IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 15, Issue 4 Ser. II (Jul – Aug 2019), PP 01-09
www.iosrjournals.org

Application of Non Standard Finite Difference Scheme on the General Non Linear First Order Dynamic Equation

Getahun Tadesse Haile[*1], Srinivasan V[2]
[1] *Lecturer, Faculty of Natural and Computational Sciences, Department of Mathematics, Gambella University, Gambella, Ethiopia*
[2] *Associate Professor, Faculty of Natural and Computational Sciences, Department of Mathematics, Gambella University, Gambella, Ethiopia*
Corresponding Author: Getahun Tadesse Haile

Abstract: *In this article it is formulated and analyzed an unconditionally stable nonstandard finite difference method for the general nonlinear first order dynamic differential equation. This work manages with the relationship between a continuous dynamical system and numerical methods for its computer simulations, viewed as discrete dynamical systems. The term 'dynamic consistency' of a numerical scheme with the associated continuous system is usually loosely defined, meaning that the numerical solution replicate some of the properties of the solutions of the continuous system.*
Keywords: *Non linear first order dynamic equation; Non-standard finite difference method; Stability.*

Date of Submission: 13-07-2019 Date of acceptance: 29-07-2019

I. Introduction

The study of epidemics has a long history with a vast variety of models and explanations for the spread and cause of epidemic outbreaks. The investigation of epidemic models may help to understand how to eradicate and control the infectious diseases. Most of the epidemic models are continuous-time models and Specifically, these models are expressed in terms of first order nonlinear ordinary differential equations[1,2]. This is presumably in substantial part because the theory of ordinary differential equations is well developed and it can be easily applied to study continuous time systems derived from Mathematical modeling. However, data collected from transmitted diseases are usually discrete. Therefore, it is necessary and important to study discrete time epidemic models.

Following the success of Kermack and McKendrick[3] in modeling a malaria epidemic in the 1930s, many mathematical models have been developed and used to study the spread and control dynamics of numerous emerging and re-emerging human diseases[4,5] . These models, often non-linear and deterministic in nature, are generally formulated by subdividing the total population into a number of mutually exclusive compartments. The non-linear and multidimensional nature of these models[6] often necessitates the use of numerical integrators for their solutions. For many decades, standard numerical methods, such as the explicit forward Euler and higher-order Runge-Kutta methods, have been frequently and easily used to solve non-linear initial-value problems (IVPs) arising from the mathematical modeling of many real-life phenomena such as those arising in disease transmission and control. Detail analysis has been shown that the use of such schemes to solve real life models may lead to scheme-dependent instabilities and/or convergence to spurious solutions. In other words, the use of some standard numerical methods may lead to numerical solutions do not correspond to features of the solutions of the continuous model. Although the afore-mentioned drawbacks can, in general, be circumvented by using small step sizes, the computing costs associated with using such step-sizes in monitoring the long-term dynamics of population models can be substantial. Thus, there is a need to construct numerical schemes that allow the use of the largest possible step-sizes that are consistent with stability in the numerical simulations. One other well-known method of getting around the stability drawbacks associated with the use of standard explicit methods is to opt for implicit formulations for the solution of non-linear IVPs[7] . Unfortunately , although they are more robust than the explicit ones in terms of stability, it will be shown in this study that some of these standard methods are not free of spurious behavior. Both standard and nonstandard finite difference schemes are used find numerical solution of a nonlinear system of ordinary differential equations. To keep the methods fully explicit, the forward Euler and the central difference approximations from the standard one for the first derivative term is applied. The nonlocal approximations will be used to handle the nonlinear terms for nonstandard finite difference method. And also make use of denominator functions which are little more complex functions of the time step-size than the classical one. Furthermore, in this article the nonstandard finite difference method preserve some key properties of the corresponding continuous model at different time step and for numerous values of n is proved. It is noted that the proposed schemes are unconditionally stable.

In this study non-standard finite difference scheme is applied by applying Mickens Rules on Non linear first order Dynamic equation given by

$$\frac{dy}{dt} = y(1-y^n), \; y_0 = 0.5 \quad (1)$$

Where n is a positive integer. The analytical solution of equation (1) is given by :

$$y(t) = \left(1 + (2^n - 1)e^{n(1-t)}\right)^{\frac{-1}{n}} \quad (2)$$

This equation can be reduced to a well known equation for a variable n, such as when n = 1, equation(1) becomes a logistic differential equation which was discussed in [8]

Approximating nonlinear first order dynamic equation using Forward Euler Scheme at different step lengths with n = 2

This numerical procedure requires that the system (1) is transformed by introducing these substitutions:

$y(t) \to y_k$ and $\frac{dy}{dt} \to \frac{y_{k+1} - y_k}{\Delta t}$ where $\Delta t = h$ is the time step.

$$\frac{y_{k+1} - y_k}{h} = y_k(1 - {y^n}_k) \quad (3)$$

Solving for y_{k+1} we get $y_{k+1} = y_k + hy_k(1 - {y^2}_k)$ (4)

The numerical solutions of equation(1) with Forward Euler method at different step lengths and also the analytical solution has been displayed graphically in the following figures.

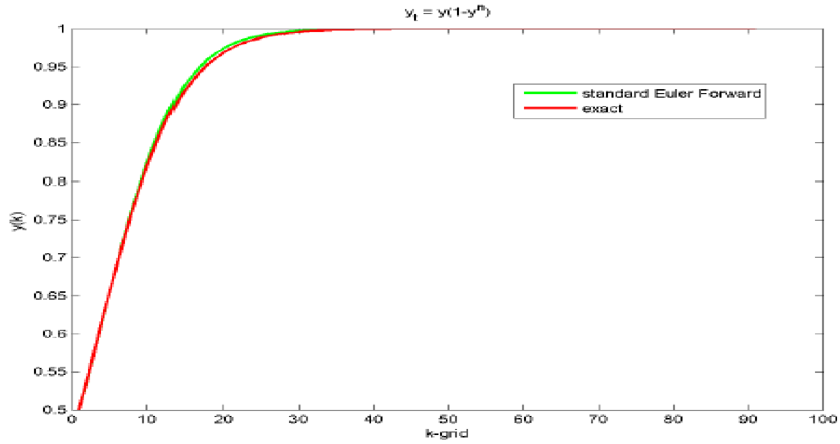

Figure:1 Approximation of Non linear dynamic equation using forward Euler method when h = 0.1 and n = 2.

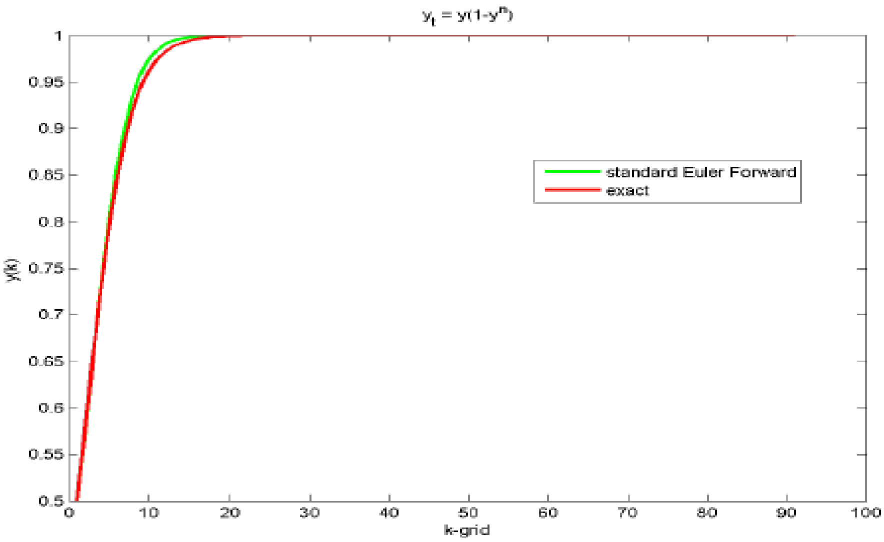

Figure:2 Approximation of Non linear dynamic equation using forward Euler method when h = 0.2 and n = 2

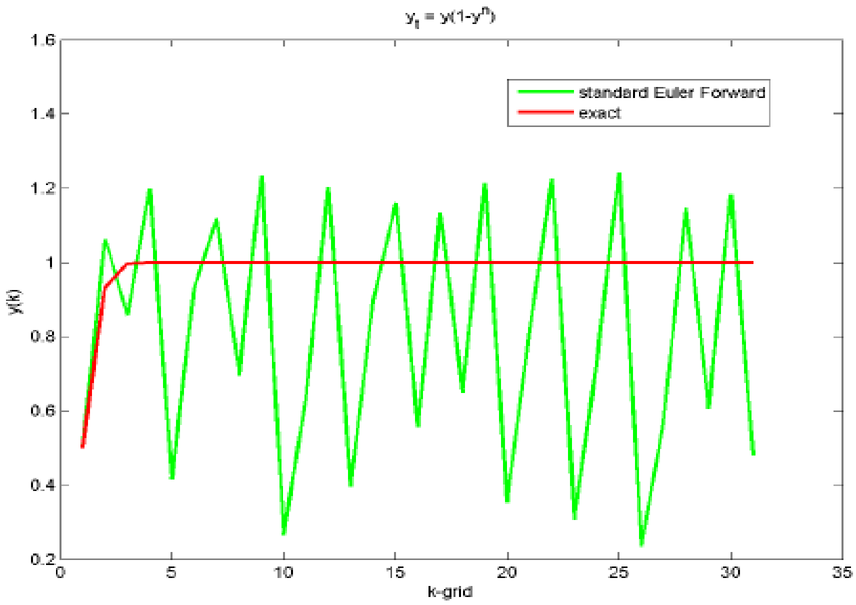

Figure:3. Approximation of Non linear dynamic equation using forward Euler method when h = 1.5 and n = 2

The numerical solution of nonlinear first order dynamic equation using Central Euler Scheme at different step lengths when n = 2

In this numerical procedure, on introducing the following substitutions to transform the system equation (1) i.e, $y(t) \to y_k$ and $\frac{dy(t)}{dt} \to \frac{y_{k+1} - y_{k-1}}{2h}$ Solving for y_{k+1} we get

$$y_{k+1} = y_{k-1} + 2hy_k(1 - y_k{}^2) \qquad (5)$$

The numerical solution of (5) with central difference method at different step lengths and also the analytical solution has been shown graphically in the following figures.

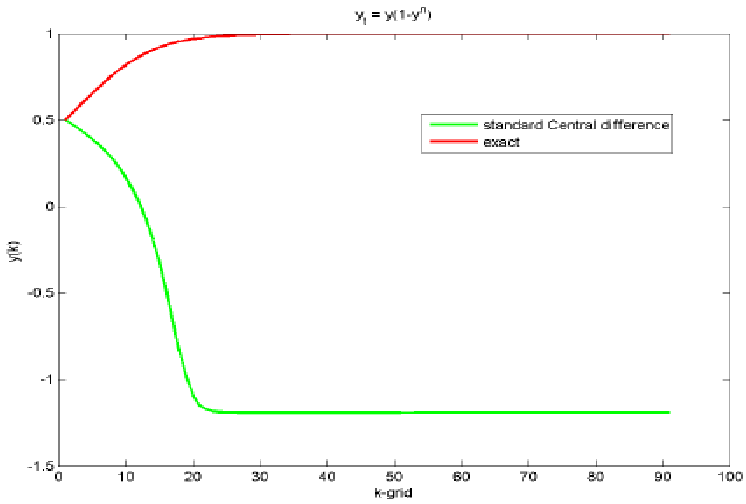

Figure:4. Approximation of Non linear dynamic equation using central difference method when h = 0.1 and n = 2

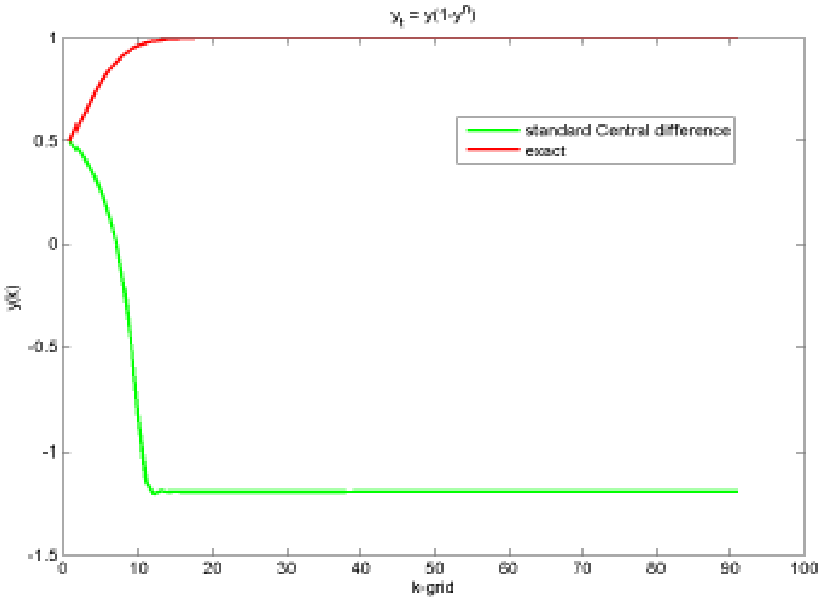

Figure:5. Approximation of Non linear dynamic equation using central difference method when h = 0.2 and n = 2

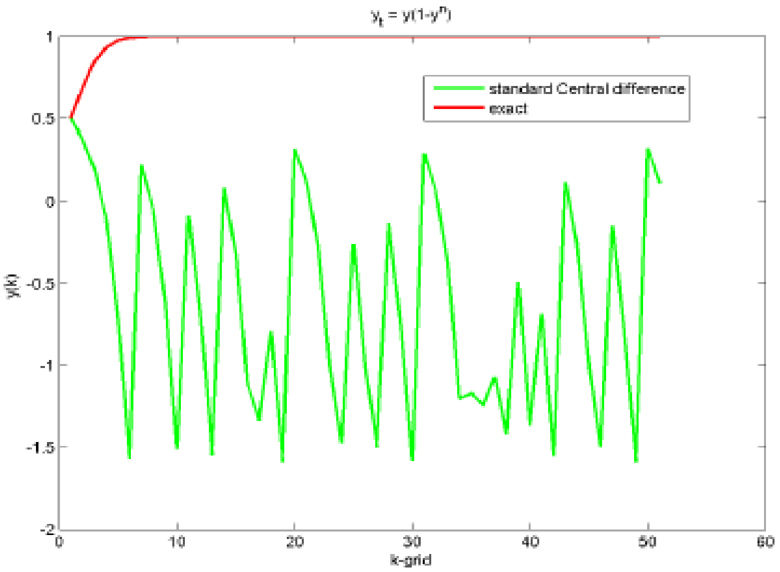

Figure:6. Approximation of Non linear dynamic equation using central difference method when h = 0.5 and n = 2

If increasing the non linearity of the dynamic equation for all step size, the central difference scheme exhibits numerical instability by converging to false steady states.

A numerical solution of nonlinear first order dynamic equation using Non Standard Finite difference method at different step lengths and different n values

Here Non standard finite difference method is applied on the general nonlinear first order dynamic equation given by eq(1) for different values of n, respecting Mickens rules. We apply the Non standard finite difference Micken's rules to transform the model in to the discrete scheme is applied.

$$\frac{\Delta y(t)}{\Phi(\Delta t)} = y(t) - (y(t))^3 \qquad (6)$$

where $\Delta y(t) = y_{k+1} - y_k$ and $\Phi(\Delta t)$ is a function of the time step $\Delta t = h$. It is called the denominator function in respect of Mickens II rule. Substituting all these in to equation (6) it becomes:

$$\frac{y_{k+1} - y_k}{\Phi(h)} = y_k(1 - y_{k+1} y_k) \qquad (7)$$

Now it is expected to derive a suitable denominator function $\Phi(h)$. One of the method in searching the denominator function for equation (7) is given by

$$\Phi(h, R^*) = \frac{\varphi(hR^*)}{R^*} \quad (8)$$

where $\varphi(z) = 1 - e^{-z}$ and

$$R^* = \mathrm{Max}|R_i|, i = 1, 2, 3, \ldots \quad (9)$$

Where R_i is :

$$R_i = \frac{df(y^*)}{dy}, f(y^*) = 0 \quad (10)$$

Let the cubic nonlinear part $f(y) = y(1 - y^2) = 0$ Clearly the dynamic differential equation for the choice of n = 2 produces three fixed points. These are $y_1{}^* = 0$, $y_2{}^* = 1$ and $y_3{}^* = -1$ by simple calculation defined in (10) , we can obtain $R_1 = 1, R_2 = R_3 = -2$

Therefore, $R^* = 2$ and this gives the suitable denominator function $\mathbf{\Phi}$ is :

$$\Phi(h, 2) = \frac{\varphi(2h)}{2} = \frac{1 - e^{-2h}}{2} = \Phi(h) \quad (11)$$

Transform (7) by using the new denominator function obtained in (11) we get

$$\frac{y_{k+1} - y_k}{\frac{1 - e^{-2h}}{2}} = y_k(1 - y_{k+1}y_k) \quad (12)$$

In the same procedure , for n = 3 equation(1) reduces to the following equation with four fixed points.

$$\frac{dy}{dt} = y(1 - y^3) \quad (13)$$

the fourth order polynomial

$f(y) = y(1 - y^3)$ has showed four fixed points given by

$$y_1{}^* = 0, y_2{}^* = 1, y_3{}^* = \frac{-1 + \sqrt{3i}}{2}, \; y_4{}^* = \frac{-1 - \sqrt{3i}}{2} \quad (14)$$

And also $R_1 = 1, R_2 = R_3 = R_4 = -3$. Therefore $R^* = 3$ and this is the suitable denominator function when n = 3 is:

$$\Phi(h, 3) = \frac{\varphi(3h)}{3} = \frac{1 - e^{-3h}}{3} = \Phi(h) \quad (15)$$

Transform (13) by using the new denominator function and replacing $\frac{dy(t)}{dt} \to \frac{y_{k+1} - y_k}{\Phi(h)}$ and $y^3 \to y_{k+1}y_k{}^2$ then

$$\frac{y_{k+1} - y_k}{\frac{1 - e^{-3h}}{3}} = y_k(1 - y_{k+1}y_k{}^2) \quad (16)$$

In general , for n > 0 , we use the following non standard discretization for the general nonlinear first order dynamic differential equation(1).

$$\frac{y_{k+1} - y_k}{\frac{1 - e^{-nh}}{n}} = y_k(1 - y_{k+1}y_k{}^{n-1}) \quad (17)$$

The numerical solutions of equation (1) with non standard finite difference method when n = 2 and at different step lengths and also the analytical solution has been shown graphically in the following figures.

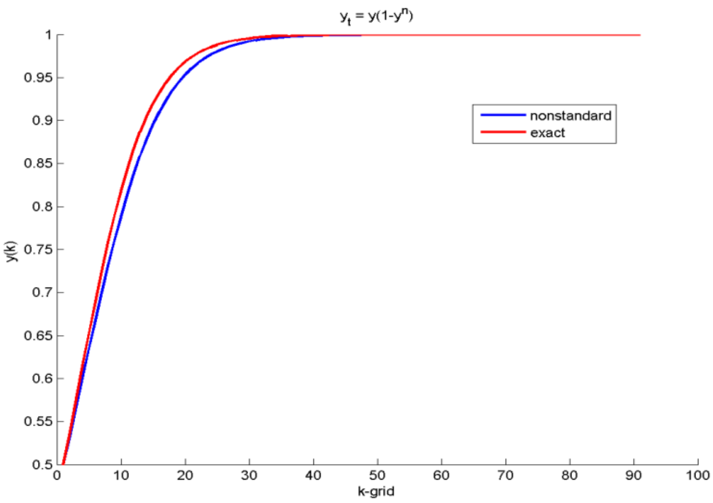

Figure:7. Approximation of nonlinear dynamic equation using non standard finite difference method when h = 0.1 and n = 2

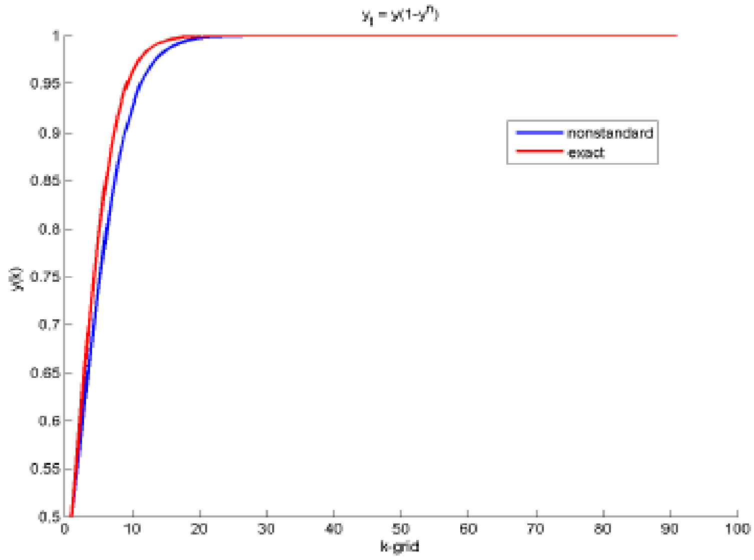

Figure:8. Approximation of nonlinear dynamic equation using non standard finite difference method when h = 0.2 and n = 2

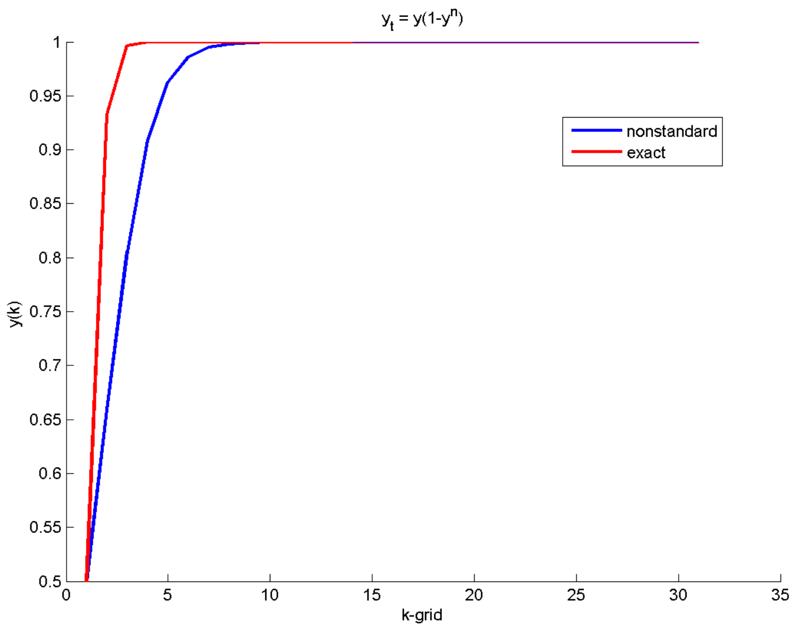

Figure:9. Approximation of nonlinear dynamic equation using non standard finite difference method when h = 1.5 and n = 2

Approximation of Non linear first order dynamic equation with Euler forward and Non standard finite difference Methods and Comparison of the solution of the two Schemes

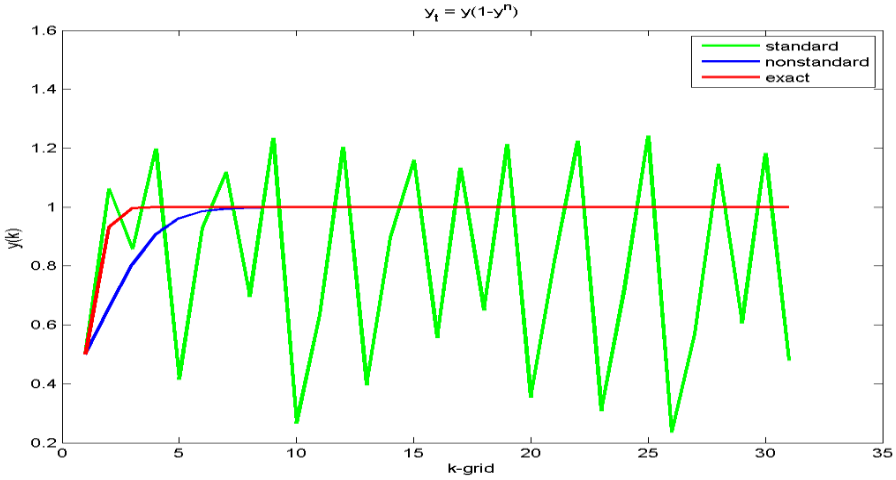

Figure:10 Comparison between the approximate solution of nonlinear dynamic differential equation using forward Euler and non standard finite difference methods when h = 1.5 and n = 2

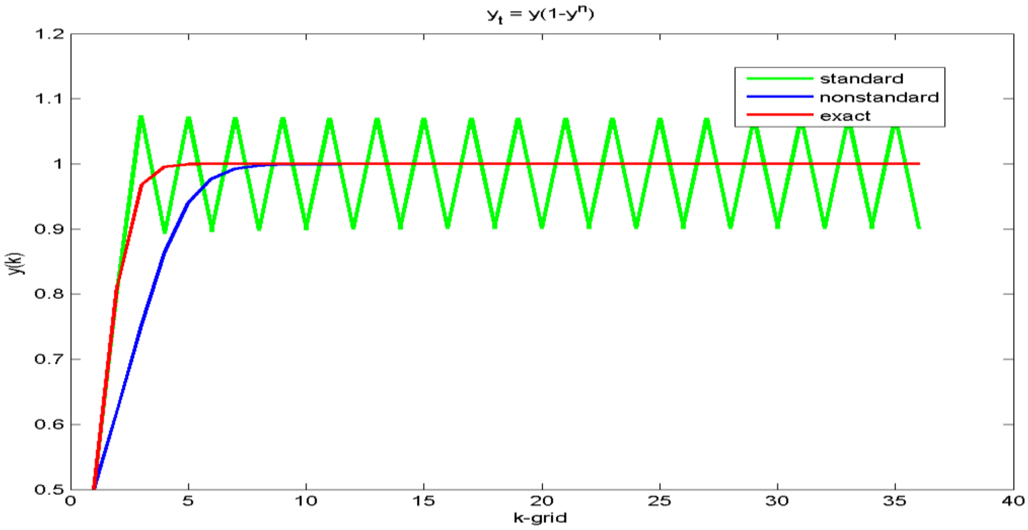

Figure:11 Comparison between the approximate solution of nonlinear dynamic differential equation using forward Euler and non standard finite difference methods when h = 0.7 and n = 3

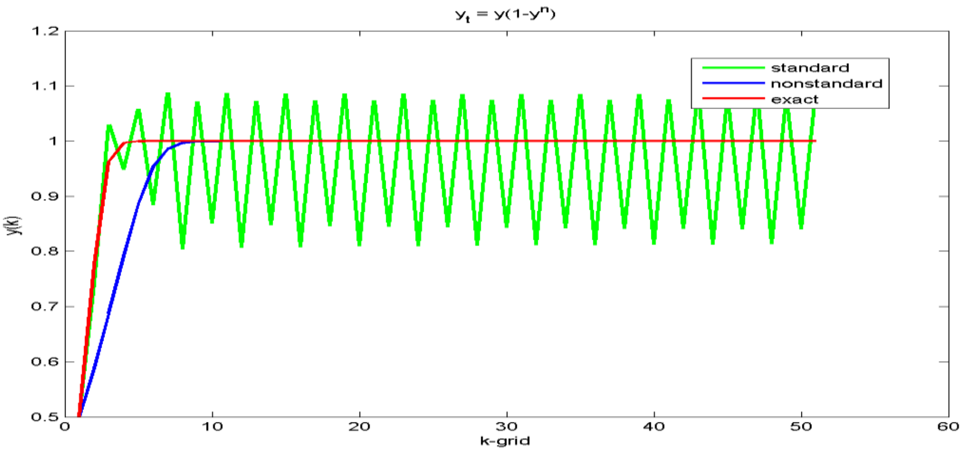

Figure:12 Comparison between the approximate solution of nonlinear dynamic differential equation using forward Euler and non standard finite difference methods when h = 0.5 and n = 5

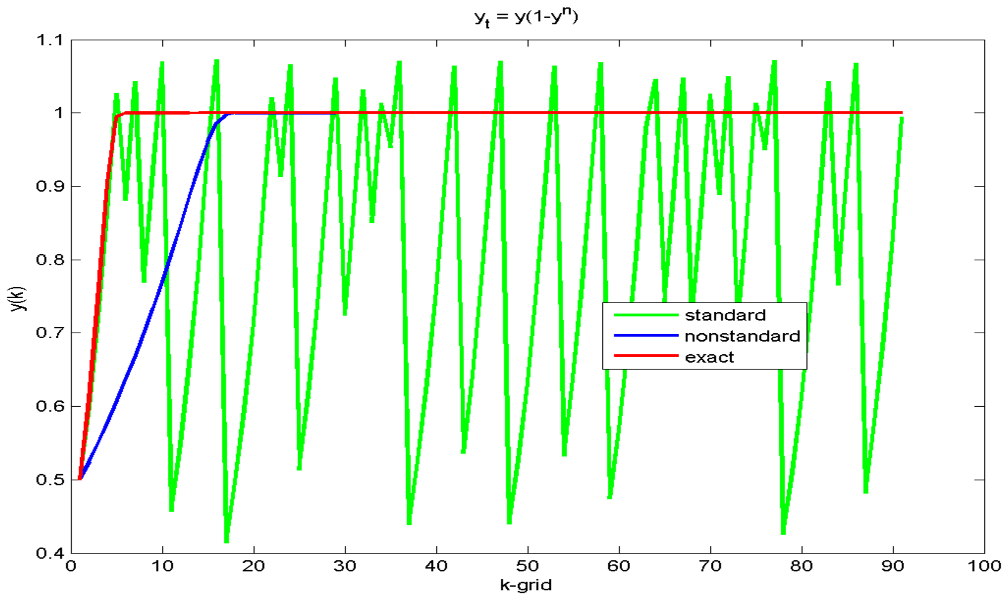

Figure:13 Comparison between the approximate solution of nonlinear dynamic differential equation using forward Euler and non standard finite difference methods when h = 0.2 and n = 20

From the comparison of standard and non standard finite difference schemes in solving non linear dynamic equation it observed that even if on using small step size (h<1) , if the non linearity of the dynamic equation increases , the standard finite difference scheme exhibits numerical instability.

II. Conclusion

The present investigation accentuates the utilization of non-standard finite difference methods to various values of n of nonlinear dynamic equations. It is considered that nonlinear dynamic equation and found the numerical solution by using nonstandard finite difference method by applying Mickens rules. For correlation also it arise similar issues and obtained the solution by using Euler Forward scheme and central difference scheme. From the figures it shown that the Euler Forward scheme is relatively convergent and more stable than the central difference scheme, so for better results it is compared the solutions of nonstandard finite difference method with the solution of Euler Forward at different step lengths by interchanging the nonlinearity of the nonlinear dynamic equation from small to big.. In spite of the fact that for h < 1, the two strategies are in a decent understanding, and for h >1, standard finite difference method exhibits the numerical instability for all n. Furthermore, when n is increased, nonstandard discrete models don't display numerical instabilities for all h. As appeared numerical simulations, the non-standard finite difference framework reproduces the dynamical features of the continuous time analogous. In particular, it is proved that the method guarantees the correct asymptotic behavior regardless from the size of the time step and the linearity of the nonlinear dynamic equation.

It was appeared, with a similar starting conditions, the standard finite difference methods prompts scheme dependent numerical instabilities for step length greater than a critical values, which manifest as jumps in the population evolution in time domain and convergence toward wrong steady state solutions. Then again, in tests that were performed, the nonstandard finite difference scheme demonstrates non of these issues, affirming the proficiency of the technique. The proposed technique is exceptionally focused. It is qualitatively stable, that is, it produces results which are dynamically consistent with those of the continuous system. These methods preserve the positivity of solutions and the stability properties of the equilibria for arbitrary step-sizes, while the solutions obtained by other numerical methods experience difficulties in either preserving the positivity of the solutions or in converging to the correct equilibria. Moreover, since huge advance sizes can be utilized, these techniques spare the calculation time and memory.

References

[1]. Michens R.E, Advances in the applications of nonstandard finite difference schemes /edited by Ronald E.Mickens, 2005.
[2]. R. E.Mickens, A nonstandard finite-difference scheme for the Lotka-Volterra system. Applied Numerical Mathematics,2003, 45, pp. 309 – 314.
[3]. Z. Lu and W. Wang. Permanence and global attractivity for Lotka-volterra difference systems. J.Math. biol., 1999, 39, pp. 269 – 282.
[4]. L. J. S. Allen. Some discrete-time SI, SIR, and SIS epidemic models. Mathematical Biosciences, 1994, 124, pp. 83 -105.

[5]. Ronald E. Mickens and Talitha M. Washington, A note on an nonstandard finite difference scheme for mathematical model of respiratory virus transmission, arxiv:1008.2314V1.[q.bio.PE].13Aug2010

[6]. Gianlunca Gabbrielline, K.C. , Non standard finite difference scheme for mutualistic interaction description, January 5,2015.

[7]. Ibijola, E., Ogunrinde, R. and Ade-Ibijola, On the theory and applications of new nonstandard finite difference methods for the solution of initial value problems in ordinary differential equations, Advances in Natural and Applied Sciences,2008, 2(3), pp. 214-224.

Getahun Tadesse Haile" Application of Non Standard Finite Difference Scheme on the General Non Linear First Order Dynamic Equation" IOSR Journal of Mathematics (IOSR-JM) 15.4 (2019): 01-09.