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An Improved Krill herd Algorithm of Nonlinear Mixed Integer Programming Problems

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Abstract: *In order to solve the Nonlinear Mixed Integer Programming Problem, an improved Krill Herd algorithm(ANRKH) based on natural selection and random disturbance is proposed in this paper. Firstly, the induced weight and foraging weight based on the time-varying nonlinear decreasing strategy is proposed to improve the induced movement and foraging movement of krill. Secondly, random disturbance factor is added into the process of generating the new generation of krill herd population. And the evolution of the survival of the fittest in natural selection mechanism enhances the quality of the individuals in the krill herd population. Those steps can effectively balance the exploration and development ability of KH. Finally through the experiments of 16 test functions, this proposed algorithm compares with other algorithms. Experimental results demonstrate that this proposed algorithm has a significant advantage.*

Key words: *nonlinear mixed integer programming, krill herd algorithm, time-varying weight, random disturbance, natural selection*

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I. Introduction

Nonlinear Mixed Integer Programming (NMIP) problem is classical mathematical programming problems with continuous real variables, discrete integer variables, the objective function and constraints are nonlinear functions. Nowadays, it has been widely used in natural science, management, financial optimization, computer, military and so on. There are two main methods to solve the NMIP problem at present: the traditional deterministic method and the new intelligent algorithm. Traditional deterministic methods include branch and bound method [1], extended cut plane method [2], and outer approximation method [3], and so on. But these methods cannot solve the complex NMIP problem, and when the dimension of the function is increased, the amount of computation will increase rapidly, and sometimes cannot search for the best results. Therefore, researchers have been widely improved new intelligent algorithms to solve the NMIP problem in recent years, which mainly including: particle swarm optimization, genetic algorithm, evolutionary algorithm, simulated annealing algorithm, and so on. These algorithms show their strong vitality and flexibility in solving the NMIP problem, but these algorithms are easily to fall into local optimum. The improved of the hybrid algorithm with differential evolution was showed in the reference [4], and it was successful used in the power system unit commitment; a new chaotic search strategy was proposed in the reference [5], and it was introduced into the standard particle swarm algorithm, to enhance the global and local search ability of the algorithm, and to improve the convergence and robustness of the algorithm, which makes the algorithm easier to find the optimal solution of the NMIP problem.

In order to solve the nonlinear mixed integer programming problem, an improved krill optimization algorithm (ANRKH) , which based on natural selection and random disturbance, is proposed in this paper. In each iteration of the krill herd algorithm [6], the time-varying nonlinear decreasing strategy is used for induced weight and foraging weight of krill during the individual movement, eliminating the adverse effects of fixed induction weight and foraging weight during each iteration; And the second, random disturbance is added into the process to generate a new generation of individual krill, making a new generation of individual krill carry more exchange information; Finally, through natural selection, the position of krill individuals with bad target function value was substituted by the position of individual krill with good target function value, and the quality of the new generation krill was improved. So as to enhance the algorithm's global search and local exploration capabilities. The effectiveness of the algorithm is illustrated by experimental simulation in the course of the study.

1 Krill Herd Algorithm

Krill Herd (KH) algorithm is a novel type of meta-heuristic method for solving optimization problems. This method is inspired by the herding of Antarctic krill swarms when searching for food and communicating with each other in nature[7]. For each krill, its position in search space is influenced by three components described as follow:

i. movement induced by other krill (induced movement);
ii. foraging motion;
iii. random diffusion.

KH adopted the following Lagrangian model as below:

$$\frac{dx_i}{dt} = N_i + F_i + D_i \quad (1)$$

Where N_i is the movement induced by other krill, F_i is the foraging motion, and D_i is the random diffusion. $i = 1,2,3,\cdots, NP$, and NP is population size.

For the component (i), (ii) and (iii) and krill i, it can be given as:

$$N_i = N^{\max}\alpha_i + w_n N_i^{old} \quad (2)$$

$$F_i = V_f \beta_i + w_f F_i^{old} \quad (3)$$

$$D_i = D^{\max}(1 - \frac{t}{t_{\max}})\delta \quad (4)$$

And $N^{\max}$ is the maximum induced speed and set to 0.01, V_f is the maximum foraging speed， $D_{\max}$ is the maximum diffusion rate. α_i is motion direction, primarily decided by a target effect α_i^{target}, and α_i^{local} is a local effect, and β_i is the feeding direction, δ is the diffusion direction. w_n is the inertia weight, called induced weight, between 0 and 1, w_f is the foraging weight. N_i^{old} is the last motion. t is the number of iterations, and $t_{\max}$ is the maximum number of iterations.

Based on the above-mentioned three components, the position of a krill from t to $t + \Delta t$ is given as below:

$$x_i(t + \Delta t) = x_i(t) + (\frac{dx_i}{dt})(\Delta t) \quad (5)$$

$$\Delta t = Ct\sum_{j=1}^{NV}(UB_j - LB_j) \quad (6)$$

Among them, Δt is the scaling factor of the velocity vector, C_t is the step-size scaling factor, and is the constant between [0,2]. Where NV is the total number of variables, and LB_j and UB_j are lower and upper bounds of the j th variables ($j = 1,2,3,\cdots, NV$), respectively. Therefore, the absolute of their subtraction shows the search space. It is empirically found that C_t is a constant number between [0,2]. It is also obvious that low values of C_t let the krill individuals to search the space carefully.

In order to improve the performance of the algorithm, the genetic operator (crossover or mutation) is performed in the algorithm, and the crossover operator is more effective by testing. The crossover operator formula is as follows:

$$x_{i,m} = \begin{cases} x_{r,m} & rand_{i,m} < Cr \\ x_{i,m} & else \end{cases} \quad (7)$$

$$x_{i,m} = \begin{cases} x_{gbest,m} + \mu(x_{p,m} - x_{q,m}) & rand_{i,m} < Mu \\ x_{i,m} & else \end{cases} \quad (8)$$

where C_r is crossover probability, M_u is a genetic operator, $rand$ Is a uniformly distributed random number in the scope of $[0,1]$, u as a constant in the scope of $[0,1]$, $r \in \{1,2,\cdots, i-1, i+1,\cdots, NP\}$.

II. ANRKH Algorithm

2.1 Time variant nonlinear decreasing strategy of foraging and induced weight

Assuming $\alpha_i = 0$ in Eq.(2) and $\beta_i = 0$ in Eq.(3) in KH, individual krill will always do the induced movement by $w_n N_i^{old}$ and foraging motion by $w_f F_i^{old}$ until to the boundary. From this we can know that larger w_n and w_f are conducive to jump out of local optimum, so that KH has the strong ability of convergence at this time. Smaller w_n and w_f are conducive to search in the local space carefully and KH has the advantage of local search at this time. After all, reasonable adjustment of w_n and w_f is the key to efficient search and avoid falling into the local optimum. Time variant nonlinear decreasing strategy of foraging and induced weight is proposed in this paper and expression is given below:

$$w_n = w_f = \frac{w_{\max} - w_{\min}}{t_{\max}} * (t_{\max} - t) + w_{\min} * rand \tag{9}$$

where t is the current number of iteration and $t_{\max}$ is the maximum number of iterations, $w_{\max}$ and $w_{\min}$ represent the maximum and the minimum value of w_n and w_f. This strategy makes w_n and w_f in the gradual decrease overall, the added $rand$ changes the monotone mode of linear decreasing, so that the algorithm can adapt well to the current search situation in the whole iteration process, thus more effective to adjust the algorithm's global search and local exploration capability.

2.2 Random Disturbance

In the KH algorithm, krill individuals were randomly distributed in various locations in the early stage. The location of the food was calculated from the current position of the individual krill. However, with the iterative process, the krill population location and food location tending to be homosexual, so that the exchange of information between individual krill populations, and the exchange of information between krill individuals and food locations, is becoming less and less important. So we add a random disturbance to the new generation of its new creation, and update the equation as follow:

$$x_i(t + \Delta t) = x_i(t) + \left(\frac{dx_i}{dt}\right) * (\Delta t) * rand \tag{10}$$

By the way of updating the random disturbance, the information contained in the individuals of the new generation krill population can be increased, so that the individuals which fall into the local optimum will jump out of the local optimal point and move toward the global optimal direction. The local exploration ability of the enhancement algorithm is improved, and the accuracy of the solution is improved.

2.3 Natural Selection

Here's the basic idea of natural selection:

After a new generation individual of krill herd, we'll assess the fitness value of the new generation, that is to sort the new generation of krill herd according to fitness value. Among the new generation krill herd, the last 1/K of those with poor fitness value is replaced by the first 1/K of those with good fitness value. On the basic of it, reserve the most optimal value of each individual's memory and increase the proportion of excellent particle in the new population so as to ensure that each individual has good performance, which also speed up the convergence of the algorithm. In order to maintain the diversity and overall of the new generation of krill population and prevent the individual of the old generation population falling into local optimum, K is set to 10 in this paper[8].

2.4 Population Initialization and Constraint Treatment

In order to ensure that the proportion of feasible solutions in the initial population is above 70%, the population initialization method and selection strategy [9] are adopted in this paper. It is very important to select the appropriate constraint processing method, when solving nonlinear mixed integer programming problems. Although there are many constraint processing mechanisms, we use the simplest Deb [10] constraint processing method here as follows:

i. If both solutions are feasible solutions, select the particles with good fitness value;
ii. If one solution is feasible and the other solution is infeasible, the feasible solution particle is selected;
iii.If both solutions are infeasible solutions, choose to violate the constraints of small particles.

III. Algorithmic Flow

The flow of ANRKH algorithm for nonlinear mixed integer programming problem is as follows:
Step1: Population initialization and parameter setting;
Step2: The population is randomly initialized at 70% of the feasible solutions;
Step3: Update the individual optimal value and the global optimal value, according to the calculated fitness value and the violation of the constraint function;
Step4: use Eq.(9), and to calculate w_n and w_f ;
Step5: Calculate the velocity component of individual movement of krill by using Eq.(2), Eq.(3) and Eq.(4) respectively;
Step6: Calculate the moving $\frac{dx_i}{dt}$ speed and position $x_i(t)$ of krill by Eq.(1) and Eq.(10) respectively;
Step7: According to the constraint processing mechanism, update the individual optimal position and the global optimum position of the particle in the population;
Step8: Natural selection of the population;
Step9: Determine whether the given termination conditions are met, If they are met, global optimal solution and global optimal value should be output, then the algorithm end; Otherwise, return to Step3 to continue the iterative search.

IV. Experiments and Results Analysis

In order to illustrate the feasibility and effectiveness of the ANRKH algorithm for solving nonlinear mixed integer programming problems, the ANRKH algorithm is tested on the test function of 16 nonlinear mixed integer programming problems with constraints (g01-g16). And compared with the three algorithms in the reference [10]. In the numerical experiments, the number of particles in the initial population is set to 100 and the maximum number of iterations is 1000. To make sure about the performance of the evaluation algorithm, each standard test function is run 100 times, compared both the accuracy of solution and the success rate of the solution.

Table 4.1 Results of ANRKH algorithm

Test function	Known optimal solution	Average value	Optimal value	Worst value
g01(min)	2.000000	2.002578	2.000000	2.151062
g02(min)	2.124	2.124507	2.124475	2.124563
g03(min)	1.07654	1.076542	1.076542	1.076542
g04(min)	-6961.81381	-6961.813881	-6961.813895	-6961.813711
g05(min)	-68	-68.000000	-68.000000	-68.000000
g06(min)	-6	-6.000000	-6.000000	-6.000000
g07(min)	99.244811	99.239565	99.239554	99.239615
g08(min)	3.557463	3.874695	3.785201	3.964213
g09(max)	32217.4	32217.426640	32217.426640	32217.426640
g10(max)	0.94347	0.952976	0.952976	0.952976
g11(min)	8	8.000000	8.000000	8.000000
g12(min)	14	14.009900	14.000000	15.000000
g13(min)	-42.632	-42.632210	-42.632210	-42.632210
g14(min)	0	9.990051e-012	3.378997e-013	6.399021e-011
g15(min)	807	807.4000000	807.000000	809.800000
g16(max)	1030361	1352439	1352439	1352439

Table 4.2 Comparison of the success rate of several algorithms

Test function	ANRKH(%)	MI-LXPM(%)	RST2ANU(%)	AXNUM(%)
g01(min)	99	86	45	87
g02(min)	100	82	60	63
g03(min)	100	50	08	42
g04(min)	93	88	03	86
g05(min)	100	100	78	92
g06(min)	100	100	100	100
g07(min)	100	55	0	42
g08(min)	0	40	22	07
g09(max)	100	100	100	100
g10(max)	100	89	97	39
g11(min)	100	100	100	93
g12(min)	99	70	36	19
g13(min)	100	97	99	95
g14(min)	100	100	100	100
g15(min)	77	93	24	11
g16(max)	100	100	100	100

From the results in Table 4.1, the ANRKH algorithm can find the optimal solutions for the standard test functions g01, g02, g03, g05, g06, g11, g12, g13, g14. For standard test function g07, g09, g10, the ANRKH algorithm presented here yields better results than the known optimal solution. For standard test functions g16, its optimal value is $f_{\max} = 1030361$ in reference [11], while the ANRKH algorithm finds the global optimal value of 1352439. It can be seen that the known optimal location is only a local extreme point. For standard test function g08, ANRKH algorithm does not find its optimal solution.

In order to better explain the feasibility and effectiveness of ANRKH algorithm in solving nonlinear mixed integer programming problems, the success rates in solving the test problem of the ANRKH algorithm, was compared with MI-LXPM algorithm, ST2ANU algorithm and AXNUM algorithm [10] in this paper. As can be seen from Table 4.2, the success rate of the ANRKH algorithm is 100% for most problems and is higher than the success rate of the other three algorithms; for problem g15, the success rate of the ANRKH algorithm is lower than that of the MI-LXPM algorithm, and higher than the other two algorithms. For the problem g08, the success rate of the ANRKH algorithm is 0, and the algorithm does not find its optimal solution.

As can be seen from Table 4.1 and Table 4.2, the ANRKH algorithm not only improves the solution precision, but also improves the success rate of the algorithm in solving most nonlinear mixed integer programming problems. It is feasible and effective to solve the nonlinear mixed integer programming problem.

V. Conclusion

In order to solve the problem of nonlinear mixed integer programming, an improved krill population algorithm (ANRKH) which based on natural selection and stochastic perturbation, is proposed in this paper. In each iteration of krill population algorithm, a nonlinear time-varying decreasing strategy is adopted to induced weight and foraging weight of krill individuals. Second, random distribution is added to the generation process of the new generation krill. Finally, the quality of the new generation krill is improved by natural selection, so as to enhance the global searching and local exploration ability of the algorithm.

At the last, after compared with the known three algorithms in 16 test functions, ANRKH has a strong feasibility and validity.

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