*IOSR Journal of Mathematics (IOSR-JM)*
*e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 5 Ser. II (sep – Oct 2019), PP 41-46*
*www.iosrjournals.org*

# On Sequences of diophantine 3-tuples generated through Pronic Numbers

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***Abstract:*** *This paper deals with the study of constructing sequences of diophantine triples $(a,b,c)$ based on two given pronic numbers such that the product of any two elements of the set added by a polynomial with integer coefficient is a perfect square.*

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Date of Submission: 11-09-2019 Date of Acceptance: 26-09-2019

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## I. Introduction

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m positive integers $\{a_1, a_2, a_3, .....,a_m\}$ is said to have the property $D(n), n \in Z-\{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuple with property $D(n)$.

Many Mathematicians considered the construction of different formulations of diophantine triples with the property $D(n)$ for any arbitrary integer n [1] and also, for any linear polynomials in n. In this context, one may refer [2-12] for an extensive review of various problems on diophantine triples.

Given two pronic numbers, this paper aims at constructing sequences of diophantine triples where the product of any two members of the triple with the polynomial with integer coefficients satisfies the required property.

## II. Method of analysis

**Sequence: 1**

Consider the Pronic numbers $Pr_n$ and $Pr_{2n}$ given by

$Pr_n = n(n+1)$ , $Pr_{2n} = 2n(2n+1)$

Let $a = 4Pr_n$, $b = Pr_{2n}$

It is observed that

$ab + n^2 = (4n^2 + 3n)^2$

Therefore, the pair $(a,b)$ represents diophantine 2-tuple with the property $D(n^2)$ .

Let $c_1$ be any non-zero polynomial in x such that

$$ac_1 + n^2 = p^2 \tag{1}$$

$$bc_1 + n^2 = q^2 \tag{2}$$

Eliminating $c_1$ between (1) and (2), we have

$$bp^2 - aq^2 = (b-a)n^2 \tag{3}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \tag{4}$$

in (3) and simplifying we get

$$X^2 = abT^2 + n^2$$

which is satisfied by $T = 1$ , $X = 4n^2 + 3n$

In view of (4) and (1), it is seen that

$$c_1 = 16n^2 + 12n$$

Note that $(a, b, c_1)$ represents diophantine 3-tuple with property $D(n^2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$c_2 = 36n^2 + 30n$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$c_3 = 64n^2 + 56n$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$c_4 = 100n^2 + 90n$

exhibits diophantine 3-tuple with property $D(n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_s = (4s^2 + 8s + 4)n^2 + (4s^2 + 6s + 2) \ , \ s = 1,2,3,...$$

Now, consider $(b, c_1)$ and employing the above procedure, it is seen that the triple $(b, c_1, c_2)$ where

$c_2 = 36n^2 + 24n$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking $(b, c_2)$ and employing the above procedure, it is seen that the triple $(b, c_2, c_3)$ where

$c_3 = 64n^2 + 40n$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking $(b, c_3)$ and employing the above procedure, it is seen that the triple $(b, c_3, c_4)$ where

$c_4 = 100n^2 + 60n$

exhibits diophantine 3-tuple with property $D(n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(b, c_s, c_{s+1})$ where

$$c_n = (2s + 2)^2 n^2 + (2s^2 + 6s + 4)n \ , \ s = 1,2,3,...$$

**Sequence: 2**

Consider the Pronic numbers $Pr_n$ and $Pr_{2n}$ given by

$Pr_n = n(n+1)$ , $Pr_{2n} = 2n(2n+1)$

Let $a = Pr_n$, $b = Pr_{2n}$

It is observed that

$ab + 2n^2 + 2n^3 = (2n^2 + 2n)^2$

Therefore, the pair $(a, b)$ represents diophantine 2-tuple with the property $D(2n^2 + 2n^3)$ .

Let $c_1$ be any non-zero polynomial in x such that

$$ac_1 + 2n^2 + 2n^3 = p^2 \tag{5}$$

$$bc_1 + 2n^2 + 2n^3 = q^2 \tag{6}$$

Eliminating $c_1$ between (5) and (6), we have

$$bp^2 - aq^2 = (b - a)(2n^2 + 2n^3) \tag{7}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \tag{8}$$

in (7) and simplifying we get

$$X^2 = abT^2 + 2n^2 + 2n^3$$

which is satisfied by $T = 1$ , $X = 2n^2 + 2n$

In view of (8) and (5), it is seen that

$$c_1 = 9n^2 + 7n$$

Note that $(a, b, c_1)$ represents diophantine 3-tuple with property $D(2n^2 + 2n^3)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$c_2 = 16n^2 + 14n$

exhibits diophantine 3-tuple with property $D(2n^2 + 2n^3)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$c_3 = 25n^2 + 23n$

exhibits diophantine 3-tuple with property $D(2n^2 + 2n^3)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_s = (s+2)^2 n^2 + (s^2 + 4s + 2) n, \; s = 1, 2, 3, ...$$

Now, consider $(b, c_1)$ and employing the above procedure, it is seen that the triple $(b, c_1, c_2)$ where

$c_2 = 25n^2 + 17n$

exhibits diophantine 3-tuple with property $D(2n^2 + 2n^3)$

Taking $(b, c_2)$ and employing the above procedure, it is seen that the triple $(b, c_2, c_3)$ where

$c_3 = 49n^2 + 31n$

exhibits diophantine 3-tuple with property $D(2n^2 + 2n^3)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(b, c_s, c_{s+1})$ where

$$c_s = (2s+1)^2 n^2 + (2s^2 + 4s + 1) n, \; s = 1, 2, 3, ...$$

**Sequence: 3**

Consider the Pronic numbers $Pr_n$ and $Pr_{4n}$ given by

$Pr_n = n(n+1)$ , $Pr_{4n} = 4n(4n+1)$

Let $a = 4Pr_n$, $b = Pr_{4n}$

It is observed that

$ab + 9n^2 = (8n^2 + 5n)^2$

Therefore, the pair $(a, b)$ represents diophantine 2-tuple with the property $D(9n^2)$ .

Let $c_1$ be any non-zero polynomial in x such that

$$ac_1 + 9n^2 = p^2 \tag{9}$$

$$bc_1 + 9n^2 = q^2 \tag{10}$$

Eliminating $c_1$ between (9) and (10), we have

$$bp^2 - aq^2 = (b-a)(9n^2) \tag{11}$$

Introducing the linear transformations

$$p = X + aT \; , \; q = X + bT \tag{12}$$

in (11) and simplifying we get

$$X^2 = abT^2 + 9n^2$$

which is satisfied by $T = 1$ , $X = 8n^2 + 5n$

In view of (12) and (9), it is seen that

$$c_1 = 36n^2 + 18n$$

Note that $(a, b, c_1)$ represents diophantine 3-tuple with property $D(9n^2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$c_2 = 64n^2 + 40n$

exhibits diophantine 3-tuple with property $D(9n^2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$c_3 = 100n^2 + 70n$

exhibits diophantine 3-tuple with property $D(9n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_s = (2s+4)^2 n^2 + (4s^2+10s+4)n \ , \ s = 1, 2, 3, ...$$

Now, consider $(b, c_1)$ and employing the above procedure, it is seen that the triple $(b, c_1, c_2)$ where

$c_2 = 100n^2 + 40n$

exhibits diophantine 3-tuple with property $D(9n^2)$

Taking $(b, c_2)$ and employing the above procedure, it is seen that the triple $(b, c_2, c_3)$ where

$c_3 = 196n^2 + 70n$

exhibits diophantine 3-tuple with property $D(9n^2)$

Taking $(b, c_3)$ and employing the above procedure, it is seen that the triple $(b, c_3, c_4)$ where

$c_4 = 324n^2 + 108n$

exhibits diophantine 3-tuple with property $D(9n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(b, c_s, c_{s+1})$ where

$$c_s = (4s+2)^2 n^2 + (4s^2+10s+4)n \ , \ s = 1, 2, 3, ...$$

**Sequence: 4**

Consider the Pronic numbers $Pr_{n-1}$ and $Pr_n$ given by

$Pr_{n-1} = n(n-1) \ , \ Pr_n = n(n+1)$

Let $a = Pr_{n-1} + Pr_n \ , \ b = 8 Pr_n$

It is observed that

$ab + 4n^2 = (4n^2 + 2n)^2$

Therefore, the pair $(a, b)$ represents diophantine 2-tuple with the property $D(4n^2)$ .

Let $c_1$ be any non-zero polynomial in x such that

$$ac_1 + 4n^2 = p^2 \qquad (13)$$

$$bc_1 + 4n^2 = q^2 \qquad (14)$$

Eliminating $c_1$ between (13) and (14), we have

$$bp^2 - aq^2 = (b-a)(4n^2) \qquad (15)$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \qquad (16)$$

in (15) and simplifying we get

$$X^2 = abT^2 + 4n^2$$

which is satisfied by $T = 1 \ , \ X = 4n^2 + 2n$

In view of (16) and (13), it is seen that

$$c_1 = 18n^2 + 12n$$

Note that $(a, b, c_1)$ represents diophantine 3-tuple with property $D(4n^2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$c_2 = 32n^2 + 16n$

exhibits diophantine 3-tuple with property $D(4n^2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$c_3 = 50n^2 + 20n$

exhibits diophantine 3-tuple with property $D(4n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_s = (2s^2+8s+8)n^2 + (4s+8)\, n \ , \ s = 1, 2, 3, ...$$

Now, consider $(b, c_1)$ and employing the above procedure, it is seen that the triple $(b, c_1, c_2)$ where

$c_2 = 50n^2 + 40n$

exhibits diophantine 3-tuple with property $D(4n^2)$

Taking $(b, c_2)$ and employing the above procedure, it is seen that the triple $(b, c_2, c_3)$ where

$c_3 = 98n^2 + 84n$

exhibits diophantine 3-tuple with property $D(4n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(b, c_s, c_{s+1})$ where

$$c_s = (8s^2 + 8s + 2)n^2 + (8s^2 + 4s)n \ , \ s = 1, 2, 3, ...$$

**Sequence: 5**

Consider the Pronic numbers $Pr_{n-2}$ and $Pr_n$ given by

$Pr_{n-2} = n^2 - 3n + 2 \ , \ Pr_n = n(n+1)$

Let $a = Pr_{n-2}$ , $b = Pr_n$

It is observed that

$ab + 1 = (n^2 - n - 1)^2$

Therefore, the pair $(a, b)$ represents diophantine 2-tuple with the property $D(1)$ .

Let $c_1$ be any non-zero polynomial in x such that

$$ac_1 + 1 = p^2 \tag{17}$$

$$bc_1 + 1 = q^2 \tag{18}$$

Eliminating $c_1$ between (17) and (18), we have

$$bp^2 - aq^2 = (b - a)(1) \tag{19}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \tag{20}$$

in (19) and simplifying we get

$$X^2 = abT^2 + 1$$

which is satisfied by $T = 1$ , $X = n^2 - n - 1$

In view of (20) and (17), it is seen that

$$c_0 = 4n^2 - 4n$$

Note that $(a, b, c_0)$ represents diophantine 3-tuple with property $D(1)$

Taking $(a, c_0)$ and employing the above procedure, it is seen that the triple $(a, c_0, c_1)$ where

$c_1 = 9n^2 - 15n + 4$

exhibits diophantine 3-tuple with property $D(1)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$c_2 = 16n^2 - 32n + 12$

exhibits diophantine 3-tuple with property $D(1)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$c_3 = 25n^2 - 55n + 24$

exhibits diophantine 3-tuple with property $D(1)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{s-1}, c_s)$ where

$$c_{s-1} = (s+1)^2 n^2 - (3s^2 + 2s - 1)n + 2(s^2 - s), \ s = 1, 2, 3, ...$$

Now, consider $(b, c_0)$ and employing the above procedure, it is seen that the triple $(b, c_0, c_1)$ where

$c_1 = 9n^2 - 3n - 2$

exhibits diophantine 3-tuple with property $D(1)$

Taking $(b, c_1)$ and employing the above procedure, it is seen that the triple $(b, c_1, c_2)$ where

$c_2 = 16n^2 - 4$

exhibits diophantine 3-tuple with property $D(1)$

Taking $(b, c_2)$ and employing the above procedure, it is seen that the triple $(b, c_2, c_3)$ where

$c_3 = 25n^2 + 5n - 6$

exhibits diophantine 3-tuple with property $D(1)$

Taking $(b, c_3)$ and employing the above procedure, it is seen that the triple $(b, c_3, c_4)$ where

$c_4 = 36n^2 + 12n - 8$

exhibits diophantine 3-tuple with property $D(1)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(b, c_{s-1}, c_s)$ where

$$c_{s-1} = (s+1)^2 n^2 + (s^2 - 2s - 3)n - 2(s-1),\ \ s = 1, 2, 3, \ldots$$

## III. Conclusion

To conclude one may search for the construction of diophantine 3-tuples based on other choices of number patterns.

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