

Common Fixed Point Theorems In Intuitionistic Fuzzy Metric Space Using General Contractive Condition Of Integral Type.

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Abstract: The aim of this paper is to prove the generalized fuzzy metric space for four discontinuous mappings in non complete intuitionistic fuzzy metric spaces using contractive condition of integral type.

Keywords: Intuitionistic fuzzy metric spaces, weakly compatible mapping, common fixed point.

AMS mathematics subject classification 47H10, 54H25.

Date of Submission: 07-10-2019

Date of Acceptance: 22-10-2019

I. Introduction

Zadeh (1965) introduced the concept of fuzzy set and till then it has been developed extensively by many authors in different fields. The role of fuzzy topology has been recognized and applied on various fields to find more accurate results. In last 50 years, this theory has wide range of applications in diverse areas. The strong points about fuzzy mathematics are its fruitful applications, especially outside mathematics, such as in quantum particle physics studied by El Naschie[6] (2004). To use this concept in topology and analysis, Kramosil and Michalek[7] (1975) have introduced the concept of fuzzy metric space using the concept of continuous triangular norm defined by Schweizer (1960). Most recently, Gregori, Morillas, and Sapena (2011) utilized the concept of fuzzy metric spaces to color image processing and also studied several interesting examples of fuzzy metrics in the sense of George and Veeramani (1994). They define a Hausdorff topology and have show that every metric induced a fuzzy metric. Form the definition of Cauchy sequence givenby Kramosil and Michalek even \mathbb{R} is not complete so George and Veeramani[3] also modify the definition of Cauchy sequence. V. Gregori and A. Sapena [4] extend the Banach fixed point theorem to fuzzy contractive mapping of complete metric space. In the recent years, many authors had dedicated themselves to the study of fixed point theory in fuzzy metric space Branciari [2] gave an integral version of the Banach contraction principles and proved fixed point theorem for a single-valued contractive mapping of integral type in metric space. Afterwards many researchers extended the result of Branciari and obtained fixed point and common fixed point theorems for various contractive conditions of integral type on different spaces. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1], it can be considered as a generalization to fuzzy sets concept due to Zadeh [5]. Later Coker introduced a topology on intuitionistic fuzzy sets. Park introduced the notion of intuitionistic fuzzy metric spaces as a generalization to fuzzy metric spaces, which is a combination between intuitionistic fuzzy sets and the concept of a fuzzy metric space given by George and Veeramani [3], many authors established some results concerning fixed point in such spaces, he obtained some common fixed point theorems for compatible mappings in such spaces. we will generalize certain definitions to intuitionistic fuzzy metric spaces in integral type to obtain some common fixed point theorems by combining the concept of \mathbb{R} - weakly sequentially continuous mappings proved a common fixed point theorem in an intuitionistic fuzzy metric space for point-wise \mathbb{R} -weakly commuting mappings using contractive condition of integral type and established a situation in which a collection of maps has a fixed point which is a point of discontinuity.

In this paper, we prove some common fixed point theorems for six mappings by using contractive condition of integral type for class of weakly compatible maps in non complete intuitionistic fuzzy metric spaces, without taking any continuous mapping.

II. Preliminaries

Definition 2.1 . A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:

- (i) $*$ is commutative and associative,
- (ii) $*$ is continuous,

- (iii) $a * 1 = a$ for all $a \in [0, 1]$,
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $a, b, c, d \in [0, 1]$.

Definition 2.2. A binary operation $\diamond : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-conorm if \diamond is satisfying the following conditions:

- (i) \diamond is commutative and associative,
- (ii) \diamond is continuous,
- (iii) $a \diamond 0 = a$ for all $a \in [0, 1]$,
- (iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, $a, b, c, d \in [0, 1]$.

Definition 2.3 ([1]). A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric spaces if X is an arbitrary set, $*$ is a continuous t-norm, \diamond is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,

- (i) $M(x, y, t) + N(x, y, t) \leq 1$,
- (ii) $M(x, y, 0) = 0$,
- (iii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
- (iv) $M(x, y, t) = M(y, x, t)$,
- (v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
- (vi) $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous,
- (vii) $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ for all x, y in X ,
- (viii) $N(x, y, 0) = 1$,
- (ix) $N(x, y, t) = 0$ for all $t > 0$ if and only if $x = y$,
- (x) $N(x, y, t) = N(y, x, t)$,
- (xi) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
- (xii) $N(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is right continuous,
- (xiii) $\lim_{t \rightarrow \infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric on X . The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t , respectively.

Remark 2.4.

(i). The concept of triangular norms (t -norms) and triangular conorms (t -conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively.

(II). every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t -norm $*$ and t -conorm

\diamond are associated, i.e., $x \diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

(III). In intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$

Example:2.5. Let (X, d) be a metric space. Define t-norm $a * b = \min\{a, b\}$ and t-conorm $a \diamond b = \max\{a, b\}$ and for all $x, y \in X$ and $t > 0$,

$$(2a) \quad M_d(x, y, t) = \frac{t}{t + d(x, y)}, N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric. On the other hand, note that there exists no metric on X satisfying (2a).

Definition 2.6 ([1]). Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

- i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n \rightarrow \infty} x_n = x$) if, for all $t > 0$.

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1, \lim_{n \rightarrow \infty} N(x_n, x, t) = 0$$

ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1, \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0$$

(III).is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 2.7([1]). Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X . If there exists a number $k \in (0, 1)$ such that

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t), \quad N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, \dots$, then $\{y_n\}$ is a Cauchy sequence in X .

Lemma 2.8 ([1]). Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1)$,

$$M(x, y, kt) \geq M(x, y, t) \quad \text{and} \quad N(x, y, kt) \leq N(x, y, t),$$

then $x = y$.

Definition 6 ([14]). Two self mappings S and T are said to be weakly compatible if they commute at their coincidence points; i.e., if $Tu = Su$ for some $u \in X$, then $TSu = STu$.

In this paper, we prove some common fixed point theorems for six mappings by using contractive condition of integral type for class of weakly compatible maps in non complete intuitionistic fuzzy metric spaces,

III. Main Result

Theorem 3.1. Let $(X, E, F, *, \diamond)$ be an intuitionistic fuzzy metric space with continuous t -norm $*$ and continuous t -conorm \diamond defined by $t * t \geq t$ and $(1 - t) \diamond (1 - t) \leq (1 - t)$ for all $t \in [0, 1]$. Let A, B, S, T, P and Q be mappings from X into itself such that

- (a) $S(X) \subset B(X)$ and $T(X) \subset C(X)$ and $U(X) \subset A(X)$.also let $A(x)$ is complete .
- (b) There exist a constant $k \in (0, 1)$, such that

$$\int_0^{E(Sx, Ty, Uz, kt)} \psi(t) dt \geq \int_0^{E(x, y, z, t)} \psi(t) dt$$

$$\int_0^{F(Sx, Ty, Uz, kt)} \psi(t) dt \leq \int_0^{F(x, y, z, t)} \psi(t) dt$$

Where $\psi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$

Is a lebesgue integrable mapping which is summable, non-negative and such that

$$\int_0^\epsilon \psi(t) dt > 0 \text{ for each } \epsilon > 0 \text{ where}$$

$$E(Sx, Ty, Uz, kt) \geq \text{Max}(E(Ax, By, Cz, t), E(Sx, Ax, Bz, t), E(Tx, Bx, Bz, t), E(Ux, Cx, Cz, t))$$

$$F(Sx, Ty, Uz, kt) \leq \text{Min} (F(Ax, By, Cz, t), F(Sx, Ax, Bz, t), F(Tx, Bx, Bz, t), F(Ux, Cx, Cz, t))$$

For all $x, y \in X$ $\alpha \in (0, 3)$ and $t > 0$

(c) If one of $S(x), A(x), B(X), CU(X), T(X)$ is a complete subspace of X then

Further And

(d) is weakly compatible

Then

(iii) $A, B, S, T, P, Q, , S$ and T have a unique common fixed point in X .

Prof : Since $S(X) \subset B(X)$ and $T(X) \subset C(X)$ and $U(X) \subset A(X)$

We can define sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$$\{y_{3n+1}\} = \{Sx_{3n}\} = \{Bx_{3n+1}\}$$

$$\{y_{3n+2}\} = \{Tx_{3n+1}\} = \{Cx_{3n+2}\}$$

$\{y_{3n+3}\} = \{Ux_{3n+2}\} = \{Ax_{3n+3}\}$ then from (1) we have

$$\int_0^{E(sx_n, Tx_{3n+1}, Ux_{3n+2}, kt)} \varphi(x) dt \geq \max \int_0^{E(Ax_{3n}, Bx_{3n+1}, Cx_{3n+2}, t), E(Sx_{3n}, Ax_{3n}, Bx_{3n+2}, t), E(Tx_{3n}, Bx_{3n}, Bx_{3n+2}, t), E(Ux_{3n}, Cx_{3n}, Cx_{3n+2}, t)} \varphi(x) dt$$

$$\int_0^{E(y_{3n+1}, y_{3n+2}, y_{3n+3}, kt)} \varphi(x) dt \geq \max \int_0^{E(y_{3n}, y_{3n+1}, y_{3n+2}, t), E(y_{3n+1}, y_{3n}, y_{3n+2}, t), E(y_{3n+1}, y_{3n}, y_{3n+2}, t), E(y_{3n+1}, y_{3n}, y_{3n+2}, t)} \varphi(x) dt$$

$$\int_0^{E(y_{3n+1}, y_{3n+2}, y_{3n+3}, kt)} \varphi(x) dt \geq \int_0^{E(y_{3n}, y_{3n+1}, y_{3n+2}, t)} \varphi(x) dt$$

Similary we have

$$\int_0^{E(y_n, y_{n+1}, y_{n+2}, kt)} \varphi(x) dt \geq \int_0^{E(y_{n-1}, y_n, y_{n+1}, t)} \varphi(x) dt$$

Hence $\{y_n\}$ is Cauchy and since X is Complete ,

there exists z in X such that $\{y_n\}$ tends to z .

$$\int_0^{F(sx_n, Tx_{3n+1}, Ux_{3n+2}, kt)} \varphi(x) dt \leq \min \int_0^{F(Ax_{3n}, Bx_{3n+1}, Cx_{3n+2}, t), F(Sx_{3n}, Ax_{3n}, Bx_{3n+2}, t), F(Tx_{3n}, Bx_{3n}, Bx_{3n+2}, t), F(Ux_{3n}, Cx_{3n}, Cx_{3n+2}, t)} \varphi(x) dt$$

$$\int_0^{F(y_{3n+1}, y_{3n+2}, y_{3n+3}, kt)} \varphi(x) dt \leq \min\{F(y_{3n}, y_{3n+1}, y_{3n+2}, t), F(y_{3n+1}, y_{3n}, y_{3n+2}, t), F(y_{3n+1}, y_{3n}, y_{3n+2}, t), F(y_{3n+1}, y_{3n}, y_{3n+2}, t)\} \varphi(x) dt$$

$$\int_0^{F(y_{3n+1}, y_{3n+2}, y_{3n+3}, kt)} \varphi(x) dt \leq \int_0^{F(y_{3n}, y_{3n+1}, y_{3n+2}, t)} \varphi(x) dt$$

Similarly we have

$$\int_0^{F(y_n, y_{n+1}, y_{n+2}, kt)} \varphi(x) dt \leq \int_0^{F(y_{n-1}, y_n, y_{n+1}, t)} \varphi(x) dt$$

Hence $\{y_n\}$ is Cauchy and since X is Complete, there exists z in X such that $\{y_n\}$ tends to z .

So the sub sequences $\{y_{3n}\}, \{y_{3n+1}\}, \{y_{3n+2}\}$, are also convergent

i.e $\lim Bx_{3n+1} = \lim Sx_{3n} = \lim Cx_{3n+2} = \lim Ax_{3n+3} = \lim Ux_{3n+2} = z$

Since $A(X)$ is complete there exist w in X such that $Aw = z$ we claim that $Sw = z$.

$$\begin{aligned} & \int_0^{E(Sw, Tx_{3n+1}, Ux_{3n+2}, kt)} \varphi(x) dt \\ & \quad E(Aw, Bx_{3n+1}, Cx_{3n+2}, t), E(Sw, Aw, Bx_{3n+2}, t), E(Tw, Bw, Bx_{3n+2}, t), \\ & \geq \max \int_0^{E(Uw, Cw, Cx_{3n+2}, t)} \varphi(x) dt \end{aligned}$$

$$\begin{aligned} & \int_0^{F(Sw, Tx_{3n+1}, Ux_{3n+2}, kt)} \varphi(x) dt \\ & \quad F(Aw, Bx_{3n+1}, Cx_{3n+2}, t), F(Sw, Aw, Bx_{3n+2}, t), F(Tw, Bw, Bx_{3n+2}, t), \\ & \leq \min \int_0^{F(Uw, Cw, Cx_{3n+2}, t)} \varphi(x) dt \end{aligned}$$

$$\begin{aligned} & \int_0^{E(Sw, y_{3n+2}, y_{3n+3}, kt)} \varphi(x) dt \\ & \quad E(z, y_{3n+1}, y_{3n+2}, t), E(Sw, z, y_{3n+2}, t), E(sw, z, y_{3n+2}, t), \\ & \geq \max \int_0^{E(Uw, Cw, y_{3n+2}, t)} \varphi(x) dt \end{aligned}$$

$$\int_0^{\infty} F(Sw, y_{3n+2}, y_{3n+3}, kt) \varphi(x) dt$$

$$\leq \min \int_0^{\infty} F(Uw, Cw, y_{3n+2}, t) \varphi(x) dt$$

$$F(z, y_{3n+1}, y_{3n+2}, t), F(Sw, z, y_{3n+2}, t), F(sw, z, y_{3n+2}, t),$$

Taking limit n tends to infinite .

$$\int_0^{\infty} E(Sw, z, z, kt) \varphi(x) dt \geq \max \int_0^{\infty} E(Uw, Cw, z, t) \varphi(x) dt$$

$$E(z, z, z, t), E(Sw, z, z, t), E(sw, z, z, t),$$

$$\int_0^{\infty} F(Sw, z, z, kt) \varphi(x) dt \geq \max \int_0^{\infty} F(Uw, Cw, z, t) \varphi(x) dt$$

$$F(z, z, z, t), F(Sw, z, z, t), F(sw, z, z, t),$$

i.e $E(Sw, z, z, kt) = F(Sw, z, z, kt) = 1$ Therefore $Sw = z = Aw$. Hence w is the coincidence point of S and A .

$S(X) \subset B(X)$ i.e $z \in S(X) \subset B(X)$, then there exist $t \in X$ such that $Bt = z$

$$\int_0^{\infty} E(Sx_{3n}, Tt, Ux_{3n+2}, kt) \varphi(x) dt$$

$$\geq \max \int_0^{\infty} \left\{ \begin{array}{l} E(Ax_{3n}, Bt, Cx_{3n+2}, t) E(Sx_{3n}, Ax_{3n}, Bx_{3n+2}, t), E(Tx_{3n}, Bx_{3n}, Bx_{3n+2}, t), \\ E(Ux_{3n}, Cx_{3n}, Cx_{3n+2}, t) \end{array} \right\} \varphi(x) dt$$

$$\int_0^{\infty} F(Sx_{3n}, Tt, Ux_{3n+2}, kt) \varphi(x) dt$$

$$\leq \min \int_0^{\infty} \left\{ \begin{array}{l} F(Ax_{3n}, Bt, Cx_{3n+2}, t) F(Sx_{3n}, Ax_{3n}, Bx_{3n+2}, t), F(Tx_{3n}, Bx_{3n}, Bx_{3n+2}, t), \\ F(Ux_{3n}, Cx_{3n}, Cx_{3n+2}, t) \end{array} \right\} \varphi(x) dt$$

$$\int_0^{E(y_{3n+1}, Tt, y_{3n+3}, kt)} \varphi(x) dt \geq \max \int_0^{E(y_{3n}, z, y_{3n+2}, t), E(y_{3n+1}, y_{3n}, y_{3n+2}, t), E(y_{3n+1}, y_{3n}, y_{3n+2}, t)E(y_{3n+1}, y_{3n}, y_{3n+2}, t)} \varphi(x) dt$$

$$\int_0^{F(y_{3n+1}, Tt, y_{3n+3}, kt)} \varphi(x) dt \leq \min \int_0^{F(y_{3n}, z, y_{3n+2}, t), F(y_{3n+1}, y_{3n}, y_{3n+2}, t), F(y_{3n+1}, y_{3n}, y_{3n+2}, t)F(y_{3n+1}, y_{3n}, y_{3n+2}, t)} \varphi(x) dt$$

Taking limit $n \rightarrow \infty$

$$\int_0^{E(z, Tt, z, kt)} \varphi(x) dt \geq \max \int_0^{E(z, z, z, t), E(z, z, z, t), E(z, z, z, t)} \varphi(x) dt$$

$$\int_0^{F(z, Tt, z, kt)} \varphi(x) dt \geq \max \int_0^{F(z, z, z, t), F(z, z, z, t), F(z, z, z, t)} \varphi(x) dt$$

$$\int_0^{E(Sx_{3n}, Tx_{3n+1}, Uv, kt)} \varphi(x) dt \geq \max \int_0^{E(Ax_{3n}, Bx_{3n+1}, Cv, t), E(Sx_{3n}, Ax_{3n}, Bv, t), E(Tx_{3n}, Bx_{3n}, Bv, t), E(Ux_{3n}, Cx_{3n}, Cv, t)} \varphi(x) dt$$

$$\int_0^{F(Sx_{3n}, Tx_{3n+1}, Uv, kt)} \varphi(x) dt \leq \min \int_0^{F(Ax_{3n}, Bx_{3n+1}, Cv, t), F(Sx_{3n}, Ax_{3n}, Bv, t), F(Tx_{3n}, Bx_{3n}, Bv, t), F(Ux_{3n}, Cx_{3n}, Cv, t)} \varphi(x) dt$$

$$\int_0^{E(y_{3n+1}, y_{3n+2}, Uv, kt)} \varphi(x) dt \geq \max \int_0^{\left\{ E(y_{3n}, y_{3n+1}, Cv, t)E(y_{3n+1}, y_{3n}, Bv, t), E(y_{3n+1}, y_{3n}, Bv, t), E(y_{3n+1}, y_{3n}, Bv, t) \right\}} \varphi(x) dt$$

$$\int_0^k F(y_{3n+1}, y_{3n+2}, Uv, kt) \varphi(x) dt \leq \min \int_0^k \left\{ \begin{array}{l} F(y_{3n}, y_{3n+1}, Cv, t) F(y_{3n+1}, y_{3n}, Bv, t), F(y_{3n+1}, y_{3n}, Bv, t), \\ F(y_{3n+1}, y_{3n}, Bv, t) \end{array} \right\} \varphi(x) dt$$

Taking limit $n \rightarrow \infty$

$$\int_0^k E(z, z, Uv, kt) \varphi(x) dt \geq \max \int_0^k \begin{array}{l} E(z, z, z, t), E(z, z, Bv, t), E(z, z, Bv, t), \\ E(z, z, z, t) \end{array} \varphi(x) dt$$

$$\int_0^k F(z, z, Uv, kt) \varphi(x) dt \leq \min \int_0^k \begin{array}{l} F(z, z, z, t), F(z, z, Bv, t), F(z, z, Bv, t), \\ F(z, z, z, t) \end{array} \varphi(x) dt$$

i.e $E(z, z, Uv, Kt) \geq 1, F(z, z, Uv, Kt) \geq 1$

Hence $Uv=z$. We have $Uv=Cv=z$, Since (A, S_0, B, T) and (C, U) are weakly compatible at coincidence points. We have $Sw=z=Aw$. Then $ASw=SAw$ i.e $Az=Sz$. Also $Tt=z=Bt$. Then $BTt=TBt$ i.e $Bz=Tz$. Since (C, U) is weakly compatible, similarly we get $Cz=Uz$.

$$\int_0^k E(Sz, Tx_{3n+1}, Uz, kt) \varphi(x) dt \geq \max \int_0^k \begin{array}{l} E(Az, Bx_{3n+1}, Cz, t), E(Sz, Az, Bz, t), E(Tz, Bz, Bz, t), \\ E(Uz, Cz, Cz, t) \end{array} \varphi(x) dt$$

$$\int_0^k F(Sz, Tx_{3n+1}, Uz, kt) \varphi(x) dt \leq \min \int_0^k \begin{array}{l} F(Az, Bx_{3n+1}, Cz, t), F(Sz, Az, Bz, t), F(Tz, Bz, Bz, t), \\ F(Uz, Cz, Cz, t) \end{array} \varphi(x) dt$$

Taking limit $n \rightarrow \infty$

$$\int_0^k E(Sz, z, Uz, kt) \varphi(x) dt \geq \max \int_0^k \begin{array}{l} E(Az, z, Cz, t), E(Sz, Az, Bz, t), 1, 1 \\ E(Sz, z, Uz, kt) \end{array} \varphi(x) dt$$

$$\int_0^{F(Sz, z, Uz, kt)} \varphi(x) dt \leq \min \int_0^{F(Az, z, Cz, t), F(Sz, Az, Bz, t), 1, 1} \varphi(x) dt$$

Thus we have $Sz=z=Uz$. Hence $Az=Sz=Cz=Uz=z$

$$\begin{aligned} \int_0^{E(Sx_{3n}, Tz, Uz, kt)} \varphi(x) dt \\ \geq \max \int_0^{E(Ax_{3n}, Bz, Cz, t), E(Sx_{3n}, Ax_{3n}, Bz, t), E(Tx_{3n}, Bx_{3n}, Bz, t), E(Ux_{3n}, Cx_{3n}, Cz, t)} \varphi(x) dt \end{aligned}$$

$$\begin{aligned} \int_0^{F(Sx_{3n}, Tz, Uz, kt)} \varphi(x) dt \\ \leq \min \int_0^{F(Ax_{3n}, Bz, Cz, t), F(Sx_{3n}, Ax_{3n}, Bz, t), F(Tx_{3n}, Bx_{3n}, Bz, t), F(Ux_{3n}, Cx_{3n}, Cz, t)} \varphi(x) dt \end{aligned}$$

Taking limit $n \rightarrow \infty$

$$\int_0^{E(z, Tz, Uz, kt)} \varphi(x) dt \geq \max \int_0^{E(z, Bz, Cz, t), E(z, z, Bz, t), E(z, z, Bz, t), E(z, z, z, t)} \varphi(x) dt$$

$$\int_0^{F(z, Tz, Uz, kt)} \varphi(x) dt \leq \min \int_0^{F(z, Bz, Cz, t), F(z, z, Bz, t), F(z, z, Bz, t), F(z, z, z, t)} \varphi(x) dt$$

$$\int_0^{E(z, Tz, Uz, kt)} \varphi(x) dt \geq \max \int_0^{E(z, Bz, Cz, t), E(z, z, Bz, t), E(z, z, Bz, t), 1} \varphi(x) dt$$

$$\int_0^{F(z, Tz, Uz, kt)} \varphi(x) dt \leq \min \int_0^{F(z, Bz, Cz, t), F(z, z, Bz, t), F(z, z, Bz, t), 1} \varphi(x) dt$$

Hence $Tz=Uz=z$. Thus $Az=Sz=Bz=Tz=Cz=Uz=z$ thus z is a common fixed point of $A, B, C, S, T & U$. To prove uniqueness, let z^1 be another common fixed point of $A, B, C, S, T & U$.

$$\int_0^k E(Sz, Tz^1, Uz, kt) \varphi(x) dt$$

$$\geq \max \int_0^k E(Az, Bz^1, Cz, t), E(Sz, Az, Bz, t), E(Tz, Bz, Bz, t), E(Uz, Cz, Cz, t) \varphi(x) dt$$

$$\int_0^k F(Sz, Tz^1, Uz, kt) \varphi(x) dt$$

$$\leq \min \int_0^k F(Az, Bz^1, Cz, t), F(Sz, Az, Bz, t), F(Tz, Bz, Bz, t), F(Uz, Cz, Cz, t) \varphi(x) dt$$

$$\int_0^k E(z, z^1, z, kt) \varphi(x) dt \geq \max \int_0^k E(z, z^1, z, t), E(z, z, z, t), E(z, z, z, t), E(z, z, z, t) \varphi(x) dt$$

$$\int_0^k F(z, z^1, z, kt) \varphi(x) dt \leq \min \int_0^k F(z, z^1, z, t), F(z, z, z, t), F(z, z, z, t), F(z, z, z, t) \varphi(x) dt$$

$$E(z, z^1, z, Kt) \geq 1.$$

$$F(z, z^1, z, Kt) \geq 1.$$

Hence $z^1=z$

IV. Conclusion

Fixed point theory has many applications in several branches of science such as game theory, nonlinear programming, economics, theory of differential equations, etc. In this paper we define generalized E- fuzzy metric space and discuss some of the properties of generalized fuzzy metric space. Also we prove common fixed point theorems in generalized fuzzy metric space. Our results presented in this paper generalize and improve some known results in fuzzy metric space.

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M.L.L. Phanikanth. " Common Fixed Point Theorems In Intuitionistic Fuzzy Metric Space Using General Contractive Condition Of Integral Type.." IOSR Journal of Mathematics (IOSR-JM) 15.5 (2019): 35-45.