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On Statistical Convergence of Double Sequence of Functions in 2n-Normed Spaces

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Abstract: *In this paper, we shall introduce the concept of statistical convergence and statistical Cauchy of double sequence of functions in 2n-normed spaces. We shall also investigate some properties and establish relationships between these concept in 2n-normed spaces.*
Keywords and Phrases: *Statistical convergence, Statistical Cauchy, Double Sequence, 2n-normed Spaces.*

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I. Introduction

Throughout the paper, N denotes the set of all positive integers R the set of all real numbers. The concept of convergence of sequence of real numbers has been extended to statistical convergence independently by Fast (1951) and Schoenberg (1959). Gökhan et al. (2012) introduced the concept of point wise statistical convergence and statistical Cauchy sequence of real-valued function. Balcerzak et al. (2007) studied statistical convergence and ideal convergence for sequence of functions. Gezer and Karakes (2005) investigated I-pointwise and uniform convergence and I^*- Point-wise and uniform convergence of functions of sequence and then they examined the relation between them. Gökhan et al. (2007) introduced the notion of point-wise and uniform statistical convergence of double sequences of real-valued function.

Dűndar and Altay (2015) and (2016) introduced and studied the concepts of point wise and uniformly I-convergence of point wise and I^*-convergence of double sequences of function and investigated some properties about them. Furthermore, Dűndar (2015) investigated some results of I_2-convergence of double sequence of functions. The concept of 2-normed spaces was initially introduced by Gähler (1963) and (1964) in the 1960's. Gűrdal and Pehlivan (2009) studied statistical convergence, statistical Cauchy sequence and investigated some properties of statistical convergence in 2-normed spaces. Sharma and Kumar (2008) introduced statistical convergence, statistical Cauchy sequence, statistical limit points and statistical cluster points in probabilistic 2-normed spaces. Savas and Gűrdal (2016), dealt with I-convergence of sequences of functions in random 2-normed-spaces and introduced the concepts of ideal uniform convergence and ideal point wise convergence in topology induced by random 2-normed spaces.

Sarabadan and Talebi (2011) presented various kinds of statistical convergence and I-convergence for sequences of functions with values in 2-normed spaces and also defined the notion of I-equi-statistically convergence and study I-equi-statistically convergence of sequences of function. Sahiner et al. (2007) and Gűrdal (2006) studied I-convergence in 2-normed spaces. Gűrdal and Acik (2008) investigated I-Cauchy and I^*-Cauchy sequence in 2-normed spaces. Brono and Ali (2016) introduced and studied the concept of λ_2-statistical convergence in 2n-normed spaces. They also defined λ_2-statistically convergent double sequences in 2n-normed linear spaces X.

In this paper, we shall introduce the concept of statistical convergence and statistical Cauchy convergence of double sequence of functions in 2n-normed spaces and establish some relationship between these two concepts.

II. Definitions, Notations And Preliminaries

Now, we recall the concepts of density, statistical convergence, 2n-normed spaces and some of fundamental definitions and notations, which would be needed in sequel.

Definition 2.1 (Brono and Ali [2016]) : A double sequence x_{jk} in $2n$ −normed spaces $(X, \|.\dots.\|_2)$ is said to be convergent to $l \in X$ with respect to 2n-norm space if for each $\varepsilon > 0$, there exists a positive integer n_0 such that $\left\|(x_{jk} - L),,.z_{i2},\dots,z_{in-1})\right\|_2 < \varepsilon$ for all $j, k \geq n_0$ and for every $z_{i1}, z_{i2}, \dots, z_{in-1} \in X$.

Definition 2.2 (Brono and Ali [2016]): A double sequence (x_{jk}) in 2n- normed spaces $(X,\|.,\dots,.\|)$ is called a Cauchy sequence with respect to the $2n$ −normed if for each $\mathcal{E} > 0$, there exists a positive integer $n_0 = n_0(\mathcal{E})$

such that $\|x_{jk} - x_{rs}, z_{i2}, \dots, z_{in-1}\| < \epsilon$ for all i, j, r, s, $\geq n_0$ and for every $z_{i1}, z_{i2}, \dots, z_{in-1} \in X$. If every Cauchy sequence in X converges to somel $\in$ X, then X is said to be complete with respect to the $2n$ $-$norm. We shall call any complete $2n$ $-$normed spaces a $2n$ $-$Banach Space.

Definition 2.3 (Brono and Ali [2016]): Let n be non-negative integer and X be a real vector space of dimension $d \geq 2n$ (d may be infinite). A real valued function$\|.\dots.\|$ from X^{2n} into R satisfying the following conditions

(i) $\|x_{i1}.,\dots, x_{in}\|_D = 0$ if and only if $x_{i1}, x_{i2}, \dots, x_{in}$: $i = 1,2,\dots,n$ are linearly independent.

(ii) $\|x_{i1},.\dots, x_{in}\|_D$, $i = 1,2,\dots,n$ is invariant under permutation

(iii) $\|\alpha x_{i1}, x_{i2}, \dots, x_{in}\|_D = |\alpha|\|x_{i1}, x_{i2}, \dots, x_{in}\|_D, i = 1,2,\dots,n$

(iv) $\|x + \overline{x_1}, x_{i2}, \dots, x_{in}\|_D \leq \|x, x_{i2}, \dots, x_{in}\| + \|\overline{x_1}, x_{i2}, \dots, x_{in}\|_D, i = 1,2,\dots,n$

Let us call this function a 2n-norm on Xand the pair $(X, \|.,\dots,.\|_D)$ a 2n-normed spaced

Definition 2.4 (Brono and Ali [2016]): A double sequence $(x_{jk})_{j,k\in N}$ is said to be point-wise statistically convergence to f, if for every $\epsilon > 0$

$$\lim_{jk\to\infty,\infty} \frac{1}{jk} \left|\{(j,k) \in N \times N: \|x_{jk} - x(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|\}\right| = 0$$

III. Main Results

In this paper, we study the concepts of convergence statically convergence and statistical Cauchy double sequence of functions and explore some properties and relationship between these concepts in 2n-norm spaces.

Throughout this paper, we let X and Y be two 2n normed spaces, $(f_{mn})_{m,n\in N}$ and $(g_{mn})_{m,n\in N}$ be two double sequence of functions and f,g be two functions from X to Y

Definition 3.1: The sequence $(f_{mn})_{m,n\in N}$ is said to be (pointwise) statistical convergence to f, if for every $\varepsilon \geq 0$,

$$\lim_{m,n\to\infty} \frac{1}{mn} \left| \{(m,n) \in N \times N: , \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \varepsilon \} \right| = 0$$

for each x$\in$ X and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in$Y, it means that for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$,

$$\|f_{m,n}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| < \epsilon \text{ a.a. } m,n$$

In this case, we write

$$St_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$$

REMARK 3.1: $(f_{mn})_{m,n\in N}$ is any double sequence of functions and f is any function from X to Y, then set

$\{(m,n) \in N \times N: \|f_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \epsilon\}$

For $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y, = 0$,

Since if $z = 0$ ($\vec{0}$ vector),$\|f_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = 0$.

So the above set is empty

THEOREM 3.1: If for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$,

$St_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$ and

$St_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$

then $\|f_{nm}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$, (i.e., $f = g$), for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in$Y.

Proof: Assume $f \neq g$ then $f - g \neq 0$ so there exists a z_{i1}, $z_{i2}, \dots, z_{in-1} \in Y$ such that f, g and z_{i1},$z_{i2}, \dots, z_{in-1}$ are lineraly independent (such a z_{i1}, $z_{i2}, \dots, z_{in-1}$ exist since $d \geq 2$). Therefore for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$.

$\|f(x) - g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = 2\epsilon$ with $\epsilon > o$

Now, for each $x \in X$ and each non zero, $z_{i1}, z_{i2}, \dots, z_{in-1} \in$Y. we get $2\epsilon\|f(x) - g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = (f(x) - f_{mn}(x) + (f_{mn}(x) - g(x),)z_{i1}, z_{i2}, \dots, z_{in-1}\| \leq \|f_{mn}(x) - g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| + \|f_{nm}(x) - g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| + \|f_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$ and so

$\{(m,n) \in N \times N: \|f_{mn}(x) - g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| < \epsilon\} \subseteq \{(m,n) \in N \times N: \| f_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1} \| \geq \epsilon\}$

But for each $x \in X$ and each nonzero, $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$.

$\delta_2\{(m,n) \in N \times N: \|f_{mn}(x) - g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| < \epsilon\}) = 0$, then contradicting the fact that

$$St_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$$

THEOREM 3.2: If $(f_{mn})_{m,n\in N}$ is a convergent double sequence of functions such that $f_{mn} = g_{mn}$ a.a.m, n then $(f_{mn})_{m,n\in N}$ is statistically convergent.

Proof: Suppose that for each x∈ X *and each non zero* $z_{i1}, z_{i2}, \dots, z_{in-1}$ ∈Y,
$\delta_2(\{(m,n) \in N \times N: f_{mn}(x) \neq g_{mn}(x)\}) = 0$ and
$\lim_{mn\to\infty} \|g_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$,
then for every ∈ >0
$\{(m,n) \in N \times N: \|\|f_{nm}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \epsilon\} \subseteq \{(m,n) \in N \times N: \|g_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \epsilon\} \cup \{ ((m,n) \in N \times N: f_{mn}(x) \neq g_{mn}(x)\}$

Therefore,
$\delta_2(\{(m,n) \in N \times N: \|\|f_{nm}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \epsilon\}) \leq$
$\delta_2(\{(m,n) \in N \times N: \|g_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \epsilon\}) +$
$\delta_2(\{(m,n) \in N \times N: f_{mn}(x) \neq g_{mn}(x)\})$-------- (1)
Since $\lim_{m,n\to\infty} \|g_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$,

for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$.
The set $\{(m,n \in N \times N: \| g_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1} \| \geq \epsilon\}$ contains finite number of integers and so $\delta_2(\{(m,n) \in N \times N: \| g_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1} \| \geq \epsilon\}) = 0$
Using inequality (1) we get for every $\epsilon > 0$
$\delta_2(\{(m,n) \in N \times N: \| g_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1} \| \geq \epsilon\}) = 0$
for each x∈ X and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1}$ ∈Y
$St_2 - \lim_{m,n\to\infty} \|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$

THEOREM 3.3:
Let α ∈ R, *if for each* $x \in X$ *and each non zero* $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$
$St_2 - lim_{m,n\to\infty}\|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$ and

$$St_2 - lim_{m,n\to\infty}\|g_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$$

Then
i. $St_2 - lim_{m,n\to\infty}\|f_{mn}(x)+g_{nm}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \|f(x) + g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$ and
ii. $St_2 - lim_{m,n\to\infty}\|\alpha f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|\alpha f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$

PROOF (1): Suppose that
$St_2 - lim_{m,n\to\infty}\|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$, and

$$St_2 - lim_{m,n\to\infty}\|g_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$$

for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$
Then $\delta_2(K_1) = 0$ *and* $\delta_2(K_2) = 0$ where
$K_1 = K_1(\epsilon, z_{i1}, z_{i2}, \dots, z_{in-1}) : \{(m,n) \in N\times N: \|f_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \frac{\epsilon}{2}\}$ and
$K_2 = K_2(\epsilon, z_{i1}, z_{i2}, \dots, z_{in-1}) : \{(m,n) \in N \times N: \|g_{mn}(x)\ g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \frac{\epsilon}{2}\}$
for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$. Let
K = $K(\epsilon, z_{i1}, z_{i2}, \dots, z_{in-1}) = \{(m,n) \in N \times N: \|(f_{mn}(x) + g_{mn}(x)) - (f(x) + g(x)), z_{i1}, z_{i2}, \dots, z_{in-1}\| \geq \epsilon\}$
To prove that
$\delta_2(K) = 0$, it suffices to show that $K \subset K_1 \cup K_2$. *Let* $m_0, n_0 \in K$ then, for each $x \in X$ and each nonzero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$
$\left\|(f_{m_0 n_o}(x) + g_{m_0 n_o}(x) - (f(x) + g(x), . z_{i1}, z_{i2}, \dots, z_{in-1})\right\| \geq \epsilon$ − − − −(2)
Suppose to the contrary, that $m_0 n_o \notin K_1$ *and* $m_0 n_o \notin k_2$. *if* $m_0 n_o \notin K_1$ *and* $m_0 n_o \notin K_2$ *then, for each* $x \in X$ *and each non zero* $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y \left\|f_{m_0 n_o}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\right\| < \frac{\epsilon}{2}\}$ and $\left\|g_{m_0 n_o}(x) -\right.$ gx,zi1, zi2,…,zin−1<∈2}

Then, we get
$\left\|f_{m_0 n_o}(x) + g_{m_0 n_o}(x) - (f(x) + g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\right\| \leq$

$$\left\|f_{m_0 n_o}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\right\| + \left\|g_{m_0 n_o}(x) - g(x), z_{i1}, z_{i2}, \dots, z_{in-1}\right\| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

For each $x \in X$ *and each non zero* $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$ which contradicts (2). Hence $m_{0,} n_o \in K_1 \cup K_2$ and so $K \subset K_1 \cup K_2$
ii. Let $\alpha \in R$ $(\alpha = 0)$ *and for each* $x \in X$ *and each non zero* $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$

St$_2-\ lim_{m,n\to\infty}\|f_{mn}(x),\ z_{i1}, z_{i2}, \ldots, z_{in-1}\|$
Then, we get $\delta_2(\{(m,n)\in N\ \times N: \|f_{mn}(x),\ z_{i1}, z_{i2}, \ldots, z_{in-1}\| \geq \frac{\epsilon}{\alpha}\}) = 0$
Therefore, for each $x \in X$ and each non zero $z_{i1}, z_{i2}, \ldots, z_{in-1} \in$Y, $we\ have$
$\{(m,n) \in N\ \times N: \|\alpha f_{mn}(x) - \alpha f(x), z_{i1}, z_{i2}, \ldots, z_{in-1}\| \geq \epsilon\}=$
$\{: \{(m,n)\ \in N\ \times\ N: |\alpha| \|f_{mn}(x) -\ f(x), z_{i1}, z_{i2}, \ldots, z_{in-1}\| \geq \epsilon\}$
$= \{(m,n)\ \in N\ \times N: \|f_{mn}(x),\ z_{i1}, z_{i2}, \ldots, z_{in-1}\| \geq \frac{\epsilon}{\alpha}\}$

Hence, the right hand side of the above equality equals 0. Therefore, each $x \in X$ and each non zero $z_{i1}, z_{i2}, \ldots, z_{in-1} \in Y, we\ have$
St$_2-\ lim_{m,n\to\infty}\|\alpha f_{mn}(x),\ z_{i1}, z_{i2}, \ldots, z_{in-1}\| = \|\alpha f(x), z_{i1}, z_{i2}, \ldots, z_{in-1}\|$,
Now, we give the concept of statistical Cauchy double sequence and investigate relationship between statistical Cauchy double sequence and statistical convergent of double sequence in 2n – normed spaces.

DEFINITION 3.2
The double sequence of (f_{mn}) is said to be statistically Cauchy if for every
$\epsilon > 0\ and\ each\ non\ zero\ z_{i1}, z_{i2}, \ldots, z_{in-1} \in Y,$ there exists a number
$K = K\ (\boldsymbol{\epsilon}, z_{i1}, z_{i2}, \ldots, z_{in-1})$ such that
$\delta_2\{(m,n)\in N\ \times\ N: \|f_{mn}(x) - f_{jk}(x), z_{i1}, z_{i2}, \ldots, z_{in-1}\|\epsilon \geq\}$=0
For each $x\ \in X\ \ i.e., \|f_{mn}(x) - f_{jk}(x), z_{i1}, z_{i2}, \ldots, z_{in-1}\| < \epsilon\ a.a.m,n$

THEOREM 3.4
Let be statistically Cauchy double sequence of functions in a finite dimensional $2n$-normed spaces (X,‖.,‖$_2$) then, there exists a convergent double sequence of functions $\{g_{mn}\}_{m,n>1}$ (X, ‖.,.‖) such that $f_{mn}\ = g_{mn}$ for a.a. m, n

PROOF:
First note that $\{f_{mn}\}_{m,n>1}$ is statistically convergence of functions in (X, ‖.‖∞) choose anatural number K(1) such that the closed ball
B_u^1 $=Bu\ (f_{jk\,(1)}, 1)$ contains $f_{mn}(x)$ a. a. m, n and for each $x \in X$.
Then choose a natural number K(2) such that the closed ball B$_2$ = $Bu\ (f_{jk\,(1)}(x), {}^1/_2)$ contains $f_{mn}(x)$ a. a. $m\ , n$ and for each x $\in X$.

NOTE That B_u^2= $B_u^1\ \ \cap B_2$ also contains $f_{mn}(x)$ for a. a m, n and for each $x \in X$. Thus, by continuing of this process, we can obtain a sequence
$\{2B_u^{m,n}\}$ m,n ≥1 of nested closed balls such that diam $(B_u^{m,n})$≤ $\frac{1}{2mn}$.
Therefore,

$$\bigcap_{m,n=1,1}^{\infty,\infty} B_u^{m,n} = \{h\ (x)\}$$

Where h is a function from X to Y. Since each $B_u^{m,n}$ contain $f_{mn}(x)$ for a. a. m, n and for each, $x \in\ X$, we can choose a sequence of strictly increasing natural numbers $\{S_{mn}\}_{m,n>1}$ such that for each $x \in X$

$\frac{1}{mn}|\ \{(m,n)\in N \times N: f_{mn}(x) \notin B_u^{m,n}\}\ \ |<\frac{1}{mn}$ if $m, n > S_{mn}$
Put
R_{mn} $=\{(m,n)\in N\ \times\ N: m, n \geq, s_{mn}, f_{mn}(x) \notin B_u^{m,n}\}\ x \in X,$ for all $m, n > 1\ and\ R\ \ U_{mn=1,1}^{\infty,\infty}R_{mn}$. Now for each $x \in\ X$, defined the sequence of function $\{g_{mn}\}m, n \geq 1$ as follows

$$g_{mn}(x) = \begin{cases} h(x), if\ m, n \in R \\ f_{mn}(x), otherwise \end{cases}$$

NOTE: That, $lim_{mn\to\infty}g_{mn}(x) = h(x),\ \ for\ each\ x \in X.$
In fact for each $\epsilon > 0$ and for each $x \in\ X$, choose natural numbers m, n such that $\epsilon > \frac{1}{m,n} > 0$. Then, for each $m, n \geq S_{mn}$ and for each $x \in X$, $g_{mn}\ (x)$ = h (x) or $g_{mn}\ (x)$= $f_{mn}\ (x)\ B_u^{m,n}$ and so in each case.
$\|g_{m.n}\ (x)$ – $h(x)\|_\infty$≤ diam $(B_u^{m,n})$ ≤ ${}^1/_{2m-1,n-1}$
Since, for each $x \in X, \{(m,n) \in\ \ N \times N: g_{mn}(x)\} \neq f_{mn}\ \ (x)\} \subseteq \{(m,n) \in$ N × N}:$f_{mn}(x)B_u^{m,n}\}$ we have

$\frac{1}{mn}\left|\{(m,n)\in N\times N: g_{mn}(x)\neq f_{mn}(x)\}\right|\leq\frac{1}{mn}$
$\left|\{(m,n)\in N\times N: g_{mn}(x)\neq f_{mn}(x)\}\right|<\frac{1}{mn}$ and so

$\delta_2(\{(m,n)\in N\times N: g_{mn}(x)= f_{mn}(x)\})=0$
Thus, $g_{mn}(x)$=$f_{mn}(x)$ for a. a. m,n and for each $x \in X\ in\ (X,\|.\|_\infty)$. Suppose that $\{U_1,..,U_d\}$ is a basis for (X, ‖.,‖). Since, for each x $\in X$,
$\lim_{m,n\to\infty,} \| g_{mn}(x) - h(x)\|_\infty = 0$ and $\| g_{mn}(x) - h(x), u_i\|_\infty$ for all I≤ i≤ d, then we have
$\lim_{m,n\to\infty,} \|g_{mn}(x) - h(x),\quad z_{i1},\ zi2 \dots z_{in-1}\|\infty = 0$
For each $x \in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$

THEOREM 3.5
The double sequence $\{f_{mn}\}$ is statically convergent if and only if and only if $\{f_{mn}\}$ is a statistically Cauchy sequence functions.

PROOF:
Assume that f be function from X to Y and
St$_2$$-\lim_{m,n\to\infty}\|f_{mn}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| = \| f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\|$
For each $x \in X\ and\ eachnon\ zero z_{i1}, z_{i2}, \dots, z_{in-1} \in Y\ and\ \epsilon > O. Then,$ each $x\in X$and each non zero$z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$, we have

$$\|f_{mn}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\| < \frac{\epsilon}{2}\ a.a.m,n$$

If K= $K(\epsilon, z_{i1}, z_{i2}, \dots, z_{in-1})$ is choosen so that for each x $\in X$ and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y, \left\|f_{jk}(x) - f(x), z_{i1}, z_{i2}, \dots, z_{in-1}\right\| < \frac{\epsilon}{2}$
And so we have
$\left\|f_{mn}(x) - f_{jk}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\right\| \leq \|f_{mn}(x) - f(x), z\_i1, z\_i2, \dots, z\_(in-1)\|+$
$\left\|f(x) - f_{jk}(x), z_{i1}, z_{i2}, \dots, z_{in-1}\right\| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$ a.a. m,n
Hence , (f_{mn}) is statistically Cauchy sequence of functions
Now, assume that $\{f_{mn}\}$ is statistically Cauchy double sequence of functions
By theorem 4 there exist a convergent sequence $(g_{mn})_{m,n\in N}$ from X to Y such that $f_{mn} = g_{mn}$ for a.a. m, n. By therorem we
$St_2 - \lim_{m,n\to\infty}\|f_{mn}(x),\quad z_{i1}, z_{i2}, \dots, z_{in-1}\| = \|f(x),\quad z_{i1}, z_{i2}, \dots, z_{in-1}\|$
For each x $\in$ X and each non zero $z_{i1}, z_{i2}, \dots, z_{in-1} \in Y$

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