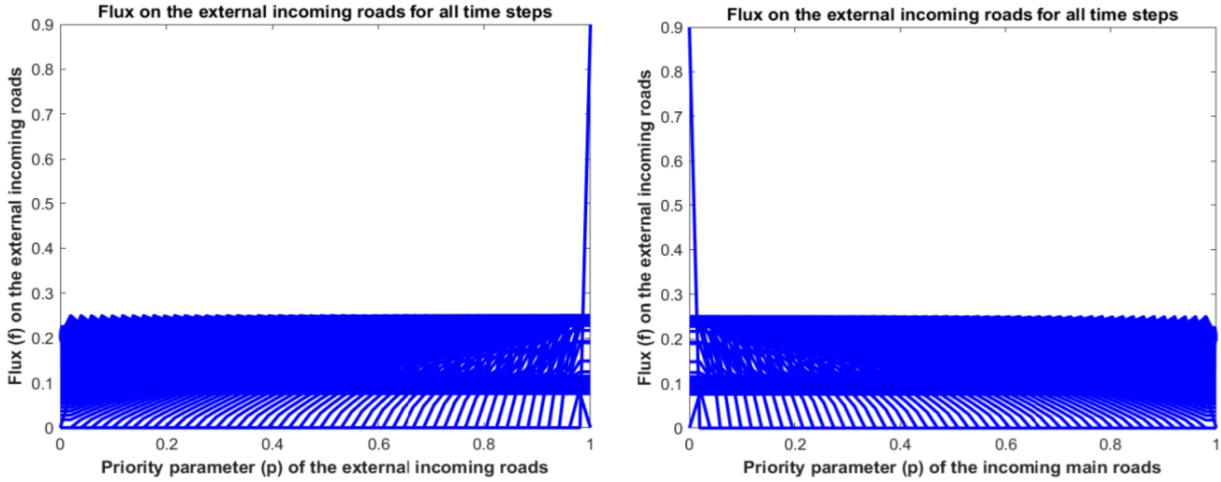


(a) Priority given for the entrance roads (b) Priority given for the incoming main roads

Figure 24: The Flux on roads

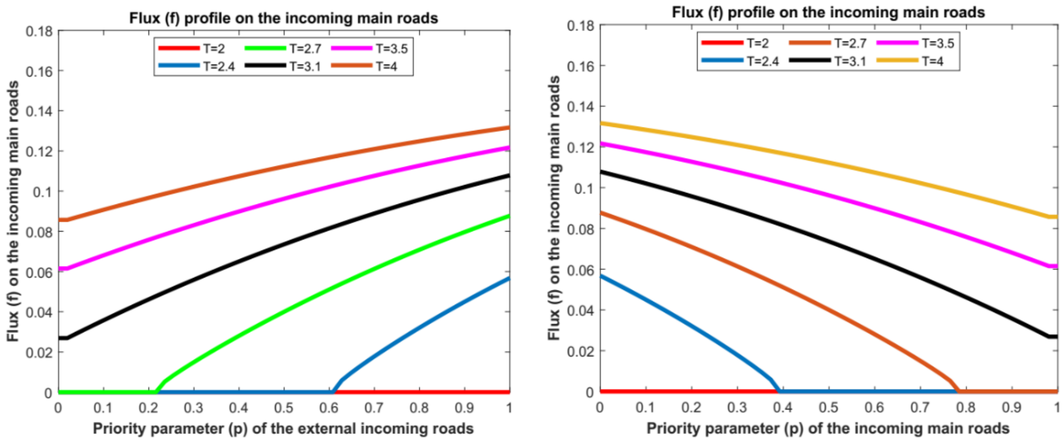
Figure 24 (a) shows the flux increases as the priority parameter grows from 0 to 1 on the Entrance roads.

Figure 20 (b) shows the density profile of the traffic flow on the Main roads against priority parameter applied on these roads. It is observed that on these roads the density falls down as the priority parameter grows from 0 to 1. Density is decreasing since the traffic flows smoothly as the priority value increases.



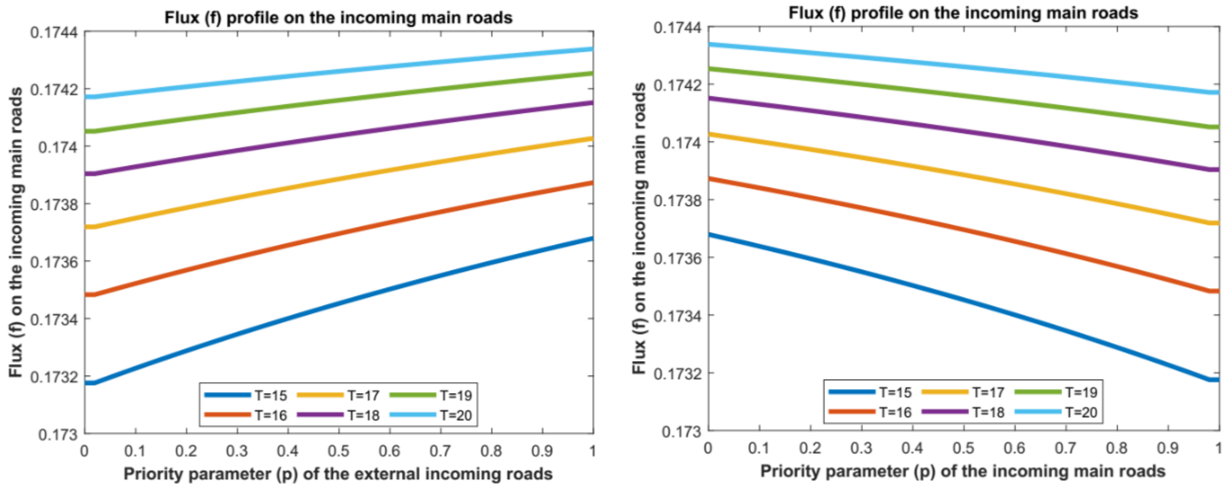
(a) Density on the entrances road (b) Density on the incoming main roads

Figure 25: The density of entrances and incoming main roads decrease as the priority given to each road increase.



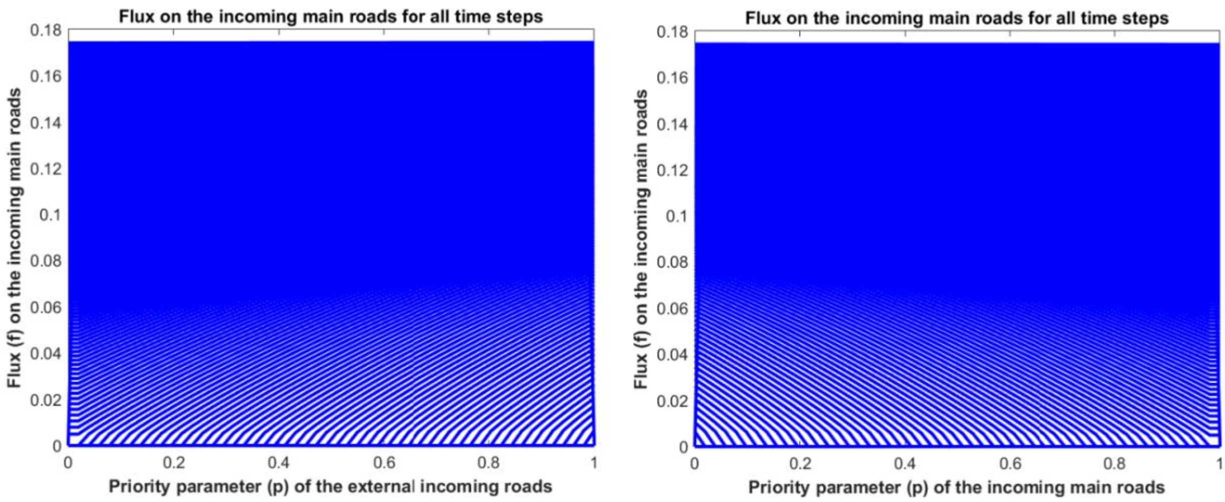
(a) Density on the entrances road (b) Density on the incoming main roads

Figure 26: The density of entrances and incoming main roads decrease as the priority given to each road increase.



(a) Density on the entrances road (b) Density on the incoming main roads

Figure 27: The density of entrances and incoming main roads decrease as the priority given to each road increase



(a) Density on the entrances road (b) Density on the incoming main roads

Figure 28: The density of entrances and incoming main roads decrease as the priority given to each road increase

V. Conclusions

In this study, the evolution of traffic flow on the road intersection of a single lane three legs roundabout is analyzed from a macroscopic point of view following Lighthill – Whitham – Richards model. Here we have considered a roundabout networking consisting of totally 12 roads and 6 junctions.

Road numbers 1, 2, and 3 are named as Entrance roads; Road numbers 4, 5, and 6 are named as Exit roads and the road numbers 7, 8, 9, 10, 11, and 12 are named as Main roads.

Similarly the three junctions denoted by J_1, J_3, J_5 are named as merging junctions and the remaining three junctions denoted by J_2, J_4, J_6 are named as Diverging junctions.

It is well known and well understood if the traffic flow on roads is low then no congestion occurs and there do not arise any study. However, the problem arises whenever the traffic flow is high.

Here the high traffic flow situation on the described roundabout networking of roads is considered and shown a solution for the congestion problem through the implementation of Priority.

Normally congestion on main roads is not desirable but if required it can be tolerated on the other roads. Thus, in this study it is attempted to reduce congestion on main roads by giving more priority to the traffic on these roads. As a result the traffic flow on the main roads is observed to flow freely. However, the traffic on the entrance roads is affected and it is tolerable.

It can be concluded that to reduce traffic congestion on main roads or important roads is to give priority for the traffic on these roads.

The priority principle will not solve the congestion problem completely but it shifts the problem from main roads to other roads and it is tolerable.

References

[1] Alberto B., Fang Y.: Continuous Riemann Solvers for Traffic Flow at a Junction, Department of Mathematics, Penn State University, University Park, PA. 16802, U.S.A. (2014).

[2] Bergersen B. D.: Numerical Solutions of Traffic Flow on Networks, Norwegian University of Science and Technology, 1, (2014).

[3] Colombo R. M.: Hyperbolic Phase Transitions in Traffic Flow, SIAM J. on Appl. Math., 63, (2002), 708-721.

[4] Colombo R. M., Goatin P., and Piccoli B.: Road network with phase transition, Journal of Hyperbolic Differential Equations, 7, (2010), 85-106.

[5] Courant R., Friedrichs K., Lewy H.: On the partial difference equations of mathematical physics, IBM journal of Research and Development, 11(2), (1967), 215-234.

[6] Feng Y., Liu Y., Deo, P. and Ruskin H. J. : Heterogeneous Traffic Flow Model For Two Lane Roundabouts and Controlled Intersection, International Journal of Modern Physics, 1, (2007), 107-117.

[7] Garavello M. and Piccoli B.: Traffic flow on road networks, SIAM J. Math. Anal., 36, (2005), 1862-1886.

[8] Garavello M. and Piccoli B.: Traffic flow on road networks Conservation Laws Models, American Institute of Mathematical Sciences, 1, (2006).

[9] Godunov S. K.: A finite difference method for the numerical computation of discontinuous solutions of the equations of fluid dynamics, Matematicheskii Sbornik,47 (1959),271-290.

[10] Hundesa T., Lemecha L., Koya P.R.: One-Dimensional Explicit Tolesa Numerical Scheme for Solving First Order Hyperbolic Equations and Its Application to Macroscopic Traffic Flow Model, Scientific Research Publishing Inc., Applied Mathematics, 10, (2019), 119-136. https://doi.org/10.4236/am.2019.103011

[11] Iompair B. E.: Geometric Design of Junctions (priority junctions, direct accesses, roundabouts, grade separated and compact grade separated junctions), Transport Infrastructure Ireland (TII) Publication, (2017).http://www.tiipublications.ie

[12] Kruzhkov S. N.: First order quasilinear equations in several independent variables, MatematicheskiiSbornik, 123(2), (1970), 228-255.

[13] Legesse L., Anne M., Semu M., and Axel K.: Modelling Pedestrians' Impact on the Performance of a Roundabout, Neural, Parallel, and Scientific Computations, 24, (2016), 317-334.

[14] LeVeque, R. J.: Finite Volume Methods for Hyperbolic Problems, Cambridge Texts in Applied Mathematics. Cambridge University Press, (2002).

[15] Lighthill M. J, Whitham G. B., Schindler P.: A Theory of Traffic Flow on Long Crowded Roads,Proceedings of the Royal Society of London, Series A. Mathematical and Physical Sciences, 229, (1955), 317-345.

[16] Pratelli A.: Design of modern roundabouts in urban traffic systems, WIT Transactions on The Built Environment, 89, (2006).doi:10.2495/UT060091

[17] Richards R. I.: Shock waves on the highway, Operations research, 4, (1956), 42-51.

[18] Yu C., Zhang J., Yao D., Zhang R. and Jin H.: Speed-Density Model of In-terruptedTra c Flow Based on Coil Data, Hindawi Publishing Corporation Mobile Information Systems, (2016). http://dx.doi.org/10.1155/2016/7968108.

[19] Marinela I.: Improving performance of roundabout intersections by optimizing traffic-flow speed. *MATEC Web of Conferences **121**, 06003, (2017).DOI: 10.1051/matecconf/20171210.*

[20] GaravelloM., Han K., and Piccoli B. Models for vehicular traffic on networks, volume 9 of AIMS Series on Applied Mathematics. *American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2016.*

[21] Holden H., and Risebro N. H. A mathematical model of traffic flowon a network of unidirectional roads. *SIAM Journal on MathematicalAnalysis*, 26(4):999-1017, 1995.

[22] Somanathan P. P., Thamizh V. A. Influence of Traffic Mix On PCU Value Of Vehicles Under Heterogeneous Traffic Conditions. *International Journal for Traffic and Transport Engineering, 3(3): 302 – 330, (2013). DOI: http://dx.doi.org/10.7708/ijtte.2013.3(3).07*.

Tolesa Hundesa. " Impacts of Priority Parameters on the Traffic Performance at a Road Intersection." IOSR Journal of Mathematics (IOSR-JM) 15.6 (2019): 09-25.