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Almost $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces

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Abstract: *In this chapter we have introduced two types of $b^{\#}$ continuous mappings namely intuitionistic fuzzy almost $b^{\#}$ continuous mappings and intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.*
Keywords: *Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy almost $b^{\#}$ continuous mapping.*

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I. Introduction

Intuitionistic fuzzy set is introduced by Atanassov in 1986. Using the notion of intuitionistic fuzzy sets, Coker [1997] has constructed the basic concepts of intuitionistic fuzzy topological spaces. The concept of $b^{\#}$ closed sets and $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces are introduced by Gomathi and Jayanthi (2018). In this chapter we have introduced two types of $b^{\#}$ continuous mappings namely intuitionistic fuzzy almost $b^{\#}$ continuous mappings and intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.

II. Preliminaries

Definition 2.1: [Atanassov 1986] An intuitionistic fuzzy set(IFS) A is an object having the form $A= \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$,where the functions $\mu_A: X \rightarrow [0, 1]$ and $\nu_A: X \rightarrow [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively , and $0\leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X) , the set of all intuitionistic fuzzy sets in X. An IFS of A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2.2: [Atanassov 1986] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$. Then the following properties hold:

i. $A\subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
ii. $A=B$ if and only if $A \subseteq B$ and $A\supseteq B$,
iii. $A^c = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$,
iv. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle: x \in X\}$,
v. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x) , \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The IFSs $0_\sim= \langle x, 0, 1\rangle$ and $1_\sim= \langle x, 1, 0\rangle$ are respectively the empty set and whole set of X.

Definition 2.3: [Coker, 1997] An intuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

i. $0_\sim,1_\sim \in \tau$
ii. $G_1\cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
iii. $\cup G_i \in \tau$ for any $\{G_i : i \in J\}\subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. Then the complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4: [Coker, 1997] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$int(A) = \cup \{G/G$ is an IFOS in X and $G \subseteq A\}$,
$cl(A) = \cap \{K/K$ is an IFCS in X and $A \subseteq K\}$.

Definition 2.5: [Gurcay, Coker and Hayder, 1997] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

i) intuitionistic fuzzy semi closed set if $int(cl(A)) \subseteq A$
ii) intuitionistic fuzzy pre closed set if $cl(int(A)) \subseteq A$

iii) intuitionistic fuzzy regular closed set if cl(int(A)) = A
iv) intuitionistic fuzzy α closed set if cl(int(cl(A))) ⊆ A
v) intuitionistic fuzzy β closed set if int(cl(int(A))) ⊆ A

Definition 2.6: [Hanafy, 2009] An IFS A=⟨x, μ_A, ν_A⟩ in an IFTS (X, τ) is said to be an intuitionistic fuzzy γ closed set if int(cl(A)) ∩ cl(int(A)) ⊆ A.

Definition 2.7: [Gomathi and Jayanthi, 2018] An IFS A = ⟨x, μ_A, ν_A⟩ in an IFTS (X, τ) is said to be an intuitionistic fuzzy $b^{\#}$ closed set (IF$b^{\#}$CS) if int(cl(A)) ∩ cl(int(A)) = A.

Definition 2.8: [Coker, 1997] Let X and Y be two non empty sets and f: X →Y be a mapping. If B = {⟨y, μ_B(y), ν_B(y) / y ∈ Y⟩} is an IFS in Y, then the preimage of B under f is denoted and defined by f^{-1}(B)= { ⟨ x, $f^{-1}(\mu_B)$(x), $f^{-1}(\nu_B)$(x) / x ∈ X ⟩ }, Where f^{-1} (μ_B)(x) = μ_B(f(x)) for every x ∈ X.

Definition 2.9: [Gurcay, Coker and Hayder, 1997] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f said to be an intuitionistic fuzzy continuous mapping if f^{-1}(V) is an IFCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.10: [Gomathi and Jayanthi, 2018] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

i) intuitionistic fuzzy $b^{\#}$ continuous mapping if f^{-1} (V) is an IF$b^{\#}$CS in (X, τ) for every IFCS V of (Y, σ).
ii) intuitionistic fuzzy contra $b^{\#}$ continuous mapping if f^{-1} (V) is an IF$b^{\#}$CS in (X, τ) for every IFOS V of (Y, σ).

Definition 2.11: [Coker and Demirci, 1995] Intuitionistic fuzzy point (IFP), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by $p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & if\ x=p \\ (0,1) & otherwise \end{cases}$. An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if α ≤ μ_A and β ≥ ν_A.

Definition 2.12: [Thakur and Rekha Chaturvedi, 2008] Two IFSs A and B are said to be q-coincident (A $_q$ B) if and only if there exist an element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.13: [Seok Jong Lee and Eun Pyo Lee, 2000] Let $p_{(\alpha, \beta)}$ be an IFP in (X, τ). An IFS A of X is called an intuitionistic fuzzy neighbourhood of $p_{(\alpha, \beta)}$ if there exist an IFOS B in X such that $p_{(\alpha, \beta)}$ ∈ B ⊆ A.

III. Almost $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces

In this chapter we have introduced and investigated intuitionistic fuzzy almost $b^{\#}$ continuous mappings, intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings, intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space. We have provided many interesting results using these spaces.

Definition 3.1: If every IF$b^{\#}$CS is an IFCS in (X, τ), then the space is called as an intuitionistic fuzzy $T_{cb^{\#}}$ space (IF$T_{cb^{\#}}$ space).

Example 3.2: Let X={a, b} and then τ = {$0_\sim$, G_1, G_2 $1_\sim$} is an IFT on X, where, G_1 = ⟨x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)⟩ and G_2 = ⟨x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)⟩. Then (X, τ) is an IF $T_{cb^{\#}}$ space.

Definition 3.3: If every IFCS is an IF$b^{\#}$CS in (X, τ), then the space is called as an intuitionistic fuzzy $T_{b^{\#}}$ space (IF$T_{b^{\#}}$ space).

Example 3.4: Let X = {a, b} and then τ = {$0_\sim$, G_1, G_2 $1_\sim$} is an IFT on X, where, G_1 = ⟨x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)⟩ and G_2 = ⟨x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)⟩. Then (X, τ) is an IF$T_{b^{\#}}$ space.

Definition 3.5: A mapping f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy almost $b^{\#}$ continuous mapping if f^{-1}(V) is an IF$b^{\#}$CS in (X, τ) for every IFRCS V of (Y, σ).

Example 3.6: Let X = {a, b}, Y = {u, v}.Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3,G_4,$1_\sim$} are IFT on X and Y respectively, where,G_1 = ⟨x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)⟩, G_2 = ⟨x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)⟩,G_3 = ⟨y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)⟩ and G_4 = ⟨y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)⟩. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an intuitionistic fuzzy $b^{\#}$ continuous mapping.

Proposition 3.7: A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping if and only if the inverse image of each IFROS in Y is an IF$b^{\#}$OS in X.

Proof: Straight forward.

Proposition 3.8: If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then for each IFP $p_{(\alpha,\ \beta)}$ of X and each A ∈ σ such that f($p_{(\alpha,\ \beta)}$) ∈ A, there exists an IF$b^{\#}$OS B of X such that $p_{(\alpha,\ \beta)}$ ∈ B and f(B) ⊆ A.

Proof: Let $p_{(\alpha,\ \beta)}$ be an IFP of X and A ∈ σ such that f($p_{(\alpha,\ \beta)}$) ∈ A, then $p_{(\alpha,\ \beta)}$ ∈ f^{-1}(A). Put B = f^{-1}(A). Then by hypothesis, B is an IF$b^{\#}$OS in X such that $p_{(\alpha,\ \beta)}$ ∈ B and f(B) = f(f^{-1}(A)) ⊆ A.

Proposition 3.9: If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then for each IFP $p_{(\alpha,\ \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha,\ \beta)})_q A$, there exists an IFb#OS B of X such that $(p_{(\alpha,\ \beta)})_q$ B and f(B)⊆ A.

Proof: Let $p_{(\alpha,\ \beta)}$ be an IFP of X and A ∈ σ such that $f(p_{(\alpha,\ \beta)})_q A$. Then $p_{(\alpha,\ \beta)q}$ $f^{-1}(A)$ put B = $f^{-1}(A)$. Then by hypothesis, B is an IFb#OS in X such that $p_{(\alpha,\ \beta)q}B$ and f(B) = f($f^{-1}(A)$) ⊆ A.

Proposition 3.10: Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then f^{-1}(int(cl(B))) ⊆ cl(int(f^{-1}(cl(B))))∪ int(cl((f^{-1}(cl(B)))) for every IFS B in Y.

Proof: Let B be any IFS in Y. Then int(cl(B)) is an IFROS in Y. By hypothesis f^{-1}(int(cl(B))) is an IFb#OS in X. Since every IFb#OS is an IFbOS, f^{-1}(int(cl(B))) is an IFb OS in X. Therefore f^{-1}(int(cl(B))) ⊆ cl(int(f^{-1}(int(cl(B))))) ∪ int(cl(f^{-1}(int(cl(B))))) ⊆ cl(int(f^{-1}(cl(B)))) ∪ int(cl(f^{-1}(cl(B)))).

Proposition 3.11: Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then cl(int(f^{-1}(int(B)))) ∩ int(cl(f^{-1}(int(B)))) ⊆ f^{-1}(cl(int(B))) for each IFRCS B of Y.

Proof: Let B be any IFS in Y. Then cl(int(B)) is an IFRCS in Y. By hypothesis f^{-1}(cl(int(B))) is an IFb#CS in X. Since every IFb# CS is an IFbCS, f^{-1}(cl(int(B))) is an IFbCS in X. Therefore cl(int(f^{-1}(int(B))))∩int(cl(f^{-1}(int(B)))) ⊆ cl(int(f^{-1}(cl(int(B))))) ∩int(cl(f^{-1}(cl(int(B))))) ⊆ f^{-1}(cl(int(B))).

Proposition 3.12: Let f: (X, τ) → (Y, σ) be an mapping, where X is an IFT $_{cb^{\#}}$ space. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then (int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B))) ⊆ f^{-1}(cl(B) for every IFRCS B in Y.

Proof: Let B ⊆ Y be an IFRCS. By hypothesis, f^{-1}(B) is an IFb# CS in X. Since every IFb#CS is an IFCS in X as X is an IFT $_{cb^{\#}}$ space, f^{-1}(B) is an IFCS in X. Therefore cl(f^{-1}(B) = f^{-1}(B). Now (int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B))) ⊆ f^{-1}(B) ∪ (int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B))) ⊆ cl(f^{-1}(B) = f^{-1}(B) ⊆ f^{-1}(cl(B). Hence (int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B))) ⊆ f^{-1}(cl(B).

Proposition 3.13: Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an IFT $_{cb^{\#}}$ space, then for each IFP $p_{(\alpha,\ \beta)}$ in X and each IFROS A in Y such that f($p_{(\alpha,\ \beta)}$) ∈ A, int(f^{-1}(cl(A))) is an intuitionistic fuzzy neighbourhood of $p_{(\alpha,\ \beta)}$ in X.

Proof: Let $p_{(\alpha,\ \beta)}$ ∈ X and let A be an IFROS in Y such that f($p_{(\alpha,\ \beta)}$) ∈ A, $p_{(\alpha,\ \beta)}$ ∈ f^{-1}(A). By hypothesis f^{-1}(A) is an IFb# OS in X. Since X is an IFT $_{cb^{\#}}$ space, f^{-1}(A) is an IFOS in X. Now $p_{(\alpha,\ \beta)}$ ∈ f^{-1}(A) = int(f^{-1}(A)) ⊆ int(f^{-1}(cl(A))). Hence intf^{-1}(cl(A)) is an intuitionistic fuzzy neighbourhood of $p_{(\alpha,\ \beta)}$ in X.

Proposition 3.14: Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an IFT $_{cb^{\#}}$ space. Then cl(f^{-1}(A)) ⊆ f^{-1}(cl(A)) for every IFSOS A in Y.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and let A be an IFSOS in Y. Then A ⊆ cl(int(A)). Now, cl(A) ⊆ cl(cl(int(A))) ⊆ cl(int(cl(A))) ⊆ cl(cl(A)) ⊆ cl(A). Therefore cl(A) = cl(int(cl(A))). This implies cl(A) is an IFRCS in Y. By hypothesis, f^{-1}(cl(A)) is an IFb#CS in X. Since every IFb#CS is an IFCS in X as X is an IFT $_{cb^{\#}}$ space, f^{-1}(cl(A)) is an IFCS in X. Therefore cl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Now, cl(f^{-1}(A)) ⊆ cl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Thus cl(f^{-1}(A)) ⊆ f^{-1}(cl(A)).

Proposition 3.15: Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an IFT $_{cb^{\#}}$ space. Then f^{-1}(A) ⊆ int(f^{-1}(int(cl(A)))) for every IFPOS A in Y.

Proof: Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and let A be an IFPOS in Y. Then A ⊆ int(cl(A)) . Since int(cl(A)) is an IFROS in Y , by hypothesis, f^{-1} (int(cl(A))) is an IFb#OS in X. Since every IFb#OS is an IFOS in X as X is an IFT $_{cb^{\#}}$ space, f^{-1} (int(cl(A))) is an IFOS in X. Therefore f^{-1} (A) ⊆ f^{-1} (int(cl(A))) = int(f^{-1}(int(cl(A)))).

Proposition 3.16: Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then f^{-1}(int(B)) ⊆ int(f^{-1}(B)) for every IFS B in Y where X is an IFT $_{cb^{\#}}$ space.

Proof: Let f be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping. Let B be an IFROS in Y. By hypothesis f^{-1}(B) is an IFb#OS in X. Since every IFb# OS is an IFOS in X as X is an IFT $_{cb^{\#}}$ space, f^{-1} (B) is an IFOS in X. Therefore f^{-1}(int(B)) ⊆ f^{-1}(B) = int(f^{-1}(B)).

Proposition 3.17: Let f: (X, τ) → (Y, σ) be a mapping where X is an IFT $_{b^{\#}}$ space. If f^{-1}(int(B)) ⊆ int(f^{-1}(B)) for every IFS B in Y, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

Proof: Let B be an IFROS. By hypothesis, f^{-1}(int(B)) ⊆ int(f^{-1}(B)) . Since B is an IFROS, it is an IFOS in Y. Therefore int(B) = B. Hence f^{-1}(B) = f^{-1}(int(B)) ⊆ int(f^{-1}(B)) ⊆ f^{-1}(B). This implies f^{-1}(B) is an IFOS in X and hence $f^{-1}(B)$ is an $IFb^{\#}OS$ in X as X is an $IFT_{b^{\#}}$ space. Thus f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

Proposition 3.18: Let f: (X, τ) → (Y, σ) be a mapping where X is an $IFT_{b^{\#}}$ space. If $cl(f^{-1}(B)) \subseteq (f^{-1}(cl(B))$ for every IFS B in Y, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

Proof: Let B be an IFRCS. By hypothesis, $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$. Since B is an IFRCS, it is an IFCS in Y. Therefore cl(B) = B. Hence $f^{-1}(B) = f^{-1}(cl(B)) \supseteq cl(f^{-1}(B)) \supseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFCS in X and hence $f^{-1}(B)$ is an $IFb^{\#}$ CS in X as X is an $IFT_{b^{\#}}$ space. Thus f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

Definition 3.19: A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an $IFb^{\#}CS$ in (X, τ) for every IFROS V of (Y, σ).

Example 3.20: Let X = {a, b}, Y = {u, v}. Then τ = {0~, G_1, G_2 1~} and σ = {0~, G_3, G_4 1~} are IFTs on X and Y respectively, where, $G_1 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$, $G_3 = \langle y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle$ and $G_4 = \langle y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle$. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

Proposition 3.21: Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping. Then for every IFRCS A in Y and for every IFP $p_{(\alpha,\ \beta)} \in X$, if $f(p_{(\alpha,\ \beta)})\ _q$ A then $p_{(\alpha,\ \beta)\ q}$ $bint(f^{-1}(A))$.

Proof: Let f be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping. Let A ⊆ Y be an IFRCS and let $p_{(\alpha,\ \beta)} \in X$. Also let $f(p_{(\alpha,\ \beta)})\ _q$ A, then $p_{(\alpha,\ \beta)\ q}$ $f^{-1}(A)$. By hypothesis, $f^{-1}(A)$ is an $IFb^{\#}OS$ in X. Since every $IFb^{\#}$ OS is an IFbOS, $f^{-1}(A)$ is an IFbOS in X. Hence $bint(f^{-1}(A)) = f^{-1}(A)$ and $p_{(\alpha,\ \beta)\ q}bint(f^{-1}(A))$.

Proposition 3.22: If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping $f^{-1}(bcl(int(B))) \subseteq bint(f^{-1}(cl(int(B))))$ for every IFS B in Y.

Proof: Let B ⊆ Y be an IFS. Then cl(int(B)) is an IFRCS in Y. By hypothesis, $f^{-1}(cl(int(B)))$ is an $IFb^{\#}$ OS in X. Since every $IFb^{\#}$ OS is an IFbOS, $f^{-1}(cl(int(B)))$ is an IFb OS in X. Therefore $f^{-1}(bcl(int(B))) \subseteq f^{-1}(cl(int(B))) = bint(f^{-1}(cl(int(B)))$.

Proposition 3.23: If f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then for each IFP $p_{(\alpha,\ \beta)} \in X$ and for each IFRCS B containing $f(p_{(\alpha,\ \beta)})$, there exists an IFbOS A ⊆ X and $p_{(\alpha,\ \beta)} \in A$ such that $A \subseteq f^{-1}(B)$.

Proof: Let B be an IFRCS in Y. Let $p_{(\alpha,\ \beta)}$ be an IFP in X such that $f(p_{(\alpha,\ \beta)}) \in B$. Then $p_{(\alpha,\ \beta)} \in f^{-1}(f(p_{(\alpha,\ \beta)})) \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an $IFb^{\#}OS$ in X. Since every $IFb^{\#}OS$ is an IFb OS, $f^{-1}(B)$ is an IFbOS in X. Now, let $A = bint(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $A \subseteq f^{-1}(B)$.

Proposition 3.24: Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then $f^{-1}(cl(int(B))) \supseteq cl(int(f^{-1}(int(B)))) \cup int(cl((f^{-1}(int(B))))$ for every IFS B in Y.

Proof: Let B be any IFS in Y. Then cl(int(B)) is an IFRCS in Y. By hypothesis $f^{-1}(cl(int(B))$ is an $IFb^{\#}OS$ in X. Since every $IFb^{\#}OS$ is an IFbOS, $f^{-1}(cl(int(B))$ is an IFbOS in X. Therefore $f^{-1}(cl(int(B)) \supseteq cl(int(f^{-1}(cl(int(B)))) \cup int(cl(f^{-1}(cl(int(B))))) \supseteq cl(int(f^{-1}(int(B)))) \cup int(cl(f^{-1}(int(B))))$.

Proposition 3.25: Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then $cl(int(f^{-1}(cl(B)))) \cap int(cl(f^{-1}(cl(B)))) \subseteq f^{-1}(int(cl(B)))$ for each IFS B of Y.

Proof: Let B be any IFS in Y. Then int(cl(B)) is an IFROS in Y. By hypothesis $f^{-1}(int(cl(B)))$ is an $IFb^{\#}CS$ in X. Since every $IFb^{\#}CS$ is an IFbCS, $f^{-1}(int(cl(B)))$ is an IFbCS in X. Therefore $cl(int(f^{-1}(cl(B)))) \cap int(cl(f^{-1}(cl(B)))) \subseteq cl(int(f^{-1}(int(cl(B))))) \cap int(cl(f^{-1}(int(cl(B))))) \subseteq f^{-1}(int(cl(B)))$.

Proposition 3.26: If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an $IFT_{cb^{\#}}$ space, then the following conditions hold:

i) $cl(f^{-1}(B)) \subseteq f^{-1}(\mathrm{int}(cl(B)))$ for every IFROS in Y.

ii) $f^{-1}(cl(\mathrm{int}(B))) \subseteq \mathrm{int}(f^{-1}(B))$ for every IFRCS in Y.

Proof: (i) Let B ⊆ Y be an IFROS. By, hypothesis $f^{-1}(B)$ is an $IFb^{\#}CS$ in X. Since every $IFb^{\#}CS$ is an IFCS in X as X is an $IFT_{cb^{\#}}$ space, $f^{-1}(B)$ is an IFCS in X. This implies $cl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(\mathrm{int}(B)) \subseteq f^{-1}(\mathrm{int}(cl(B)))$.

(ii) Let B ⊆ Y be an IFRCS. By hypothesis, $f^{-1}(B)$ is an $IFb^{\#}OS$ in X. Since every $IFb^{\#}OS$ is an IFOS in X as X is an $IFT_{cb^{\#}}$ space, $f^{-1}(B)$ is an IFOS in X. This implies $\mathrm{int}(f^{-1}(B)) = f^{-1}(B) = f^{-1}(cl(B)) \supseteq f^{-1}cl(\mathrm{int}(B))$.

References

[1]. **Atanassov, K.,** Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, 1986, 87- 96.
[2]. **Coker, D.,** An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88, 1997, 81 - 89.
[3]. **Coker, D.** and **Demirci, M.,** On intuitionistic fuzzy points, Notes on Intuitionistic Fuzzy Sets, 1, 1995, 79-84.
[4]. **Gomathi, G.,** and **Jayanthi, D.,** Intuitionistic fuzzy $b^{\#}$ continuous mapping, Advances in Fuzzy Mathematics, 13, 2018, 39 - 47.
[5]. **Gomathi, G.,** and **Jayanthi, D.,** $b^{\#}$ Closed sets in Intuitionistic Fuzzy Topological Spaces, International Journal of Mathematical Trends and technology, 65, 2019, 22-26.
[6]. **Gurcay, H., Coker, D.** and **Hayder, Es, A.,** On fuzzy continuity in intuitionistic fuzzy topological spaces, The Journal of Fuzzy Mathematics, 5, 1997, 365-378.
[7]. **Hanafy, I. M.,** Intuitionistic fuzzy γ continuity, Canad, Math. Bull, 52, 2009, 1- 11.
[8]. **Seok Jong Lee.,** and **EunPyo Lee.,** The Category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc., 37, 2000, 63-76.
[9]. **Thakur, S. S.,** and **Dhavaseelan, R.,** Nowhere dense sets in intuitionistic fuzzy topological spaces, Proceedings of National Seminar on Recent Developments in Topology, 2015, 17-21.

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