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A Class of New Computational Methods for the Solutions of Telegram Variable Coefficients Parabolic Equations

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Abstract: *In this work, a class of new computational methods for solution of variable coefficients partial differential equation was developed at step numbers j = 3; resulting into a Trapezoidal rule spectral based computational scheme as reported in Lambert (1973). The accuracy, consistency, stability and convergence properties of these methods were determined. The methods were implemented on some sampled problems that involve both constant and, variable coefficients parabolic partial differential equations; and evaluated by comparing them with some existing difference methods. The results obtained are found to be more rapidly converging as the step lengths h and k approaches zeros. This work provides better alternative numerical solutions to a class of dynamical problems having time dependent boundary conditions. Higher ordered telegram parabolic partial differential equations with defined theoretical solutions to given boundary conditions can be solved directly using this method.*

Key Words*; Trigonometric Functions, Taylor Series Expansion, Time, Finite& Space Difference*

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I. Introduction

Exponential growth in speeds and memory of digital computers had been at the origin of an explosive development of numerical methods. Several areas of mathematics such as differential geometry have benefited from their interaction with Partial Differential Equations. Mathematical modeling of many physical system leads to partial differential equation in various fields of physics and engineering. Non - linear differential equation in engineering and applied mathematics have been a topic of intensive research for many years. Partial differential equation system is applied in the study of mechanical system and field of another science such as description of wave propagation [Aytac and Ibrahim, 2008], dynamical behavior of structures such as beam and plates under the action of moving load [Oni and Awodola, 2009]

In recent decades, there has been great development in the numerical analysis and exact method of solving partial differential equations. Many mathematical formulations of physical phenomena containing integro - differential equations arise in many fields like fluid dynamics, biological models and chemical kinetics. Integro-differential equations are usually difficult to solve analytically so it is required to obtain an efficient approximate solution.

Non-linear phenomena are fundamentally important in various fields of science and engineering. The nonlinear models of real-life problems are still difficult to solve either numerically or theoretically. There has recent been much attention devoted to the search for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical to nonlinear models. Discontinuous Gelakin and mixed finite element methods are applied to a variety of linear and non-linear problems including stokes' problem from fluid mechanics and non –linear elliptical equations of the Monge Ampere types.

In this work; it was noted that most of these cited researchers focused on the use of Orthogonal, Legendre or Chebyshev polynomials and developed their models using weighted residual methods but ignored the use of global spectral functions involving both sinusoidal and cosinusoidal identities as the basis function which could be adopted in the sequel of developing some spectral based computational methods for the solutions of second order parabolic partial differential equations. The methods developed in this work using global spectral functions as the bases function converges more rapidly when compared to the existing finite difference methods.

II. Research Methodology

In this work, a general second order linear non-homogenous partial differential equation of the form
$$a(x,t)U_{tt}+2b(x,t)U_{xt}+c(x,t)U_{xx}=d(x,t,u,U,U_x,U_t) \quad (1.1)$$

Is considered where a, b, c, d are constants, as either characteristics or canonical; especially for a second order equation in which the derivatives of second order are all linear; with coefficients only depending on the independent variables is modified into a general linear second order one space dimensional parabolic partial differential equation of the form (1.1) by assuming a = b = 0; in the above equation; but c does not.
$$c(x,t)U_{xx}=d(x,t,u,U,U_x,U_t) \quad (1.2)$$
$$where;\ U_t=U_t(x,t,u),$$
$$U_{xx}=U_{xx}(x,t,u),$$
$$with\ initial\ condition\ 0<x<L;\ \forall\, t\geq 0;\ c(x,0)=1$$
$$and\ boundary\ conditions\ ,\ U(0,t)=U(L,t)=0$$
$$the\ region\ xt-plane\ in\ which\ the\ solution\ is\ sought\ is\ described\ by\ inequality$$
$$0\leq x\leq L;\ and\ t\geq 0\ and,\ c(x,0)=f(x)\ \forall\ x\epsilon(q,r);\ q,r,L\ are\ set\ of\ integers,$$
$$c(x,t)\ is\ a\ variable\ coefficient.$$
[Cheney et. al (2004)Ancona(2002), Jain(1992), William et. al(1992)]

III. Development of the methods

Assuming the theoretical solution of equation (1.1) is in form of the basis spectra function
$$U_{(x_{m+i},t_{n+j})}=aCos(x_{m+i},t_{n+j})+bSin(x_{m+i},t_{n+j}),\ldots\quad i,j=0,1,2,3\ldots \quad (1.3)$$
and time (t) and distance (x) as independent variables.

By considering time (t) - as an independent variable and, assume distance(x) to be stable then the equation (1.3) becomes

At step j = 3; equation (1.3) becomes equation (1.4)
$$\text{At}\quad j=3;\qquad U_{(x_m,t_{n+3})}=aCos(x_m,t_{n+3})+bSin(x_m,t_{n+3}), \quad (1.4)$$
Simplifying further to obtain;
$$U_{(x_m,t_{n+3})}-U_{(x_m,t_{n+2})}=-2\left[\sin\left(x_m,\frac{k}{2}\right)\right]\left[a\sin\left(x_m,\frac{(2t_n+5k)}{2}\right)-bcos\left(x_n,\frac{2t_n+5k}{2}\right)\right] \quad (1.5)$$
Take the first derivative of equation (1.5) with respect to t at each collocated points to obtain
$$f_{(m,n+3)}+f_{(m,n+2)}=-2\left[\cos\left(x_m,\frac{k}{2}\right)\right]\left[a\sin\left(x_m,\frac{(2t_n+5k)}{2}\right)-bcos\left(x_n,\frac{2t_n+5k}{2}\right)\right] .$$
(1.6)

Divide equation (1.6) by equation (1.5) to obtain equation (1.7)
$$\frac{U_{m,(n+3)}-U_{m,(n+2)}}{f_{(m,n+3)}+f_{(m,n+2)}}=\frac{-2\left[\sin\left(x_m,\frac{k}{2}\right)\right]\left[a\sin\left(x_m,\frac{(2t_n+5k)}{2}\right)-bcos\left(x_n,\frac{2t_n+5k}{2}\right)\right]}{-2\left[\cos\left(x_m,\frac{k}{2}\right)\right]\left[a\sin\left(x_m,\frac{(2t_n+5k)}{2}\right)-bcos\left(x_n,\frac{2t_n+5k}{2}\right)\right]}=\tan\left(x_m,\frac{k}{2}\right)=tan\,\frac{k}{2}$$
(1.7)

Adopting Taylor series in equation (1.7) and simplify to obtain
$$U_{x,(n+3)}=U_{x,(n+2)}+\frac{k}{2}\left(f_{(m,n+3)}+f_{(m,n+2)}\right) \quad (1.8)$$
Simplifying (1.8) to have
$$U_{x,(n+3)}=U_{(m,n)}+\frac{k}{2}\left[f_{(m,n+3)}+2f_{(m,n+2)}+2f_{(m,n+1)}+f_{(m,n)}\right] \quad (1.9)$$
Equation (1.9) is similar to Trapezoidal Rule three – step method [Lambert, 1973]

By reconsidering equation (1.2) and theoretical solution (1.3) for the second derivative in terms of time-space independent variables
$$U''=f'(t,x,u)=f_t+ff_x \quad (1.10)$$
$$g_{n,t}=U''_{n,t}=-(aCosx_n,t_n+bSinx_n,t_n) \quad (1.11)$$
$$g_{(n+1),t}=U''_{(n+1),t}=-(aCosx_{n+1},t_n+bSinx_{n+1},t_n), \quad (1.12)$$
$$g_{(n+2),t}=U''_{(n+2),t}=-(aCosx_{n+2},t_n+bSinx_{n+2,}t_n), \quad (1.13)$$
Add equation (1.12) and equation (1.11) to have;

$$g_{(n+1),t} + g_{n,t} = -2\cos\left(\frac{h}{2}, t_n\right)\left[a\cos\left(\frac{2x_n+h}{2}, t_n\right) + b\sin\left(\frac{2x_n+h}{2}, t_n\right)\right]. \quad (1.14)$$

Add equation (1.13) and equation (1.14) to obtain;

$$g_{(n+2),t} + g_{(n+1),t} = -2\cos\left(\frac{h}{2}, t_n\right)\left[a\cos\left(\frac{2x_n+3h}{2}, t_n\right) + b\sin\left(\frac{2x_n+3h}{2}, t_n\right)\right] \quad (1.15)$$

Add equation (1.14) and equation (1.15) to obtain

$$g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t} = 4\cos^2\left(\frac{h}{2}, t_n\right)\left[a\cos((2x_n+2h), t_n) + b\sin((2x_n+2h), t_n)\right]$$

(1.16)

Simplifying to obtain

$$\frac{U_{(m,n+2)} - 2U_{(m,n+1)} + U_{(m,n)}}{g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t}} = \frac{4\sin^2\left(\frac{h}{2}, t_n\right)[a\cos 2h + b\sin 2h].}{4\cos^2\left(\frac{h}{2}, t_n\right)[a\cos((2x_n+2h), t_n) + b\sin((2x_n+2h), t_n)].}$$

$$= \tan^2\left(\frac{h}{2}, t_n\right) = \tan^2\frac{h}{2} \quad (1.17)$$

Adopting Taylor series in equation (1.17) and simplify to obtain

$$U_{(m,n+2)} - 2U_{(m,n+1)} + U_{(m,n)} = h^2\left(\frac{1}{2} + \frac{h^2}{24} + \frac{h^4}{240}\right)^2\left(g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t}\right) \quad (1.18)$$

For sufficiently small values of h, equation (1.18) becomes

$$U_{(m,n+2)} = 2U_{(m,n+1)} - U_{(m,n)} + \frac{h^2}{2^2}\left(g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t}\right) \quad (1.19)$$

By adopting the condition for equilibrium and quasi-equilibrium to obtain

$$U_{(m,n+2)} = U_{m,n} + k[f_{x,(n+1)} + f_{x,n}] + \frac{h^2}{4}\left(g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t}\right) \quad (1.20)$$

Put equation (1.20) in equation (1.9) to obtain spectral based computational three - steps scheme in equation (1.21)

$$U_{(m,n+3)} = U_{m,n} + \frac{k}{2}\left(f_{x(,n+3)} + f_{x,(n+2)} + 2[f_{x,(n+1)} + f_{x,n}]\right)$$
$$+ \frac{h^2}{4}\left(g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t}\right). \quad (1.21)$$

Write equation (1.11) for j = 3 as shown in equation (1.21)

$$g_{(n+3),t} = U''_{(n+3),t} = -\left(aCos x_{n+3}, t_n + bSin x_{n+3}, t_n\right)_{,} \quad (1.22)$$

Add equation (1.13) to equation (1.22) to obtain

$$g_{(n+3),t} + g_{(n+2),t} = -2\cos\left(\frac{h}{2}, t_n\right)\left[a\cos\left(\frac{2x_n+5h}{2}, t_n\right) + b\sin\left(\frac{2x_n+5h}{2}, t_n\right)\right] \quad (1.23)$$

Add equation (1.15) to equation (1.23) to obtain

$$g_{(n+3),t} + 2g_{(n+2),t} + g_{(n+1),t} =$$
$$4\cos^2\left(\frac{h}{2}, t_n\right)\left[a\cos((2x_n+2h), t_n) + b\sin((2x_n+2h), t_n)\right] \quad (1.24)$$

Simplifying to obtain

$$g_{(n+3),t} + 3g_{(n+2),t} + 3g_{(n+1),t} + g_{(n,,t)} = 8\cos^2\left(\frac{h}{2}, t_n\right)\begin{bmatrix} a\cos((2x_n+3h), t_n) \\ +b\sin((2x_n+3h), t_n) \end{bmatrix} \quad (1.25)$$

Expand equation (1.3) in x direction and hold t to be constant as follows

$$U_{(x_m, t_n)} = aCos(x_{m,} t_n) + bSin(x_m, t_n) \quad (1.26)$$

$$U_{(x_{m+1}, t_n)} = aCos(x_{m+1,} t_n) + bSin(x_{m+1}, t_n)$$

$$U_{(m+1,n)} - U_{(m,n)} = -2\sin\left[x_m, \frac{h}{2}\right]\left[a\sin\left[\frac{(2x_m+3h)}{2}, t_n\right] - bCos\left[\frac{2x_m+3h}{2}, t_n\right]\right] \quad (1.27)$$

$$U_{(m+2,n)} - U_{(m+1,n)} = -2\sin\left[x_m, \frac{h}{2}\right]\left[a\sin\left[\frac{(2x_m+5h)}{2}, t_n\right] - bCos\left[\frac{2x_m+5h}{2}, t_n\right]\right] \quad (1.28)$$

Subtract equation (1.27) from equation (1.28)

$$U_{(x_{m+3}, t_n)} - 2U_{(x_{m+2}, t_n)} + U_{(x_{m+1}, t_n)} = -4\sin^2\left(\frac{h}{2}, t_n\right)\begin{bmatrix} a\cos((2x_n+2h), t_n) \\ +b\sin((2x_n+2h), t_n) \end{bmatrix}$$

(1.29)

Subtract equation (1.18) from (1.29) to obtain equation (1.30)

$$U_{(x_{m+3},t_n)} - 3U_{(x_{m+2},t_n)} + 3U_{(x_{m+1},t_n)} - U_{(x_m,t_n)} = 8sin^2\left(\frac{h}{2},t_n\right)\begin{bmatrix} acos((2x_n+3h),t_n) \\ +bsin((2x_n+3h),t_n) \end{bmatrix}$$

(1.30)

Divide equation (1.30) by equation (1.25) and simplifying as follows to obtain equation (1.31)

$$\frac{U_{(x_{m+3},t_n)} - 3U_{(x_{m+2},t_n)} + 3U_{(x_{m+1},t_n)} - U_{(x_m,t_n)}}{g_{(n+3),t} + 3g_{(n+2),t} + 3g_{(n+1),t} + g_{(n,,t)}} = \frac{8sin^2\left(\frac{h}{2},t_n\right)\begin{bmatrix} acos((2x_n+3h),t_n) \\ +bsin((2x_n+3h),t_n) \end{bmatrix}}{8cos^2\left(\frac{h}{2},t_n\right)\begin{bmatrix} acos((2x_n+3h),t_n) \\ +bsin((2x_n+3h),t_n) \end{bmatrix}}$$

$$\frac{U_{(x_m,t_{n+3})} - 3U_{(x_m,t_{n+2})} + 3U_{(x_m,t_{n+1})} - U_{(x_m,t_n)}}{g_{(m+3),t} + 3g_{(m+2),t} + 3g_{(m+1),t} + g_{(m,,t)}} = \frac{8sin^3\left(\frac{h}{2},t_n\right)}{8cos^3\left(\frac{h}{2},t_n\right)} = tan^3\left(\left(\frac{h}{2},t_n\right)\right) \quad (1.31)$$

Adopting Taylors' series expansion and obtain

$$U_{(x_m,t_{n+3})} = \left(3U_{(x_m,t_{n+2})} - 3U_{(x_m,t_{n+1})} + U_{(x_m,t_n)}\right) + \frac{h^2}{8}\left(g_{(m+3),t} + 3g_{(m+2),t} + 3g_{(m+1),t} + g_{(m,,t)}\right)$$

(1.32)

Simplifying by substitutions to obtain

$$U_{(x_m,t_{n+3})} = 3\left\{U_{m,n} + k[f_{m,(n+1)} + f_{m,n}] + \frac{h^2}{4}\left(g_{(m+2),t} + 2g_{(m+1),t} + g_{m,t}\right)\right\}$$

$$-3\left\{\begin{matrix} U_{(m,n)} + \frac{k}{2}[f_{(m,n+1)} \\ +f_{(m,n)}] \end{matrix}\right\} + U_{m,n} + \frac{h^2}{8}\left(g_{(m+3),t} + 3g_{(m+2),t} + 3g_{(m+1),t} + g_{(m,,t)}\right) \quad (1.33)$$

$$U_{(x_m,t_{n+3})} = U_{m,n} + \frac{3k}{2}[f_{(m,n+1)} + f_{m,n}] + \frac{h^2}{8}\left(g_{(m+3,t)} + 5g_{(m+2,t),} + 7g_{(m+1,t)} + 3g_{(m,,t)}\right)$$

(1.34)

Equation (1.34) is a spectral based three – step computational method for solution of second order parabolic partial differential equations.

Schemes developed for solving telegram problems are:

$$(i)U_{(x_m,t_{n+3})} = U_{m,n} + \frac{k}{2}\left(f_{x(,n+3)} + f_{x,(n+2)} + 2[f_{x,(n+1)} + f_{x,n}]\right)$$

$$+\frac{h^2}{4}\left(g_{(m+2),t} + 2g_{(m+1),t} + g_{m,t}\right)$$

(ii)$U_{(x_m,t_{n+3})} = U_{m,n} + \frac{3k}{2}[f_{(x,n+1)} + f_{x,n}] + \frac{h^2}{8}\left(g_{(m+3),t} + 5g_{(m+2),t} + 7g_{(m+1),t} + 3g_{(m,,t)}\right)$

These methods are found to be accurate, consistent, stable and convergent.

The methods were evaluated as follows

Numerical Example ; Consider the fourth order parabolic partial differential equation of the form

$$U_{tt}(x,t) + (1+x)U_{xxxx}(x,t) = \left(x^4 + x^3 - \frac{6}{7!}x^7\right)cost, \qquad 0 < x < 1, \qquad t > 0$$

with the following initial and boundary conditions

$$U(x,0) = \frac{6}{7!}x^7;\ U_t(x,0) = 0;\ U(0,t) = 0;\ U(1,t) = \frac{6}{7!}cost,$$

$$U_{xx}(0,t) = 0;\ \ U_{xx}(1,t) = \frac{6}{20}cost$$

The theoretical solution to this problem is $U(x,t) = \left(\frac{6}{7!}x^7\right)cost,$

[Olayiwola et al. (2010), and Wazwaz (2001)]

Table 1: Results of **Scheme 1** on Numerical Example 2

time (t)	X	Exact Result [ER]	Scheme 1	Olayiwola (2010)	Absolute Error (Scheme 1)	Absolute Error (Olayiwola)
0.03	0.1	1.18994E-10	1.18994E-10	1.19011E-10	6.73600E-17	1.66976E-14
	0.2	1.52312E-08	1.52312E-08	1.52316E-08	2.33783E-14	3.51848E-13
	0.3	2.60240E-07	2.6024E-07	2.60242E-07	2.69310E-13	2.27091E-12
	0.4	1.94960E-06	1.9496E-06	1.94961E-06	3.40758E-12	1.00513E-11
	0.5	9.29641E-06	9.2964E-06	9.29645E-06	9.01229E-12	3.77047E-11
	0.6	3.33107E-05	3.33107E-05	3.33108E-05	1.44717E-11	1.25717E-10
	0.7	9.79967E-05	9.79966E-05	9.79971E-05	7.97296E-11	3.75281E-10
	0.8	2.49549E-04	0.000249548	2.49550E-04	6.36170E-10	1.00966E-09
	0.9	5.69145E-04	0.000569144	5.69147E-04	7.08982E-10	2.47453E-09
0.04	0.1	1.18952E-10	1.18952E-10	1.19015E-10	2.74700E-16	6.26517E-14
	0.2	1.52373E-08	1.52259E-08	1.52273E-08	1.14088E-11	1.00357E-11
	0.3	2.60240E-07	2.60149E-07	2.60158E-07	9.13752E-11	8.24071E-11
	0.4	1.94892E-06	1.94891E-06	1.94895E-06	5.93135E-12	3.55230E-11
	0.5	9.29316E-06	9.29315E-06	9.29328E-06	1.35392E-11	1.20295E-10
	0.6	3.32991E-05	3.3299E-05	3.32994E-05	7.60302E-11	3.66160E-10
	0.7	9.79624E-05	9.79623E-05	9.79634E-05	8.68386E-11	1.02186E-09
	0.8	2.49461E-04	0.000249461	2.49464E-04	7.86879E-13	2.62666E-09
	0.9	5.68946E-04	0.000568945	5.68946E-04	9.57641E-10	4.30396E-15
0.05	0.1	1.18899E-10	1.18899E-10	1.19066E-10	2.78380E-16	1.67215E-13
	0.2	1.52191E-08	1.5219E-08	1.52231E-08	6.36321E-14	4.00571E-12
	0.3	2.60032E-07	2.60032E-07	2.60057E-07	4.95807E-13	2.49677E-11
	0.4	1.94804E-06	1.94804E-06	1.94814E-06	3.34491E-12	9.78266E-11
	0.5	9.28897E-06	9.28896E-06	9.28928E-06	7.37310E-12	3.10965E-10
	0.6	3.32841E-05	3.3284E-05	3.32850E-05	6.74634E-11	8.84279E-10
	0.7	9.79183E-05	9.79182E-05	9.79206E-05	9.00973E-11	2.32708E-09
	0.8	2.49349E-04	0.000249349	2.49355E-04	3.08148E-10	5.72036E-09
	0.9	5.68689E-04	0.000568689	5.68703E-04	1.00331E-10	1.31777E-08

Figure 1: Graph of Exact Result, Scheme 1 and Olayiwola's Result against X at t = 0.03

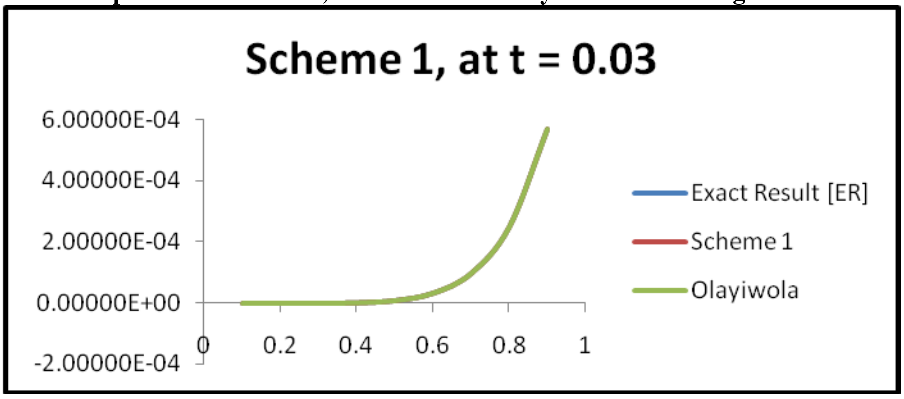


Figure 2: Graph of Exact Result, Scheme 1 and Olayiwola's Result against X at t = 0.04

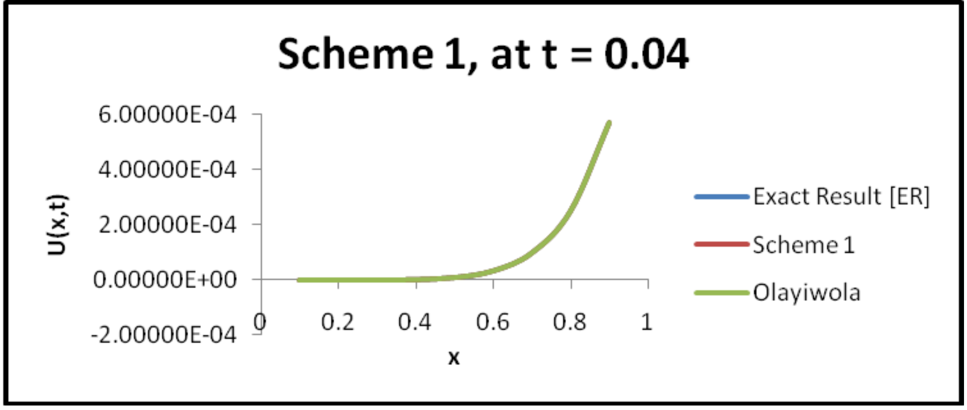


Table 2: Result of Scheme 2 on Numerical Problem

time (t)	X	Exact Result [ER]	Scheme 2	Olayiwola (2010)	Absolute Error (Scheme 2)
0.01	0.1	1.19042E-10	2.60100E-10	1.19042E-10	1.41058E-10
	0.2	1.52373E-08	1.65014E-08	1.52373E-08	1.26415E-09
	0.3	2.60344E-07	2.65093E-07	2.60344E-07	4.74871E-09
	0.4	1.95038E-06	1.96284E-06	1.95038E-06	1.24559E-08
	0.5	9.30013E-06	9.32692E-06	9.30013E-06	2.67914E-08
	0.6	3.33240E-05	3.33748E-05	3.33242E-05	5.07928E-08
	0.7	9.80359E-05	9.81239E-05	9.80359E-05	8.79897E-08
	0.8	2.49648E-04	2.49791E-04	2.49649E-04	1.43239E-07
	0.9	5.69373E-04	5.69593E-04	5.69373E-04	2.19863E-07
0.02	0.1	1.19024E-10	2.60061E-10	1.19026E-10	1.41037E-10
	0.2	1.52350E-08	1.64990E-08	1.52351E-08	1.26397E-09
	0.3	2.60305E-07	2.65053E-07	2.60305E-07	4.74794E-09
	0.4	1.95009E-06	1.96254E-06	1.95009E-06	1.24514E-08
	0.5	9.29874E-06	9.32552E-06	9.29874E-06	2.67822E-08
	0.6	3.33190E-05	3.33698E-05	3.33319E-05	5.07860E-08
	0.7	9.80212E-05	9.81092E-05	9.80213E-05	8.79696E-08
	0.8	2.49611E-04	2.49754E-04	2.49611E-04	1.42767E-07
	0.9	5.69287E-04	5.69507E-04	5.69288E-04	2.20415E-07
0.03	0.1	1.18994E-10	2.59996E-10	1.19011E-10	1.41002E-10
	0.2	1.52312E-08	1.64948E-08	1.52316E-08	1.26365E-09
	0.3	2.60240E-07	2.64987E-07	2.60242E-07	4.74667E-09
	0.4	1.94960E-06	1.96205E-06	1.94961E-06	1.24507E-08
	0.5	9.29641E-06	9.32319E-06	9.29645E-06	2.67805E-08
	0.6	3.33107E-05	3.33614E-05	3.33108E-05	5.07423E-08
	0.7	9.79967E-05	9.80846E-05	9.79971E-05	8.79386E-08
	0.8	2.49549E-04	2.49691E-04	2.49550E-04	1.42319E-07
	0.9	5.69145E-04	5.69365E-04	5.69147E-04	2.20017E-07
0.04	0.1	1.18952E-10	2.59905E-10	1.19015E-10	1.40953E-10
	0.2	1.52373E-08	1.64891E-08	1.52273E-08	1.25177E-09
	0.3	2.60240E-07	2.64894E-07	2.60158E-07	4.65390E-09
	0.4	1.94892E-06	1.96136E-06	1.94895E-06	1.24438E-08
	0.5	9.29316E-06	9.31993E-06	9.29328E-06	2.67665E-08
	0.6	3.32991E-05	3.33498E-05	3.32994E-05	5.06625E-08
	0.7	9.79624E-05	9.80503E-05	9.79634E-05	8.78992E-08

	0.8	2.49461E-04	2.49604E-04	2.49464E-04	1.42902E-07
	0.9	5.68946E-04	5.69166E-04	5.68946E-04	2.19682E-07

Figure 3 Graph of Exact Result, Scheme 2 and Olayiwola's Result against X at t = 0.01

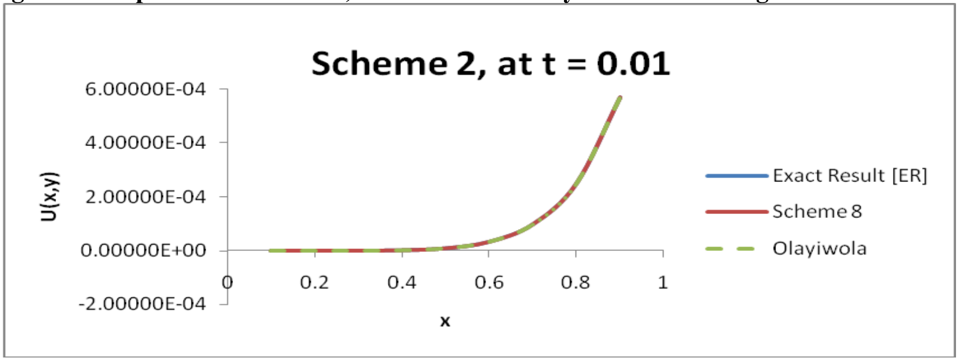


Figure 4: Graph of Exact Result, Scheme 2 and Olayiwola's Result against X at t = 0.02

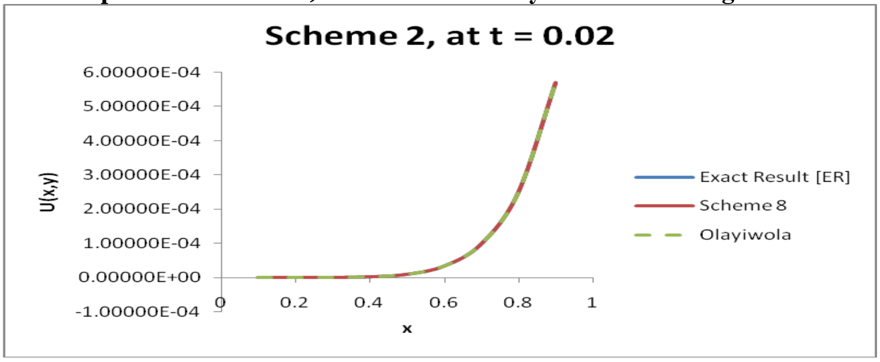


Figure 5: Graph of Exact Result, Scheme 2 and Olayiwola's Result against X at t = 0.03

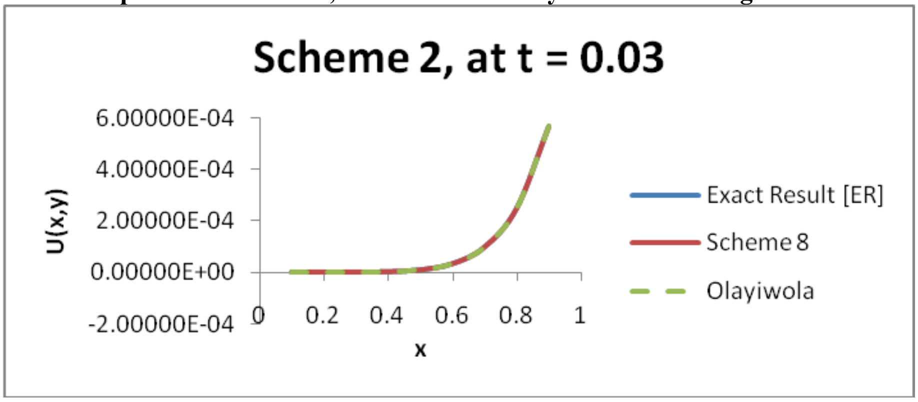
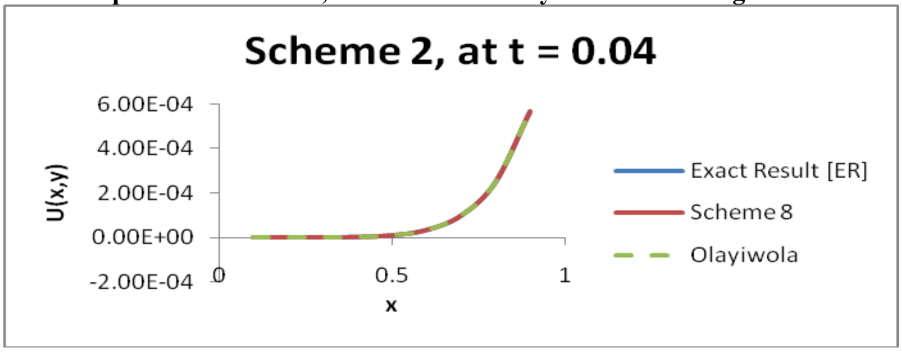


Figure 5: Graph of Exact Result, Scheme 2 and Olayiwola's Result against X at t = 0.04



IV. Conclusion

The results obtained are found to be more rapidly converging as the step lengths h and k approaches zeros. Degree of accuracy of **Scheme 1** on table 1 is higher and more rapidly converging at some of the selected points compared to Olayiwola's results and; the method in **Scheme 1** compete favorably with the existing method as shown on figures 1 & 2.

Table 2 shows the degree of accuracy of Exact Result, **Scheme 2** and Olayiwola's Result collocated at t = 0.01, 0.02, 0.03, & 0.04. They are perfectly correlated, indicating a better approximation and consistency. **Schemes 1 & 2** are direct methods of solution which requires no reduction of order. Figures 3, 4, 5, 6 of Exact Result, **Scheme 2** and Olayiwola's Result against x at t = 0.01, 0.02, 0.03, & 0.04 overlaps one another indicates perfect correlation, better approximation and consistency of the newly proposed methods.

Recommendation

- This work provides better alternative numerical solutions to a class of dynamical problems having time dependent boundary conditions.
- Higher ordered telegram parabolic partial differential equations with defined theoretical solutions to given boundary conditions can be solved directly using this method.

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